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CONTRIBUTIONS

FROM THE

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VOLUME V

NOS. 83–105

WITH NINE PLATES

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ON THE CHANGE OF SPECTRUM AND COLOR INDEX WITH DISTANCE AND ABSOLUTE BRIGHTNESS. PRESENT STATE OF THE QUESTION

By J. C. KAPTEYN[1]

In what follows I have brought together whatever evidence, published or unpublished, has come to my knowledge on the question indicated in the title of this note.

The evidence bears on the real existence of the two following observed phenomena:

I. OBSERVED PHENOMENA

(1) On the average the apparently fainter stars are redder than the brighter ones.

(2) Apparent magnitude and spectral lines being the same, the stars are redder the farther away they are.

II. POSSIBLE EXPLANATIONS

The phenomenon (1)—if real—is explainable in one of the three following ways:

(3) By the predominance of the later spectral types among the fainter stars.

(4) By an influence of the absolute brightness on the color index.

(5) By selective absorption (or scattering) of light in space.

For the phenomenon (2)—if real—there are only the two explanations (4) and (5).

III. INVESTIGATIONS REQUIRED

The three following investigations are therefore necessary to settle the reality of, and to determine quantitatively, the phenomena in question:

(6) Investigation of the *relative frequency of the several spectral classes* among the stars of the fainter magnitudes.

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(7) Investigation of the influence of *absolute brightness* on the spectrum.

(8) Investigation of the influence of *distance* on the spectrum.

IV. REMARK UPON (3)

It is important to note that a predominance of later-type stars among the fainter stars does not necessarily mean such a predominance among the more distant stars. If the spectral types are equally mixed throughout the stellar system, and if there is no space absorption nor any influence of absolute brightness, we shall find relatively more later types among the fainter stars. As a consequence, the fainter stars will be redder on the average than the brighter ones.

In order to see this at once,[1] imagine for a moment that the stellar system is sharply limited at a certain distance from the sun, and suppose, for the sake of simplicity, that we have to do only with stars of the first and second spectral classes, and that the proportion of the two classes does not change with distance. As the first-type stars are much more luminous, and as they are at the same time more nearly equal, the very faintest second-type stars must be much fainter than the least luminous first-type stars. Let the difference be a magnitudes. The very faintest stars that we see in the sky will therefore be the least luminous second-type stars which stand at the limit of the system. Let their apparent magnitude be μ. The stars of apparent magnitude μ will thus be exclusively second-type stars. So also will be the stars of magnitude $\mu-1$, $\mu-2$ . . . . up to $\mu-a$. At this last magnitude we begin to find some of the least luminous of the first-type stars and from this magnitude upward the number of first types, relatively to that of the second, will steadily increase. Evidently there is a predominance of second-type stars for the fainter magnitudes, without any thinning-out of the first-type stars with distance.

If we do not assume a sharp limit for the stellar system the same will still hold, at least if, as is found by all investigators,[2] the star-density diminishes with increasing distance. In order to show this and at the same time to obtain at least some quantitative idea, I computed the relative numbers of the first- and second-type stars for different magnitudes on the suppositions: (*a*) that for the fifth magnitude the number of stars for the two types is the same; (*b*) that for the first type the color index is zero, and for the second type +1.0 mag. I adopted the luminosity-curves found in *Groningen Publication*, No. 11, p. 31 (Sol. B). They are well represented by the formula:

---

[1] My attention was drawn to this consideration by Professor Hertzsprung.

[2] On the supposition, though, of the non-existence of appreciable space absorption.

$$\text{Number of stars} = Ce^{-h^2(M-k)^2} \tag{9}$$

in which M = absolute magnitude and

$$\text{for the first type,}\quad h=0.243 \qquad k=\ 8.95\ \text{mags.} \tag{10}$$

$$\text{for the second type,}\quad h=0.247 \qquad k=10.30\ \text{mags.}^{1} \tag{11}$$

The computation was further carried out by means of the very convenient formulae given by Schwarzschild.[2] I found the results given in Table I.

---

[1] An attempt is now being made by Dr. Kohlschütter and myself to obtain the luminosity-curves for each of the spectral classes, B, A, F, G, K, M, separately. As these are not yet available I had no choice but to adopt the curves of *Groningen Publication*, No. 11, though well aware of the fact that they are less satisfactory than what could be obtained by the use of more recent data. It seems probable (as implicitly assumed a moment ago) that with these better data we should find the value of h for the first-type stars very appreciably greater than for those of the second type.

[2] *Astronomische Nachrichten*, No. 4557. According to Schwarzschild:

$$\log \text{number of stars} = \text{const.} - \alpha_1 m - \alpha_2 m^2 \tag{P}$$

where

$$\alpha_1 = \frac{a_2(b_1+0.4) - b_2(a_1+0.6)}{a_2+b_2} \qquad \alpha_2 = \frac{a_2 b_2}{a_2+b_2}\,.$$

The values of b_1 and b_2 must be found from the luminosity-curves. We have

$$\frac{b_1+0.4}{2b_2} = k \qquad b_2 = h^2 \times \text{Mod.}$$

It is necessary in formulae (10) and (11) to diminish the values of k by 5 magnitudes in order to comply with Schwarzschild's definition of absolute magnitude, which differs by 5 mags. from that adopted in the *Groningen Publications*. The quantities a_1 and a_2 depend on the star-density according to the formula:

$$\log D\ (r) = \text{const.} + 5a_1 \log r - 25a_2 (\log r)^2$$

The first constant, only, is supposed to differ for the two types. For the other constants Schwarzschild finds:

$$a_1 = +0.097 \qquad a_2 = +0.0088$$

The constants being thus found and substituted in (P), we get the numbers whose ratios are in Table I.

The numbers in this table may be appreciably in error owing to the confessedly defective data on which it rests. At all events, however, it is evident that the apparent magnitude approaching infinity, the average color index will asymptotically approach 1.000, the maximum change thus being half a magnitude.

TABLE I

m	First Type / Second Type	Average Color Index
5	1.000	0.50
10	0.778	0.56
15	0.609	0.62
20	0.477	0.68

We thus see that the average color index of the faint stars is almost of necessity different from that of the brighter ones. In consequence of this it will be practically impossible to define a scale of magnitudes for the stars too faint for special classification in such a way that on the average the photographic and the visual scales will agree. Furthermore, unless observation yields a change in the relative numbers of the several spectral classes or in the amount of the color index materially exceeding that shown in Table I, we shall not be justified in concluding, from the observed phenomenon (1), that there is a relative increase in the later types with distance, nor shall we be able to conclude that either causes (4) or (5) are active.

Meanwhile the assumption that "the spectral types are equally mixed throughout the stellar system" is probably wrong. In *Groningen Publ.*, No. 11, Table 8, it was found that the density of the first-type stars does not decrease nearly so quickly with the distance as that of the second-type stars. This cause acts strongly in the opposite direction of the Hertzsprung effect. The phenomenon (2) is of course entirely independent of the latter effect.

V. IMPORTANCE OF THE INVESTIGATIONS (6)–(8)

Suppose that the investigation (6) leads to the result that the change in the relative frequency of the various spectral classes varies more than can be accounted for in the foregoing manner.

We shall then conclude that the later types are more frequent among the more distant stars. But if this is so, the luminosity-curve for *all the stars together* will change with distance. For the luminosity-curve of the mixture of stars of different spectral classes is of course dependent on the proportional numbers in each class. These proportional numbers changing with distance, the luminosity-curve of the mixture must necessarily change with distance. This means that our present theories about the arrangement of stars in space will need revision, for they all start from the supposition that the luminosity-curve is constant. Such a state of affairs would emphasize the necessity of an investigation of the arrangement in space *for each class of spectrum separately*. Independently of such a change, and for many obvious reasons, such a separate treatment is, in my opinion, one of the urgent desiderata of science. The investigation (6) will furnish one of the most indispensable data for such a treatment.

A well-determined influence of absolute brightness (7), besides being important in a study of the physics of stellar atmospheres, would furnish the means of determining parallaxes, especially for the distant stars, where the other methods break down.

An influence of absorption, if it exists, must introduce elements in the spectra of the distant stars which have nothing to do with the chemical and physical properties of the stellar atmospheres. For a fundamental study of the star-spectra the knowledge of any distance effect is therefore urgently required. Besides, as with the former influence, it will furnish means of getting data for very great stellar distances. In fact, the simple result of the direct observation (2), if once well established and investigated, will do this independently of the question to what extent it is to be explained by either of the causes (4) and (5). For the rest, the determination of a selective loss of light in space would be the first step toward the determination of the total loss of light. A good illustration of the absolutely fundamental importance of this latter quantity in the investigation of the structure of the stellar system is furnished by the fact that a loss of 0.18 magnitude for every unit of distance (32.6 light years), which has been assumed by Comstock, leads to a star-density, at the distance of 3,000 light years, hundreds of millions times greater than the supposition that no such loss exists.[1] It must be clear from this that, unless we succeed in determining the absorption constant within at least something like a thousandth part of a magnitude, no really definitive results can be obtained for the arrangement of stars in space.

VI. INFLUENCE THAT WOULD BE EXERTED BY SELECTIVE ABSORPTION OF LIGHT

Before adducing the evidence already existing, I will give a little table showing to what extent selective absorption of light by an interstellar medium (if homogeneous) would make itself felt, on the average, for stars of different apparent magnitudes. (The mean parallaxes have been assumed in accordance with what, in my opinion, are the best data at present available.)

TABLE II

Vis. Mag.	Change in Color Index	Vis. Mag.	Change in Color Index
6.0	0.04 Mag.	14.0	0.42 Mag.
8.0	0.07	16.0	0.75
10.0	0.13	18.0	1.30
12.0	0.24	20.0	2.40

These numbers are in accordance with the "absorption coefficient" derived in *Astrophysical Journal*, **30**, 195, 1909,[2] where *c* was taken to be vanishing. For other values of the coefficient our numbers will have only to be multiplied by a constant factor. The table shows well how an absorption, even when small and hardly appreciable for the stars below magnitude 9, may still be all-important for the very faint stars—how, therefore, its accurate determination must almost of necessity be made dependent on these latter stars. For the dependence on absolute brightness we cannot make such a table because of our ignorance as to the law which would express such a dependence.

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[1] *Astronomical Journal*, No. 566.

[2] *Mt. Wilson Contr.*, No. 42.

VII. EVIDENCE EXISTING ON THE PHENOMENON (1): THE FAINTER STARS ARE REDDER ON THE AVERAGE THAN THE BRIGHTER

a) *Fath's result.*[1]

> The result of the comparison of 76 pairs of stars on the five plates showed that, in the mean, the fainter stars are distinctly redder than the brighter ones.

b) *King's result.*[2] King finds

$$d=+0.019\pm0.010 \text{ mag.} \tag{12}$$

d representing the change of color index over unit of distance (32.6 light years). His conclusion is:

> From the correspondence and agreement of the results as viewed from different standpoints, I believe that we are dealing here with an actual factor, which, if not real absorption, produces effects similar to absorption.[3]

There is a more recent paper of King's giving

$$d=+0.023 \tag{13}$$

I quote the result from *Observatory*, February 1914, because I have not as yet seen the original paper.

c) *Turner's result.*[4]—From various considerations the author finds:

$$d=+0.030 \tag{14}$$

d) *Four clusters.*[5]—The globular clusters offer a case promising a separation of the two causes (4) and (5). For on small-scale plates, or on larger-scale plates if we operate out of focus, we can compare the color index of the cluster with that of a star of equal average spectrum and magnitude, without introducing perturbing photographic effects. At the same time, if the star has a sensible proper motion, we may be sure that the difference in distance is very considerable. As there is little reason for assuming that the separate stars of the cluster are, on the average, of a luminosity much different from that of the comparison star, a possible effect of the absolute magnitude is avoided. The results are as follows:

---

[1] *Ibid.*, No. 63, p. 5; *Astrophysical Journal*, **36**, 366, 1912.

[2] *Annals Harvard College Observatory*, **59**, 182, 1912.

[3] *Ibid.*, p. 183, 1908.

[4] *Monthly Notices*, **69**, 61–71, 1908.

[5] *Mt. Wilson Contr.*, No. 42, p. 33; *Astrophysical Journal*, **30**, 316, 1909.

N.G.C. 7076	Color index $=S+1.0$ mag.
N.G.C. 7089	$S+0.45$
Nucleus Great Andromeda Nebula	$S+1.0$
Hercules cluster	$S+0.9$

where $S=$color index of the comparison star, which is of the same magnitude and the same average spectrum (according to Fath) as the cluster. For the Hercules cluster I assumed the average spectrum to be F1, which is the average of the spectra recently found by Pease for 19 stars in the cluster.[1]

These determinations are rather rough, and the adopted average spectra of the clusters perhaps not unobjectionable. Still, as the differences are so large and the clusters almost certainly very far away, the qualitative evidence seems pretty strong.

e) Limiting photographic magnitudes as determined by Professor Pickering.—In *Harvard Circular*, No. 170, Professor Pickering gives for several instruments the magnitudes of the faintest stars obtained on the present rapid plates by an exposure of 60 minutes. I have myself derived the visual magnitudes of these faintest stars by the aid of the data in *Groningen Publication*, No. 18. In comparing the results we have to take into account the difference of the scales. Pickering's photographic magnitudes are on the International scale, the visual magnitudes on the Harvard visual scale. The International scale of photographic magnitudes is identical with the Harvard visual scale for the stars of spectrum A0, magnitude 6.0. For the stars of other spectra there is a difference. Let

$$\Delta=\text{average difference: }\left.\begin{array}{r}\text{Phot. Mag. (Int. scale)}\\ \textit{minus}\text{ Visual Mag. (Harv. scale)}\end{array}\right\}\qquad(15)$$

For the stars of visual magnitude 6 to 7 (Harv. scale) I found from Parkhurst's *Yerkes Actinometry*[2]

$$\Delta=+0.66\text{ mag.}\qquad(16)$$

---

[1] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, No. 13, p. 219, 1913.

[2] *Astrophysical Journal*, **36**, 169, 1912.

If, therefore, the fainter stars are as red as the brighter ones, we must find for them the same average difference. If they are redder, the difference must be greater. As a matter of fact I find the results given in Table III.

TABLE III

Faintest Stars on Rapid Plates, 60 Minutes Exposure

	Phot. (Int. Scale)	Vis. (Harv. Scale)	Phot.−Vis. −0.66
12-inch refractor	15.7	13.8	+1.24
16-inch refractor	16.5	14.9	+0.94
36-inch reflector	18.9	16.1	+2.14
60-inch reflector	19.5	17.4	+1.44
Means	17.65	15.55	+1.44

If the stars of these magnitudes were no redder than those of magnitude 6.5 (visual) the numbers in the last column ought to be *zero*. Still, they are probably not to be explained altogether by the increased color index of the fainter stars. Systematic errors in the scale of Pickering's photographic Polar Sequence and in the visual determinations of Parkhurst in his *Photometric Researches*—on which the determinations of *Groningen Publication*, No. 18, depend almost exclusively for the very faint stars—may have very materially contributed to the results. Seares has made a few direct comparisons of one of Parkhurst's fields with the visual Polar Sequence (considerably prolonged by Seares himself). These certainly indicate a very considerable divergence. But even if we allow for such divergences and if we take into account what has been said in Section IV, there must still remain a considerable part of the difference found between the color indices of the stars of magnitudes 15.5 and 6.5 which is to be explained either by a preponderance of the later types among the far-away stars or by one of the causes (4) or (5).

f) Limiting magnitude of the Franklin Adams plates.—Among the data of *Groningen Publication*, No. 18 (p. 11), occur fifteen Franklin Adams plates, which have been counted on an average down to visual magnitude 13.5, Harvard scale. The average limiting magnitude cannot well be fainter by more than half a magnitude, the plates being purposely counted close to the faintest

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magnitude shown. As, however, there are three plates on which the counts extended to 14.1, 14.3, 14.5, Harvard scale, I will adopt

$$\left.\begin{array}{r}\text{Vis. Mag. faintest}\\ \text{stars F. A. plates}\end{array}\right\} = 14.3 \text{ Harv. scale} = 14.96 \text{ Int. scale} \qquad (17)$$

According to a private letter from Professor Dyson, direct comparison at Greenwich of 17 Franklin Adams regions with Pickering's Polar Sequence gave in the mean:

$$\text{Phot. Mag. faintest stars on F. A. plates} = 16.51 \qquad (18)$$

The difference noted for visual magnitude 14.3 (Harvard) is +1.55 magnitudes. The possible explanations are the same as in the preceding case.

g) *Barnard's estimates* (unpublished).—In a letter to Seares, Barnard gives some estimates made by himself at the 40-inch refractor. I have added (in brackets) the photographic magnitudes given by Pickering in *Harvard Circular*, No. 170:

> The faintest star I could see on an ordinary night was 19s [18.30]. It could have been measured if necessary. I am sure I glimpsed 21s [18.67]. . . . . 17s·[16.97] was very noticeable. My estimate of 17s was 14½ or 15. . . . . 34 [17.11] is very easy. Estimated 15^{m}. I hope to get a trial on a good night.

Parkhurst's determination of the limiting visual magnitude of the 40-inch refractor, made with "good seeing," ocular 750, is 16.8 Harvard scale.[1] It is in connection with this determination that we have to take Barnard's estimates. The possibilities are as under *e*) and *f*).

h) *Hertzsprung's results* (*unpublished*).—In the summer of 1912 Professor Hertzsprung took a great number of photographs with the Mount Wilson 60-inch reflector, stopped down to 40 inches in order to increase the field. A coarse grating was placed before the opening of the tube, which of course produced spectra on both sides of the main image. The distance between the first-order spectra, which are so short that they can hardly, if at all, be distinguished from ordinary star images, is about 1 millimeter. This distance must be different for different "effective wave-lengths."

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[1] *Photometric Researches*, p. 189.

Careful measurement of this distance thus furnishes a measure of the color index. Up to the present only the photographs of a single region, that around N.G.C. 1647, have been completely measured and reduced. The total number of stars is 206. The color index increases very gradually and steadily with the magnitude. From magnitude 9 to about 14.5 the increase is about 0.7 or 0.8 magnitude. Both the stars in and outside the cluster show the phenomenon.

i) Seares's result.[1]—Seares gives results for two areas, one covering the region of Pickering's Polar Sequence, the other the region round S Cygni. For the stars of these regions both the photographic and the "photovisual" magnitudes (obtained by the use of isochromatic plates and yellow filter) were carefully determined. The number of stars included in the first area is 107, in the second about 200. The color index is found for both to increase very gradually with increasing magnitude.

> The real change in the minimum index (that is, of the color index for the whitest stars) between the 6th and the 17th magnitudes, is probably about one magnitude. Beyond the 15th magnitude there appear no stars with indices less than +0.5 mag.

The change of the average color index is 0.34 magnitudes.

j) Harvard result.—The Harvard results for the Polar Sequence give no systematic difference between the photographic and visual magnitudes of the white stars from magnitude 4.5 to 13.3. It is the only case I know in which the fainter stars were not found redder.

Meanwhile it seems quite possible that this is simply the consequence of the method of reduction adopted. A full explanation has not yet been published, but the following words occur in *Harvard Circular*, No. 170:

> An absolute scale of magnitudes was derived separately from each of about 60 plates taken by the above method, *the starting-point in every case being the mean photometric magnitude of such stars in the Polar Sequence, given in Table I, as were measured on that plate.*

---

[1] *Mt. Wilson Contr.*, No. 81. Read at the Astronomical and Astrophysical Society meeting at Atlanta, December 1913.

The words that I have here italicized seem to indicate that the agreement of the photographic and the photometric (i.e., visual) scales is a forced one. If this is really the meaning of the words, then of course the Harvard results yield no data for the present inquiry.

VIII. EVIDENCE ALREADY EXISTING ON PHENOMENON (2)

a) *Kapteyn's first paper on absorption.*[1]—Miss Maury divides her spectral class XV*a* (=K) into two subdivisions.[2]

> In the first the general absorption is slight; in the second, it is more conspicuous both in the regions of the violet above mentioned and beyond wavelengths 3889, where the photographic spectrum generally appears to be suddenly cut off.

It is found that with this difference in the spectrum corresponds a difference in distance, which is manifest by the difference in proper motion (μ):

	Average 100 μ	Percentage of Stars with 100 $\mu > 30''$	
α Cassiopeia stars (weak in violet)	11″.4 (45 stars)	0	(19)
α Boötis stars (strong in violet)	47.1 (25 stars)	48	

b) *Kohlschütter's result* (*unpublished*).—In his classification of the Mount Wilson spectra Kohlschütter sometimes noted the fact that the violet is exceptionally faint. The nine stars thus marked are listed in Table IV.

TABLE IV

Boss No.	Mag.	Spectrum	μ	Aver. μ
2335	4.70	G	0″.007	0″.300
3039	6.17	K	0.018	0.100
3217	5.36	K	0.002	0.130
3336	5.67	G5	0.008	0.200
3542	5.71	G	0.009	0.230
4612	5.42	B	0.005	0.023
4613	6.02	B1	0.010	0.023
5003	5.04	B	0.004	0.023
5583	5.54	A	0.026	0.056

On consulting Boss's *Preliminary Catalogue* it was found that in every case the proper motion was much below the average proper motion of the spectral class to which the star belongs (see last two columns). In fact, in nearly all cases the proper motion is quite insensible.

---

[1] *Mt. Wilson Contr.*, No. 31; *Astrophysical Journal*, **29**, 46, 1909.

[2] *Annals Harvard College Observatory*, **28**, I, 39, 1897.

c) Kapteyn's second paper.[1]—In this paper were treated the color indices of the stars whose spectra were determined by Miss Maury and Miss Cannon,[2] and whose photographic magnitudes could be taken from the *Draper Catalogue*. The visual magnitudes were from the *Revised Harvard Photometry*. A first attempt was made to separate the influence of the two causes: absolute brightness and distance. The data, however, proved inadequate for a satisfactory discrimination. If, therefore, we try to find only the change of color index with distance, and if we call

$$\left.\begin{array}{c} d=\text{change of color index for a change of distance} \\ \text{of } 32.6 \text{ light years } (\pi=0\rlap{.}''1) \end{array}\right\} \qquad (20)$$

we get

$$d=+0.0031\pm0.0006 \text{ mag.}^3 \qquad (21)$$

If objection is taken to the distances assumed for the derivation in accordance with the table in *Groningen Publication*, No. 8, we may write the result in terms of proper motion. In this form at least the value (21), small as it is, seems well guaranteed within the limits set by the probable error. For it was derived from stars having the same apparent magnitude and the same spectral class and differing only in proper motion. It is hard to see how systematic error, either in the spectrum or in the magnitude, could creep in. Observers of spectrum and magnitude, who at the time of their observation were certainly not aware of the amount of the proper motion, could not have introduced such error, even had they wanted to do so. Here then is a case where, if ever, we may fully trust the verdict of the probable error, and this being the case, a value over five times its probable error deserves some confidence.

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[1] *Mt. Wilson Contr.*, No. 42; *Astrophysical Journal*, **30**, 284, 1909.

[2] *Annals Harvard College Observatory*, **28** and **56**.

[3] The correction indicated in *Astrophysical Journal*, **30**, 398, 1909, has been applied.

d) *Van Rhyn's results for Parkhurst's stars between* $\delta=+73^\circ$ *and* $\delta=+90^\circ$ (*unpublished*).—The data here are somewhat scanty, but of much better quality than those available in the preceding investigation. As yet Van Rhyn has used only those of Parkhurst's stars which are in Boss. By adding those for which good proper motions are available in Groombridge and Carrington the material will be increased in the ratio of 2 to 1.[1] The results thus far are shown in Table V.

TABLE V

P.M.	Average P.M.	Average Mag.	Sp.	Average Color Index	No. Stars	Average π
<0″.050.....	0″.026	6.0	A3	+0ᵐ.12	33	0″.0070
>0.050.....	0.100	5.8	A3	+0.09	24	0.0187
<0.150.....	0.078	6.2	F3	+0.46	10	0.0150
>0.150.....	0.263	5.7	F3	+0.36	8	0.0373
<0.100.....	0.047	5.8	G3	+0.96	20	0.0108
>0.100.....	0.218	5.5	G3	+0.89	10	0.0335
<0.068.....	0.034	5.6	K4	+1.56	21	0.0088
>0.068.....	0.103	5.1	K4	+1.39	6	0.0204

The values of π were found from the magnitudes and proper motions by the table for "all stars" in *Groningen Publication*, No. 8. From these data I find, assuming that the factor required for passing from the average parallaxes to average distances is the same as in *Astrophysical Journal*, **30**, p. 398:

$$d=+0.0050\pm0.0019\text{[2]} \qquad (22)$$

The result is decidedly confirmatory of (21). Combining the two we have:

$$d=+0.0033\pm0.0005^5 \qquad (23)$$

the probable error now being only a sixth of the amount.

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[1] As this paper is going through the press, Van Rhyn communicates, as the provisional result of both Boss and Groombridge stars,

$$d=+0.0079\pm0.0023$$

[2] A more refined solution will lower this probable error. For it has been shown in my second paper that the values yielded by the spectra B to F must be increased, those found by the spectra G to M diminished. It is evident by simple inspection of Table V that a correction in this sense must greatly increase the agreement of the four separate results.

e) The results of *a* and *b* rest on spectra taken on different plates, at different times, which were separately developed. It is well known that under these circumstances photographs of the same star will often show great differences in the general absorption in the violet.

In a refined investigation it is necessary to compare stars of nearly the same magnitude, taken at the same zenith-distance, on the same plate. Some work in this direction was done by Van Rhyn at Mount Wilson. He photographed on one plate the spectra of two stars, of which the one has a large, the other a small, proper motion. Of the six pairs taken, three show the small proper motion star to be decidedly weak in the violet. The pairs are: Boss 3524 and 2750 (Sp. F8.); Boss 4042 and 4228 (Sp. G0); Boss 3922 and 4032 (Sp. K0).

f) *Adams' work.*—The matter was taken up very thoroughly a few months ago by Adams. He compared the spectra of stars of great and small proper motions in the way just described. Of the results it is stated:[1]

> Out of 20 pairs of stars investigated, two pairs are of type B8, one A0, one F4, one F7, two G5, two G6, one G8, seven K0, one K2, one K4, and one K6. Of these, the pairs of stars of types B8, A0, and F4 show no appreciable relative difference between the two ends of the spectrum, and the same is true of one pair of type G6 and one of type K0. The remaining fourteen pairs all show a marked difference, which in some cases is very great. In every case the star which is relatively faint in the violet end of the spectrum is the star of small proper motion . . . . in no case is the more distant star relatively stronger in the violet portion of the spectrum.

The phenomenon is demonstrated *ad oculos* by the reproduction of the spectra of five pairs of the observed stars.

Afterward Adams made a comparison—as yet unpublished—at the two ends of the spectrum of those stars of large and small proper motion, which have been observed for radial velocity on Mount Wilson. Photographs taken at great zenith-distances and on hazy nights have been rejected. The results are as contained in the following table, which gives the differences: density for stars of large proper motion *minus* density for stars of small proper motion at the wave-lengths indicated, the density at the red end of the spectrum having been reduced to equality for all.

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[1] *Mt. Wilson Contr.*, No. 78; *Astrophysical Journal*, **39**, 89, 1914.

TABLE VI

Type	Average Proper Motion		Difference in Density at		
			λ 4195	λ 4220	λ 4265
F0–F9	0″66 (23 stars)	0″012 (10 stars)	0.07	0.07	0.08
G0–G4 ...	0.64 (30 ")	0.009 (8 ")	0.11	0.11	0.11
G5–G9 ...	0.64 (22 ")	0.011 (14 ")	0.17	0.16	0.14
K0–K4 ...	0.70 (20 ")	0.011 (24 ")	0.16	0.15	0.15
K5–K9 ...	2.18 (5 ")	0.012 (5 ")	0.13	0.12	0.11

Adams thinks that the progression in the differences of density for F0 to K4 is probably real.

IX. FINAL REMARKS

a) Though many of the investigations summarized in what precedes are only in their beginning, the conclusion seems already very strong that both the phenomena (1) and (2) are real. Only in the latter it is provisionally necessary to read "are redder *on the average*, the farther away they are," the words in italics being added to the original formulation.

b) The various methods have their own peculiar advantages and disadvantages. The most direct and at the same time the most sensitive method appears to be that followed by Adams. It has the disadvantage of being, provisionally at least, restricted to the brighter stars, that is, to those in which in general any distance effect must be, relatively speaking, little pronounced. Meanwhile we may hope to get down to much fainter stars by diminishing the dispersion and using an objective prism or an equivalent arrangement on the reflector.

With the method followed by Seares very faint stars are reached. The interpretation of the results, however, is complicated by a lack of knowledge of the spectra.

If the relative frequencies of the several spectral classes among the fainter stars were known, the two methods would help each other in discriminating between the two causes (4) and (5). For suppose we found that this frequency is the same for the stars of all magnitudes. Then from the two phenomena (1) and (2) we should conclude that absorption (scattering) must be the main cause and not absolute magnitude. To see this, suppose for an instant that absolute magnitude were the only factor. From the phenomenon (1) we should conclude that the absolutely *fainter* stars must be the redder. For it cannot well be doubted but that the apparently fainter stars are also, on the average, the less luminous. From the phenomenon (2), on the other hand, it would follow that the absolutely *brighter* stars must be the redder. As the two are contradictory, we conclude that the premise must be wrong. Absolute magnitude could not, therefore, be the only, or even the main, cause.

c) For this separation of the effects of distance and absolute magnitude little has been done up to the present. Adams is working on the problem;[1] his results are still inconclusive, owing partly to the short time as yet devoted to the matter, and partly to the difficulty of finding suitable objects in sufficient number.

d) May not another phenomenon exist, which, side by side with other advantages, will in many cases permit an immediate separation of the two causes? In *Contributions from the Mount Wilson Solar Observatory*, No. 31, p. 3,[2] the question was put whether there might not be a gaseous absorption. In a private letter Adams states that there seem to be indications of a strengthening of the hydrogen lines in the spectra of many of the distant stars of the later types. If further investigation confirms these indications there will be another case that may be conceived as an effect either of distance or of absolute magnitude. The two will, however, now be separable, at least in many cases. For any distance effect must be explained by the presence of hydrogen in interstellar space. Now, as generally the radial velocity of the gas and the star will be different, there will be an unsymmetrical widening of the hydrogen lines, which must give rise to a displacement of the centers of these lines, different from that of the purely stellar lines; conversely such a difference will prove the distance effect, that is, the absorption or scattering of light in space.

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[1] *Loc. cit.*

[2] *Astrophysical Journal*, **29**, 48, 1909.

In this summary of what has been done up to the present, I have deemed it unnecessary to enter into details of investigations already published. For the still unpublished data somewhat more detail was obviously desirable. It must be evident how greatly the latter have contributed in establishing the reality of the phenomena under consideration. I feel the deepest obligation to the astronomers who have helped me in my task by permitting me to use their results.

GRONINGEN
February 1914

Addendum to KAPTEYN, "On the Individual Parallaxes of the Brighter Galactic Helium Stars, ..." *Mt. Wilson Contr.*, No. 82. After line 6, p. 37, insert:

For 7 of the remaining stars in List 3 ($100\,\mu \leqq 1''6$), viz., Boss 2187, 2342, 2217, 2267, 2408, 2575, 2880,[1] for which the divergence $p_o - p_c$ exceeds 50°, the probability is very great that the excessive values are caused in great part by observation error. It is for this reason that, before computing the parallax, I diminished the value of $p_o - p_c$ by one-third of its amount. This certainly is a somewhat arbitrary way of dealing with the matter, but it is to be remarked:

a) That the adopted change in the position angle is, in all cases but one, smaller than its probable error.

b) That, most likely, the procedure must bring us nearer to the truth.

c) That the value of the parallaxes of the stars in question—whether we take the corrected or the uncorrected values—is next to nothing, as the probable errors range from 0.48 to 1.06 times the whole amount of the parallax.

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[1] For Boss 2880 instead of $p_o - p_c = -40°$, read $-60°$.

Contributions from the Mount Wilson Solar Observatory, No. 84
Preprinted from the *Astrophysical Journal*, Vol. XL, 1914

A VERTICAL ADAPTATION OF THE ROWLAND MOUNTING FOR A CONCAVE GRATING

By ARTHUR S. KING

A concave-grating spectrograph recently mounted in the Pasadena laboratory of the Mount Wilson Solar Observatory embodies some new structural features which will be described in this paper. It provides for a grating of 15 ft. (4.57 m) radius, and, optically considered, is the Rowland mounting with the plane of the focal circle vertical. This permits placing the grating in a pit beneath the laboratory floor, thereby obtaining the constancy of temperature which has proved highly advantageous with plane-grating spectrographs, while the slit and the plate-holder are at a convenient height above the floor. The apparatus is so constructed that it may be operated in a fully lighted room and occupies a minimum of floor space for an instrument of this kind.

The portion of the spectrograph above the laboratory floor is shown in Plate I. The plate-holder moves on a horizontal track, 50.5 cm above the floor, supported by a frame of channel iron placed over a slot in the cover of the pit used for the vertical Littrow spectrograph. When the plate-holder carriage is as near the slit as possible, the spectrum may be photographed as far as λ 2000 in the first order. When at the extreme end of the track, λ 7000 in the second order is reached, the third order being thus covered for the range to which ordinary plates are most sensitive.

The instrument is shown in elevation in Fig. 1. The frame carrying the two vertical rails is of angle-iron fastened firmly to the concrete wall of the pit. The grating-holder is a cast-iron box, with an extension at one side which is bolted to the web of the I-beam girder, 10 cm wide, connecting with the plate-holder carriage. This girder extends in a line with the side of the grating box for 185 cm, then offsets 6.5 cm inward in order that the web may align with the inside of the plate-holder carriage to which the upper end of the girder is fastened. Within the grating-holder is an iron plate 18.5×27 cm on which the grating rests, the latter

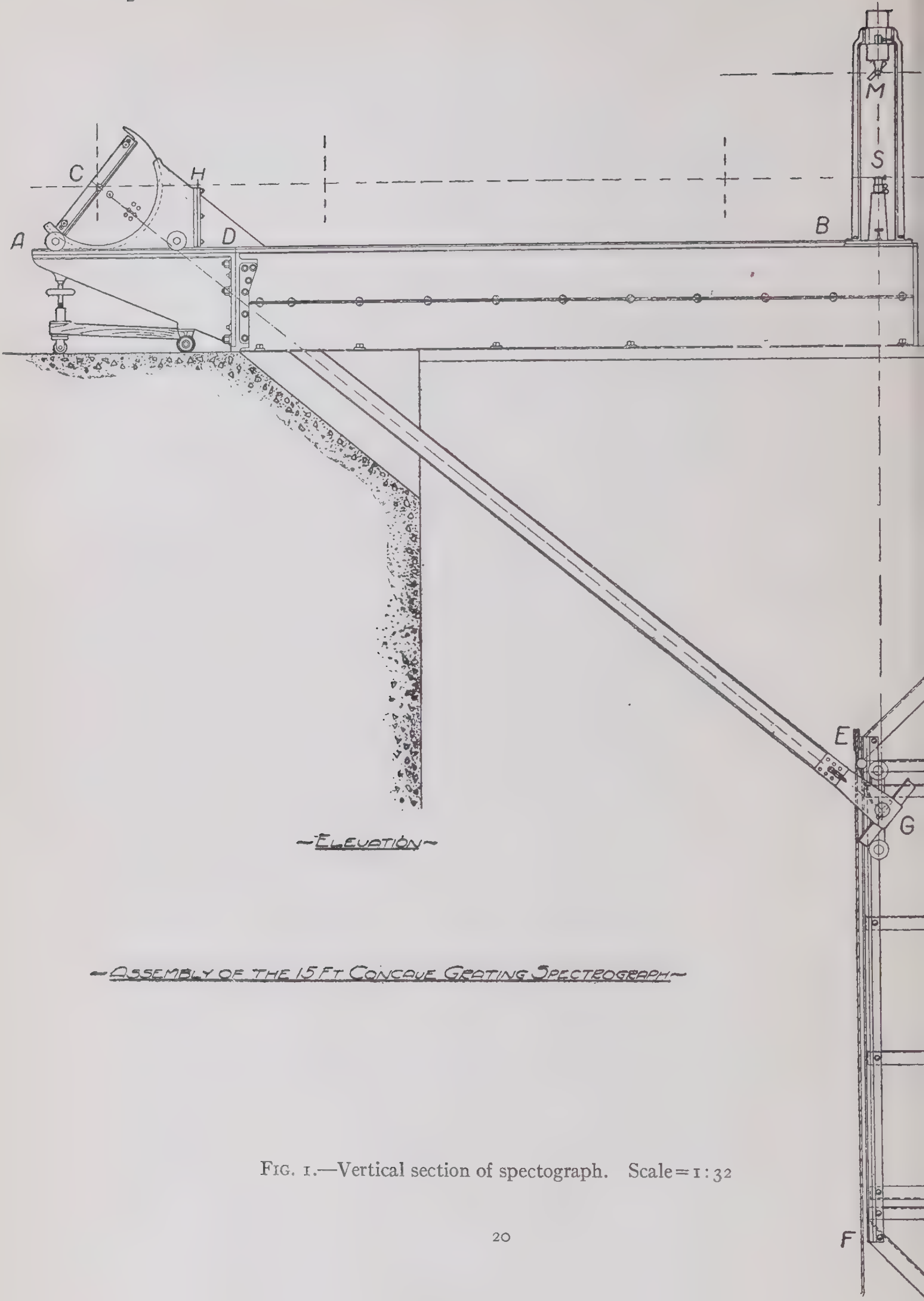

FIG. 1.—Vertical section of spectograph. Scale = 1 : 32

being held in place by four brass strips screwed to the plate. At each corner of the plate are push-and-pull screws, which serve to adjust the inclination. The grating is centered and adjusted for orientation by screws passing through the inclosing brass strips, their ends resting lightly against the edge of the grating.

In this mounting, a counterweight for the grating carriage is required, and must be of variable weight, since, as the plate-holder carriage is moved outward from the slit, the pull exerted in raising the grating becomes more nearly normal to the vertical track. Steel cables are fastened to each side of the grating box (see Fig. 1), pass at an angle to small idlers and thence over pulleys at *E* to the counterweight below *F*. The lower portion of the system is shown in Fig. 2. *W* is a mass of iron forming the constant weight, and below this is suspended a cylindrical tank *T*. From the bottom of *T*, a flexible metal tube passes to a fixed reservoir *R*, containing a quantity of heavy oil and suspended below a platform used for the adjustment of the spectrograph. The height of this reservoir is so adjusted that when the grating is on the lower portion of its track, where only the weight *W* is needed, *T* is so high that all of the oil is in *R*. When the grating rises to the point where a larger counterweight is required, the descent of *T* causes oil to run into it from *R*, the weight increasing in the proportion needed to facilitate the movement of the grating as it is pulled to the top of its track.

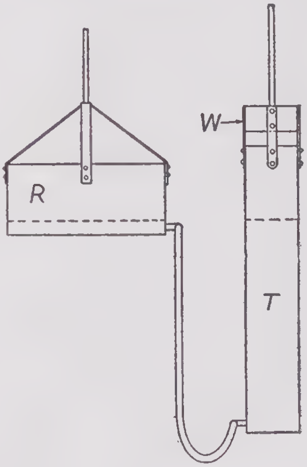

Fig. 2.—Variable counterweight system for grating-holder.

The slit is at the end of the horizontal track and vertically above the grating. A hollow iron casting has at its upper end a brass collar which in turn holds the slit-tube and permits the regulation of the height of the slit. A key-way prevents turning within the collar, the orientation of the slit with respect to the grating rulings being adjusted by turning the collar within the iron casting. This movement is controlled by a tangent screw, and the angle, marked by a scale and pointer, may be read to 0°05. The slit was made by Hilger. Its divided head reads to 0.005 mm and its length is adjustable to 18 mm by means of a wedge-shaped opening. As the light from a source in the laboratory must in general be reflected to the slit, a holder for a mirror or a total reflecting prism is mounted as shown in Fig. 1.

The plate-holder carriage (Plate I and Fig. 1) consists of a hollow iron casting moving on four grooved wheels. The sloping extension toward the slit, usually required in order that light may reach all parts of the plate, can be replaced by a vertical iron plate (at *H*, Fig. 1) when a close approach to the slit is desired. The opening between the tracks is rendered light-tight by overlapping boards and felt curtains attached to the ends of the carriage by means of spring rollers.

The side walls of the carriage are cut away along a circle as shown in Fig. 1, and a ledge at each side, 2.5 cm below the edge of this opening, supports a rotating section to which the girder is attached and on which the plate-holder lies. This is shown in section, with side and end elevations, in Fig. 3. The I-beam girder connecting the plate-holder with the grating carriage is attached to the inside of the rotating section by four bolts in slotted holes, and the distance from plate-holder to grating may be altered by means of the push-and-pull screws at *J*.

The plate-holder is carried on a sliding brass bed above an opening 4×52 cm in the top of the rotating section of the carriage and moved by means of the screw *F*. Ledges projecting from each side of the casting (see section at right of Fig. 3) carry a shutter used to limit the width of spectrum admitted to the plate. Hinged connections at the ends of the blades of the shutter (that at one end being shown in the top plan of Fig. 3) enable them to be brought into contact or separated to a distance of 3.3 cm by moving the handle *B* in or out. The whole shutter, with its blades at a fixed distance apart, may be moved across the spectrum by means of the screw *C*, so that in addition to regulating the length of spectrum lines in ordinary photographs, successive spectra for close comparison of wave-lengths may be taken by moving the shutter as a

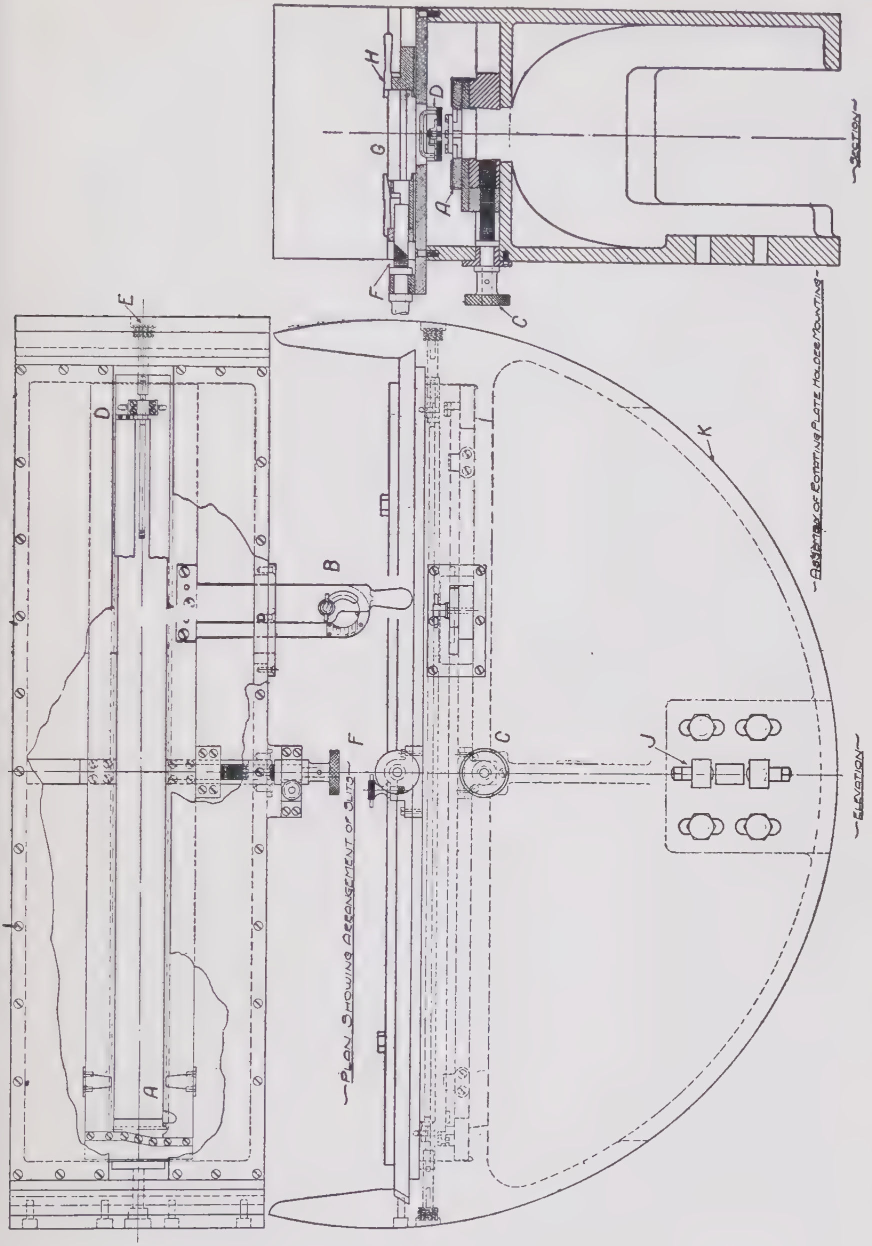
E
D
A
B
F
C
K
J
H
G
~PLAN SHOWING ARRANGEMENT OF SLITS~
~ASSEMBLY OF ROTATING PLATE HOLDER MOUNTING~
~SECTION~
~ELEVATION~

FIG. 3.—Sections of plate-holder carriage. Scale = 1 : 5

whole, or a narrow strip of spectrum may be superposed on any part of a wide strip.

A further attachment for photographing parts of a spectrum on each side of another spectrum without risk of instrumental shift consists in a brass plate which may be placed just above the shutter. A slot contains a rectangular brass frame which may be rotated through 90° about its longitudinal axis, admitting first a strip of spectrum in the middle and then one at each side. The rotation is accomplished by means of a key inserted at *E* through a hole in the rotating section of the carriage. If the hole at one end is covered owing to the position of the carriage on the track, a similar hole at the other end will be exposed. When the key is not in use, the hole is closed by a screw plug.

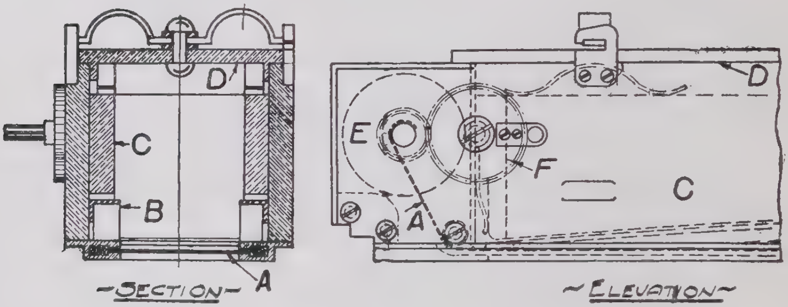

FIG. 4.—Design of holder for photographic plate. Scale = 1 : 2

The plate-holder is of brass and fits into the sliding bed already described. A ledge *B* (see Fig. 4), curved to a radius half that of the grating, supports a photographic plate 4.7×46 cm which is bent into position by the rectangular brass frame *C* held down by springs attached to the under side of the cover *D* when the latter is clamped in position. In case a celluloid film is used, the ends may be inserted under spring clamps at each end of *C* which hold the film stretched while *C* is put in position.

Instead of a sheet-metal slide in front of the plate, the movement of which would be interfered with by the construction of the carriage, a strip of opaque black cloth is used, whose edges run in deep grooves in the face-plate of the holder (*A*, Fig. 4). Tapes attached to the edges of the cloth connect with spools, *E*, at the ends of the plate-holder. A geared wheel *F*, attached to a pin rotated by hand, permits each spool to be turned rapidly in opening or closing the plate-holder.

The mounting retains all of the good features of the regular Rowland form. In addition there is a great gain in the excellent temperature control given by the pit. The grating and the connecting girder, the two parts most sensitive to temperature changes, are protected to an extent difficult of attainment in a laboratory room. During the two months following the completion of the instrument the temperature at the level of the grating changed but 1°5 C. Thermometers hung at the upper and lower ends of the grating's run have never shown a difference of more than 0°2 C. and often agree within less than 0°1. The rigidity of the apparatus has been tested several times by furnace exposures lasting three hours or more, and by one photograph of the iron arc spectrum which was divided over a seven-hour period, extending from morning till evening, the exposures being made at the beginning, middle, and end of the time. No difference in definition could be detected between this divided exposure and short exposures on adjacent portions of the plate taken at the beginning and end of the test.

As a minor advantage, this form of mounting requires but little floor-space, a narrow space against a wall, or a hallway, being sufficient. As no darkening of the room is required, the operation of the instrument need not interfere with other work being carried on in the laboratory.

The reflection of light to the slit from a source under examination, while usually convenient, is not always necessary. Thus a vacuum tube or an arc or spark used horizontally may be placed directly over the slit. When this is not feasible, the observer will select the reflecting surface giving the greatest efficiency for the region of spectrum under examination. Mirrors of silver and of speculum metal and right-angled prisms of glass and of quartz have been used thus far.

The grating used is one of exceptional quality, ruled by Dr. J. A. Anderson on the Rowland machine. The ruled surface is 5.1×10.6 cm, with 590 lines to the millimeter. While the light is largely in the first order on one side, the second and third orders are sufficiently bright to give good efficiency for the range allowed by the spectrograph.

For aid in planning the instrument, the writer is greatly indebted to Mr. Pease, who designed the main structural features, and to Mr. Nichols, of the draughting department, who introduced numerous devices which have added to the convenience of operation. The construction in the machine-shop and the mounting in the laboratory have been under the care of Mr. Ayers and Mr. Shumway, who have spared no pains to make the parts of the spectrograph accurate and easy of operation.

Mount Wilson Solar Observatory
January 26, 1914

Contributions from the Mount Wilson Solar Observatory, No. 85
Preprinted from the *Astrophysical Journal*, Vol. XL, 1914

SOME ELECTRIC FURNACE EXPERIMENTS ON THE EMISSION OF ENHANCED LINES IN A HYDROGEN ATMOSPHERE

By ARTHUR S. KING

The following experiments have been carried out in order to test the hypothesis that the presence of hydrogen may facilitate the emission of those lines which are intensified in the spectrum of the electric spark. That hydrogen may have such an influence was suggested by the experiments of Crew,[1] confirmed later by Hartmann,[2] on the arc in a hydrogen atmosphere, which has a distinct effect in strengthening the enhanced lines of metals. It is known, however, that the arc, with either metallic or carbon terminals, is maintained with much more difficulty in hydrogen than in air, the same length of arc requiring higher potentials.[3] In a later work, Crew[4] showed by graphical methods that a higher potential gradient was produced by the presence of hydrogen around the arc, which thus furnished an approach to the conditions of the spark discharge. Although the evidence has thus indicated from the first that the action of hydrogen was to alter the discharge conditions, occasional reference has been made, the latest being in a paper by S. A. Mitchell,[5] to the possibility that hydrogen in stellar atmospheres affects by its presence in some unknown manner the emission of enhanced lines by metallic vapors.

The arc is obviously unsuited to test this question, on account of the changes mentioned. The electric furnace, on the other hand, seems especially adapted, since the graphite tube, when raised to a given temperature, may be expected to perform its functions as an exciting source independently, to a large degree, of the surrounding

[1] *Astrophysical Journal*, **12**, 167, 1900.

[2] *Ibid.*, **17**, 273, 1903.

[3] See C. D. Child, *Electric Arcs*, p. 83.

[4] *Astrophysical Journal*, **20**, 274, 1904.

[5] *Ibid.*, **38**, 407, 1913.

gas. leaving the effects of mixed vapors, density, and total pressure to produce any effects that may be observed.

The experiments have included the production of the enhanced lines with the furnace in a partial vacuum and in hydrogen at varying pressures up to one atmosphere, and the use of greatly differing amounts of titanium. Thirty-three spectrograms were made, each of the important conditions being tested at least twice and in some cases several times.

EXPERIMENTAL METHOD

The tubes used in the electric furnace were of specially purified graphite, 12.5 mm inside diameter, 19 mm outside diameter, and 30.5 cm long, and were charged with pulverized titanium carbide. The heated portion of the tube was inclosed by a protecting tube of graphite of 3.2 cm internal diameter, which prevented a rapid loss of heat by the furnace tube. A potential of 25 volts with a current of about 1500 amperes brought the tube to a temperature close to 2600° C. in one minute, with little change in temperature thereafter, even when the hydrogen was at atmospheric pressure. A Gaede oil pump reduced the furnace chamber to a pressure of less than 1 mm. Observations were made for this pressure when the residue was air and also when the furnace had previously been filled with hydrogen. In both cases, a considerable quantity of occluded gas was driven off, and at low pressure the pump was not able to remove this fast enough, the pressure gradually rising during the run of the furnace to about 10 mm, which is the lowest final pressure used in the experiments. Various initial pressures of hydrogen up to one atmosphere were used, and, provided the pressure desired was as much as 10 mm, it was possible to secure constancy by frequently opening the connection with the pump. The hydrogen was prepared from zinc and sulphuric acid and passed through vessels containing sulphuric acid and caustic potash. Before a trial with hydrogen, the furnace was usually flushed three times with the gas, and again pumped out, after which hydrogen was admitted to the desired pressure.

The spectra were photographed with the second order of a concave grating of 15 ft. (4.5 m) radius, mounted in the vertical spectrograph.[1] The region covered was from λ 3900 to λ 4700 and all adjustments of the spectrograph remained the same throughout the series. Seed "23" plates were employed, giving sufficient contrast to bring out the faint enhanced lines in spite of the rather strong continuous ground given at the high temperature. The exposure times ranged from one to two minutes according to whether or not the tube and graphite protector were hot from a previous run.

The titanium enhanced lines brought out most distinctly by the furnace were λλ 4300.211, 4395.201, 4443.976, 4468.663, 4501.448, 4534.139. These are the strongest of the enhanced lines in this region and their behavior may be taken as typical of the group. Several plates were first taken with the furnace chamber pumped out, which confirmed the previous observations[2] as to the appearance of the enhanced lines in the high-temperature spectrum. The series of photographs with a hydrogen atmosphere was then begun with initial pressures at less than 1 mm and at 5 mm, the pressure rising during the run as has been noted. These were followed by plates for which the pressure was held constant at 10, 20, 40, 100, 200, 400 mm, and finally at atmospheric pressure. In case several pressures were used successively with the same tube, the chamber was pumped out and fresh hydrogen admitted between successive exposures. Tests with the same pressures in a different order were then made with a new tube. Pyrometer measurements were taken regularly and during the latter half of the exposure the readings under all conditions indicated a temperature differing not more than 50° from 2600° C.

RESULTS

1. *Effect of hydrogen at various pressures.*—No distinct effect on the intensity of the enhanced lines resulted from the use of hydrogen up to a pressure of 100 mm. At 200 mm, a weakening was perceptible, which became quite distinct at 400 mm, while five photographs in hydrogen at atmospheric pressure showed that the limiting pressure had been almost reached for enhanced lines

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[1] *Mount Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 1914.

[2] *Mount Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

to appear at 2600° C., only faint traces being seen of the two strongest lines in the group. The efficiency of the furnace in emitting the arc lines appeared to be practically unchanged by the hydrogen atmosphere. The same exposure times were used throughout, and the titanium arc lines came out strongly, even increasing in general intensity as the pressure of hydrogen rose, an effect due largely to a general widening of the arc lines at higher pressures and to more numerous reversals. The enhanced lines do not seem to be especially sensitive to changes in the amount of hydrogen present as long as the pressure is low, but increasing the pressure from 100 mm up to atmospheric pressure exerts a progressive effect in suppressing them.

2. *Effect of vapor-density.*—The influence on enhanced lines of the amount of metallic vapor present has frequently been considered in studies of the arc and spark; but in these sources a change in density of the vapor results in altering the character of the discharge. The conditions in the furnace should remain more nearly constant with varying amounts of vapor.

With this point in view, experiments were made with less than 0.01 gm of powdered titanium and with the regular charge of from 1.5 to 2 gm. The chamber was pumped out and held by the pump to a low pressure in each case.

As nearly as could be judged, the enhanced lines maintained the same intensity relatively to the arc lines whether a large or small quantity of titanium was used. Certainly there was no difference in favor of the smaller amount. The whole spectrum was stronger at higher vapor-density, with more continuous spectrum, but, allowing for this, the intensity of the enhanced lines appeared to be little influenced by changes in the amount of vapor.

DISCUSSION

The experiments indicate that neither the presence of hydrogen at low pressure nor the density of the radiating vapor materially affects the strength of enhanced lines, but that the total pressure is very important. The lines of the arc spectrum are not thus affected by the pressure, as they maintain their strength, with increasing ease of reversal, up to atmospheric pressure in hydrogen, and experiments[1] have shown that a rich spectrum of titanium arc lines is given by the furnace with pressures as high as 16 atmospheres. The evidence thus indicates that if the vapor is raised to a sufficiently high temperature in the furnace the arc lines will appear, while experiments with the furnace and with the tube arc and the ordinary arc at low pressure have shown that the enhanced lines are brought out best in a partial vacuum, a condition favorable to all of the phenomena of electro-luminescence.

The probable manner of the production of enhanced lines in stellar atmospheres appears much clearer since the discharge of electrons from hot bodies has been investigated for the relatively low temperatures of the electric furnace, this action having been shown by Harker and Kaye[2] to be strong at atmospheric pressure and by the writer[3] to persist at much higher pressures. The high stellar temperatures, especially when combined with low pressure, may thus be expected to duplicate the electronic speeds obtained in the arc and spark by steep potential gradients, a condition established by much evidence as favorable to the enhanced lines.

The relative strength of enhanced lines in the chromosphere would thus seem to follow from the rarefied condition at these levels of the solar atmosphere, which allows a high speed to be retained by the electrons expelled from the heated matter below and striking any vapor particles which may be present above. The conditions of rarefied vapor and presumably lower temperature are unfavorable for the arc lines, which are relatively weak in the chromosphere.

This point of view leaves the strength of enhanced lines as a valuable criterion for the temperatures prevailing in stars and in different regions of the solar photosphere at the levels where the electrified particles are produced. Thus the reduced strength of enhanced lines in sun-spot spectra seems valid as evidence of a lower temperature for those regions, since such a reduced temperature, if the pressure is not materially different, would result in the production of lower-speed electrons and in reducing the velocity

[1] *Mount Wilson Contr.*, No. 60; *Astrophysical Journal*, **35**, 183, 1912.

[2] *Proceedings of the Royal Society*, **86 A**, 379, 1912.

[3] *Mount Wilson Contr.*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

of those drawn in from the surrounding regions. The view involves no contradiction to the hypothesis advanced by Mr. Hale[1] that electrons are drawn in by solar vortices so as to be very numerous in sun-spots, as the production of enhanced lines appears to be governed by the speed and not by the number of the electrons.

SUMMARY

1. The experiments here described have failed to show any effect of a hydrogen atmosphere in strengthening enhanced lines. They appear in the furnace at low pressures with equal ease whether hydrogen is present or whether the furnace contains a residue of air.

2. Widely different amounts of titanium vapor at low pressure and the same temperature have shown no material effect on the relative intensities of enhanced lines.

3. Increasing the pressure of hydrogen, the temperature being held as nearly constant as possible, causes a progressive weakening of the titanium enhanced lines, until at atmospheric pressure only traces of the strongest are visible in the furnace spectrum.

Mount Wilson Solar Observatory
March 21, 1914

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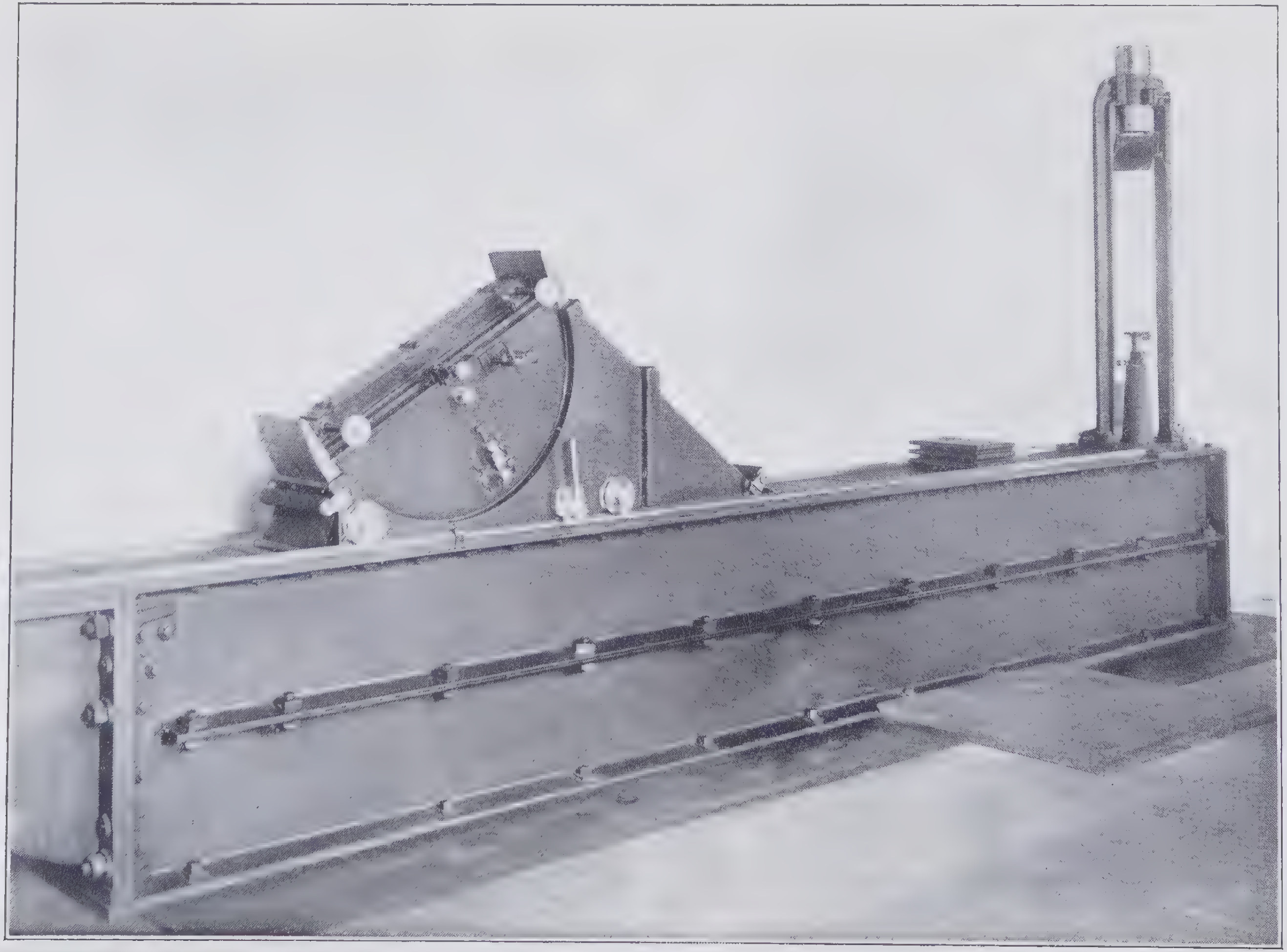
[1] *Mount Wilson Contr.*, No. 71; *Astrophysical Journal*, **38**, 27, 1913.

PLATE I



Contributions from the Mount Wilson Solar Observatory, No. 86
Preprinted from the *Astrophysical Journal*, Vol. XL, 1914

INTERMEDIATE DEGREES OF DARKENING AT THE LIMB OF STELLAR DISKS WITH AN APPLICATION TO THE ORBIT OF ALGOL[1]

By HARLOW SHAPLEY

Examples have been given[2] by the writer of the solution for the orbital elements of eclipsing binaries on the hypothesis that the stellar disks are darkened to zero at the limb according to the cosine law given in a later paragraph. Such a degree of darkening for light-emission in the visual part of the spectrum is somewhat in excess of that measured on the sun. If we assume an intermediate degree of darkening, for instance, one that would make the brightness at the limb one-third or two-thirds that of the center, it is not difficult to derive the orbital elements from a light-curve when the orbit has already been computed on the limiting assumptions of uniformly luminous and completely darkened disks. In fact, it will be shown that the interpolation of "intermediate" solutions will be sufficiently precise when uniform and darkened[3] elements are known.

The method of treating this problem has been outlined in a former paper.[4] In this note the details of numerical solutions are omitted and the results only are presented in order to show in what manner the various sets of elements differ from each other, and to what extent the orbit of a well-observed star is uncertain because of our lack of knowledge of the amount of atmospheric absorption on stellar surfaces.

I have undertaken for this investigation a discussion of the orbit of Algol, partly because of the intrinsic interest of the system and partly because of the high quality of the light-curve by Stebbins.[5]

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[1] *Mt. Wilson Contr.*, No. 86.

[2] *Astrophysical Journal*, **36**, 269, 1912; **37**, 154, 1913.

[3] "Darkened" used in this sense means "completely darkened," that is, darkened to zero at the limb.

[4] *Astrophysical Journal*, **36**, 401, 1912.

[5] *Ibid.*, **32**, 189, 1910.

A number of more or less extensive memoirs have been written concerning Algol's light-curve and photometric orbit, notably those by Pickering,[1] Harting,[2] Scheiner,[3] Pannekoek,[4] Rödiger,[5] and Stebbins.[6] All of the early solutions for orbital elements naturally neglected the secondary minimum and the reflection effect, and consequently it is not surprising that the photometric orbit derived by Stebbins and those presented in this note should be radically different from all that precede. The orbit of the system is, however, as yet by no means definitely solved, notwithstanding the enormous amount of labor—photometric, spectroscopic, and computational—that has been devoted to it, and the high accuracy with which the light-curve is now known. Stebbins' solution was based on the assumption of uniformly luminous disks and gave as one result that the faint companion is larger than its primary. My uniform solution, derived by another method, gives, as would be expected, elements practically the same as those obtained by Stebbins; but making the better assumption that the stars are considerably darkened at the limb, the size of the two components is reversed, the fainter one becoming the smaller.

It will be seen in Table I, which contains the elements computed for various degrees of darkening, that the primary star increases in relative size as we increase the darkening coefficient x, becoming just equal to its faint companion for $x=2/3$, that is, when the distribution of luminosity is similar to that of the sun. The adopted law of darkening at the limb referred to above is

$$J=J_0(1-x+x\cos\gamma),$$

where J is the apparent brightness of any point on the disk, and γ is the inclination to the line of sight of the normal to the stellar surface.

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[1] *Proceedings of the American Academy of Arts and Sciences*, **16**, 1, 1880.

[2] *Untersuchungen über den Lichtwechsel des Sternes β Persei*, Munich, 1889.

[3] *Untersuchungen über den Lichtwechsel Algols*, Bonn, 1882; *Populäre Astrophysik*, p. 625, 1908.

[4] *Untersuchungen über den Lichtwechsel Algols*, Leiden, 1902.

[5] *Untersuchungen über das Doppelsternsystem Algol*, Königsberg, 1902.

[6] *Op. cit.*

The recent spectroscopic work at the Allegheny Observatory indicates that the orbit of Algol is sensibly circular.[1] The value of the reflection effect found by Stebbins is adopted for this work, and the intensities given by him have been "rectified" to allow for this rarely observed phenomenon so that the light-curve, for the purpose of study, is of the normal eclipse form. The loss of light due to primary eclipse is 0.643 in the unit of the maximum light of the system; the value for the secondary is 0.055. In addition to those given in the summary two other solutions might be mentioned.

TABLE I

SUMMARY OF SOLUTIONS

ELEMENTS	STEBBINS	DEGREE OF DARKENING			
		Uniform	Intermediate		Complete
Darkening coefficient	0	0	1/3	2/3	1
Ratio of radius bright star to radius faint star	0.877	0.915	0.961	1.008	1.052
Fraction of the light of bright star eclipsed at primary	....	0.709	0.703	0.698	0.695
Radius bright star*	0.210	0.208	0.219	0.230	0.241
Radius faint star*	0.239	0.228	0.228	0.228	0.229
Inclination of orbit	82° 18′	83° 2′	82° 56′	82° 51′	82° 35′
Light of bright star	0.897	0.907	0.916	0.922	0.926
Light of faint star (bright side)	0.103	0.093	0.084	0.078	0.074
Relative surface intensity J_b/J_f	11.4	11.6	11.8	11.6	11.4
Semi-duration of primary eclipse	4^h54^m	4^h48^m	4^h55^m	5^h2^m	5^h10^m
"Equal-mass" density of bright star	(0.089)	0.091	0.078	0.068	0.059
"Equal-mass" density of faint star	(0.060)	0.070	0.070	0.070	0.069

* Unit of length is the radius of the relative orbit.

1. From the peculiar nature of the problem, a central annular eclipse on the hypothesis of complete darkening becomes a possibility and an orbit based on such an assumption was computed. The resulting light-curve, however, did not give a sufficiently good representation of the observed curve as it now stands, though it would not necessitate a very marked change in the observed points to make this computed orbit quite satisfactory.

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[1] *Science*, N.S., **37**, 34, 1913.

2. The accurate photometric curve of the principal minimum by Müller at Potsdam,[1] as worked over by Pannekoek,[2] was made the basis of an independent solution for uniform elements, assuming a secondary minimum of $0^m.06$. The maximum light-observations are not definite, but if considered of much weight would show ellipticity rather than reflection. The elements derived from this solution are:[3] $k=1.00$, $a_0=0.717$, $r_b=r_f=0.23$, $\cos i=0.102$, $L_b=0.925$.

TABLE II

NORMAL MAGNITUDES NEAR PRINCIPAL MINIMUM AND RESIDUALS FROM COMPUTED CURVES

PHASE	MAGNITUDE DIFFERENCE	STEBBINS O−C	SIN θ	RECTIFIED INTENSITY	OBSERVED−COMPUTED			
					$x=0$	$x=\frac{1}{3}$	$x=\frac{2}{3}$	$x=1$
-5^h18^m....	0.18	$+0^m.01$	−0.465	0.998	$0^L.00$	$0^L.00$	$0^L.00$	$0^L.00$
−4 22.....	0.20	− .02	− .388	.981	+ .01	+ .01	+ .01	+ .02
−3 54	0.28	.00	− .349	.915	− .01	− .01	− .01	− .01
−2 41	0.55	.00	− .243	.723	− .02	− .02	− .02	− .01
−1 39	0.88	− .01	− .150	.545	.00	.00	.00	.00
−1 08	1.09	.00	− .103	.458	.00	.00	.00	.00
−0 41	1.21	− .05	− .062	.415	+ .02	+ .02	+ .02	+ .02
−0 08	1.35	− .02	− .012	.370	+ .01	+ .01	+ .01	+ .01
+0 25	1.37	+ .04	+ .038	.364	− .01	− .01	− .01	− .01
+0 48	1.15	− .06	+ .073	.436	+ .02	+ .02	+ .02	+ .03
+1 13	1.08	+ .02	+ .111	.462	− .01	− .01	− .01	− .01
+1 34	0.97	+ .04	+ .143	.505	− .03	− .03	− .03	− .02
+2 03	0.69	− .06	+ .186	.641	+ .02	+ .03	+ .03	+ .02
+2 34	0.64	+ .05	+ .232	.669	− .04	− .04	− .04	− .04
+2 55	0.42	− .07	+ .263	.810	+ .03	+ .04	+ .03	+ .03
+3 16	0.42	+ .02	+ .294	.810	− .02	− .02	− .03	− .03
+3 50	0.30	+ .01	+ .343	.899	− .02	− .02	− .02	− .02
+4 14	0.23	.00	+ .377	.955	− .01	.00	.00	.00
+4 42	0.19	+ .01	+ .416	.989	− .01	.00	.00	.00
+6 14	0.17	0.00	+0.539	1.007	+0.01	+0.01	+0.01	+0.01

The normal magnitudes for the principal minimum of Algol are given in Table II. The co-ordinates of the mean observed points, taken from Stebbins' paper, have been transformed for convenience from time and magnitude difference into $\sin\theta$ and rectified light-intensity, where θ is the phase angle counted from minimum. The sum of the squares of the residuals from my computed uniform curve is 0.0066; from Stebbins' computed curve, reducing the residuals from magnitude to intensity, it is 0.0088. The last four columns of the table show that the representation of the observations is equally satisfactory for all degrees of darkening—in fact, the curves coincide for the greater part of the eclipse. From the light-curve alone we cannot hope to distinguish between these various sets of elements, and within these limits, at least, the orbit is not only entirely indeterminate, but must remain so until the question of darkening is answered.

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[1] *Astronomische Nachrichten*, **156**, 177, 1901.

[2] *Op. cit.*, p. 223.

[3] For the meaning of the notation see *Astrophysical Journal*, **36**, 404, 1912.

Referring to the summary of solutions, we see that in this typical case satisfactory intermediate solutions can be obtained from the uniform and darkened elements by linearly interpolating for the radii of the stars, the cosine of the inclination, and the duration of the eclipse. The remaining elements are then readily computed without the necessity of using the light-curve.

NOTE ON A THIRD BODY IN THE SYSTEM OF ALGOL

From the spectroscopic work by Curtiss[1] and Schlesinger[2] there seems to be little doubt that there is a third body in the system of Algol with a period of 1.9 years. If the distant component has a sensible brightness, its light will affect the computation for the orbit of the closer eclipsing pair. It may be appropriate to show briefly in what manner such a condition may be met, and to what extent the unknown quantity of light would alter the orbital elements computed above.

Uniform solution.—We proceed from the following data which were obtained from the solution of the rectified light-curve:

$$1-\lambda_p=0.643,\ 1-\lambda_s=0.055,\ \chi(k,\ \alpha_0,\ \tfrac{1}{4})=1.912,\ \sin^2\theta'=0.1793$$

(usual notation; see *Astrophysical Journal*, **36**, 404 ff., 1912). The difference in brightness of the two sides of the close companion is 0.045 so that the light at principal minimum, in terms of the greatest light of the system just outside the secondary minimum, is actually 0.312. We may assume any value of α_0, derive k from the χ-function, and then L_1 and L_2 by the equations $L_2=\frac{1-\lambda_p}{\alpha_0}$, $L_1=\frac{1-\lambda_s}{k^2\alpha_0}$

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[1] *Astrophysical Journal*, **28**, 156, 1908.

[2] *Publications of the Allegheny Observatory*, **1**, 20, 1908; *Science*, N.S., **37**, 34, 1913.

(the bright star is found to be the smaller); the remaining elements may then be derived in the usual manner. The difference $1-(L_1+L_2)=L_3$ will represent for the various assumed values of a_0 the light of the distant companion. Ordinarily, when such a contributing light-source is not suspected, the adopted values of a_0 and k are those unique values that give $1-(L_1+L_2)=0$. L_1 is the light of the bright side of the close faint companion; that of the side seen at principal minimum is $L_1-0.045$. The light remaining at that time is, then,

TABLE III

UNIFORM SOLUTIONS WITH CONSIDERATION OF THE LIGHT OF A THIRD BODY

a_0 (ASSUMED)	k	L_2	L_1	L_3	RADIUS		INCLINATION	DENSITY	
					Faint	Bright		Faint	Bright
1.00	0.733	0.643	0.102	0.255	0.247	0.181	84° 41′	0.055	0.14
0.95	.74	.677	.106	.217	.248	.183	84 52	.054	.14
0.90	.76	.715	.106	.179	.246	.187	84 15	.056	.13
0.80	.83	.804	.100	.096	.238	.198	83 27	.062	.11
0.709	0.915	0.907	0.093	0.000	0.228	0.208	83 2	0.070	0.09

$$L_3+(1-a_0)L_2+L_1-0.045=0.312$$

which is a relation that can be used to check the computations. We thus derive in Table III various sets of elements, each one being the best uniform solution for the corresponding value of the light of the third component, L_3. If the principal eclipse is assumed total, the third body may have as much as one-fourth of the whole light of the system, but under no condition more than that amount. The last solution is the one that gives $L_1+L_2=1$, and therefore yields the same set of elements as given in Table I.

Darkened solution.—The equations in this case are

$$\chi(k, a_0', \tfrac{1}{4})=1.912,\ L_2=\frac{0.643}{a_0'},\quad L_1=\frac{0.055}{a_0'Q(k,a_0')},\quad L_3=1-L_2-L_1$$

if we assume that the small star is the brighter and is eclipsed at principal minimum; but if the larger star is the brighter, we have

$$\chi(k, a_0'', \tfrac{1}{4})=1.912,\ L_2=\frac{0.055}{a_0'},\quad L_1=\frac{0.643}{a_0'Q(k,\ a_0')},\quad a_0''Q(k,\ 1)=a_0'Q(k,\ a_0').$$

The derivation of the sets of elements is then similar to the preceding case.

SUMMARY

1. If the orbits for an eclipsing binary have been computed when the darkening coefficient is zero (uniform disks) and when it is unity (completely darkened disks), solutions based on intermediate degrees of darkening can be obtained by linear interpolation for some of the elements (Table I).

2. The system of β Persei has been studied from this standpoint with the result that a considerable amount of indeterminateness is found to be inherent in the solution. For uniform disks the bright star is the smaller, while for completely darkened disks it is slightly the larger. Quantitative knowledge of the degree of darkening is necessary to remove this uncertainty (Table II).

3. An outline is given of the method of computing orbits for partially eclipsing variables when consideration is made of the possibility that a third body contributes a portion of the light, and an application is made to Algol to allow for the possible light of the distant companion whose period is 1.9 years (Table III).

Mount Wilson Solar Observatory
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THE SPECTRA OF FOUR OF THE TEMPORARY STARS

By W. S. ADAMS and F. G. PEASE

An important observation by Hartmann[1] in 1907 upon the spectrum of Nova Persei of 1901 showed that the principal nebular lines at λ 4960 and λ 5007, which were very prominent in the spectrum of the star when last observed during its brighter stages, had disappeared, and that the spectrum was essentially identical with that of the Wolf-Rayet star B.D. +35°4001. The interesting question thus raised, whether the disappearance of the principal nebular lines is characteristic of temporary stars in their later history, makes it desirable to secure spectra of such other Novae as can be obtained. Four such stars have been shown by the observations of Professor Barnard[2] to be of the fourteenth magnitude or brighter. With the magnitudes which he gives they are as follows:

	Magnitude
Nova Aurigae of 1891	14
Nova Persei of 1901	12.4
Nova Lacertae of 1910	12.5
Nova Geminorum No. 2 of 1912	10

We have succeeded in obtaining photographs of the spectra of all of these stars with a small slit spectrograph at the primary focus of the 60-inch reflector. The spectrograph consists of a 60° prism of ultra-violet flint glass used with collimating and camera lenses of about 16 cm focal length by Steinheil. The instrument is mounted in the opening of the double-slide plate-holder, and after the star has been brought upon the slit the guiding is carried on as in the case of direct photography by keeping auxiliary stars upon the cross-wires of two eyepieces, one at either side of the field, the spectrograph being moved bodily by the guiding screws. A diagonal prism mounted in a tube with an eyepiece may be slipped back and forth behind the slit, and enables the observer to bring the star accurately upon the slit at the beginning of the exposure.

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[1] *Astronomische Nachrichten*, **177**, 113, 1908.

[2] *Ibid.*, **194**, 401, 1913.

The photographs in the case of Nova Aurigae and Nova Lacertae required exposure times extending over more than one night. The observational data are given in Table I.

TABLE I

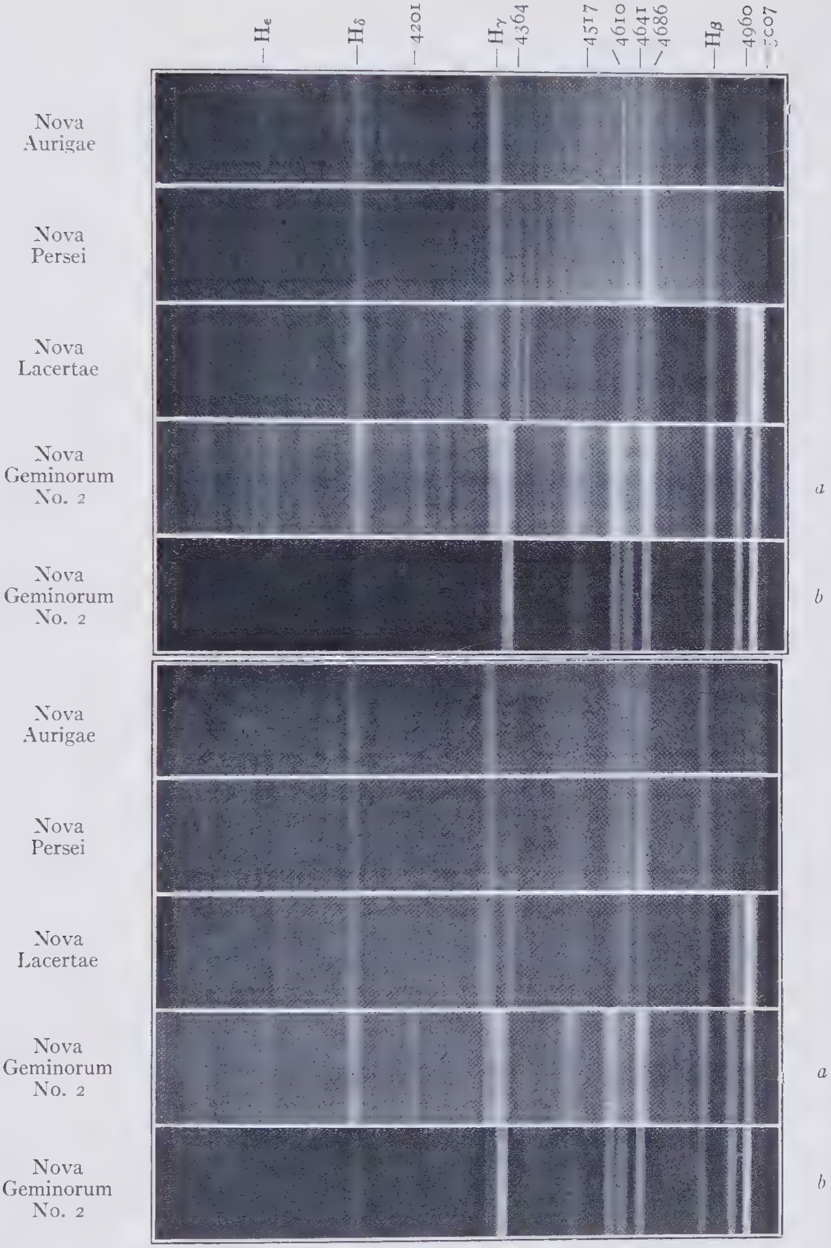
	Exposure Time	Date	Slit Width
Nova Aurigae	16 hours	1914, March 18–22	0.062 mm.
Nova Persei	8	1913, November 27	0.044
Nova Lacertae	11.5	1913, October 29–31	0.044
Nova Geminorum No. 2	2	1914, February 22	0.044

Reproductions of the enlarged and widened spectra are shown in Plate II. In addition a photograph of the spectrum of Nova Geminorum No. 2, taken with the Cassegrain spectrograph on October 12, 1913, is added for comparison. Owing to the small scale of the photographs and the consequent enlargement of grain, many false lines were introduced in the process of widening. Accordingly, two reproductions are shown. The first is a direct copy of the widened spectra. The second is from a print on which many of the false lines have been touched out by hand. Care has been taken in this case to leave untouched all parts of the spectrum in which true lines are seen on the original negatives. In addition, all of the real lines are marked on the reproductions.

The approximate intensities of the lines in the spectra are indicated in Table II upon a scale of 0 to 10. Probably several very faint lines are lost in the continuous spectrum which is particularly strong in the case of the first three stars. A dash in the table means that the line is not seen.

The single most interesting feature of these results is without doubt the absence of the chief nebular lines λ 4960 and λ 5007 from the spectra of Nova Aurigae and Nova Persei, and of the nebular line λ 4364, which is usually associated with them. The agreement of this observation in the case of Nova Persei with that of Hartmann in 1907 makes it appear that a spectrum in which these lines are

PLATE II



Upper Reproduction from Untouched Copy of Widened Spectra;
Lower Reproduction from a Retouched Print

absent must be considered as probably the permanent spectrum of this star, and from the similar result for Nova Aurigae it seems reasonable to conclude that the absence of the nebular lines is characteristic of Novae in their later history. It is about twenty-three years since the discovery of Nova Aurigae and thirteen since that of Nova Persei, but only four and two in the cases of Nova Lacertae and Nova Geminorum No. 2.

TABLE II

	Nova Aurigae	Nova Persei	Nova Lacertae	Nova Geminorum No. 2 Plate II*a*	Nova Geminorum No. 2 Plate II*b*
H_δ	2	2	2	3	3
4201	–	Trace?	Trace	1	1
H_γ	3	3	3	3	2
4364	–	–	Trace?	5	8
4517	–	Trace?	Trace?	2	2
4610	Trace	Trace	–	5	4
4641	1	1	3	Trace	3
4686	2	5	3	8	5
H_β	2	2	2	3	3
4960	–	–	5	4	5
5007	–	–	10	7	8
Continuous spectrum	Very strong	Very strong	Very strong	Strong	Trace

The extraordinary intensity of the principal nebular line at λ 5007 in the spectrum of Nova Lacertae is its most important feature, but a peculiar fact in connection with it is the weakness or entire absence of the nebular line at λ 4364. These two lines have been ascribed by Professor Nicholson[1] to the same vibrating atom, and their opposite behavior in the spectrum of this star is certainly an unusual feature.

The three lines at λ 4610, λ 4641, and λ 4686 show remarkable differences of intensity in the different spectra, and even on the two photographs of Nova Geminorum there are large variations. Reference has been made by one of us in an earlier communication[2] to some of these differences in the case of Nova Geminorum. The first line of the principal series of hydrogen at λ 4684 seems to be especially subject to fluctuations, and it is interesting to note the great difference in its intensity in Nova Persei as compared with Nova Aurigae.

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[1] *Monthly Notices*, **74**, 488, 1914.

[2] *Ibid.*, **73**, 742, 1913.

The intensity of the continuous spectrum of these stars appears to depend in general on their age, being greatest for the older Novae. The two photographs of Nova Geminorum show how very rapidly it may change, the earlier photograph showing hardly more than a trace of a continuous background.

As Hartmann has noted in the case of Nova Persei, and as the results show for both Nova Aurigae and Nova Persei, these spectra are essentially identical with the spectra of some of the Wolf-Rayet stars. With the disappearance of the principal nebular lines and the increase in intensity of the continuous spectrum the only marked points of difference have gone. This identity of spectrum, taken in connection with the well-known agreement of distribution relative to the Milky Way of Novae and Wolf-Rayet stars, makes it probable that at least a portion of the latter are temporary stars in the later stages of their history.

In the absence of knowledge as to the physical conditions under which the principal nebular lines are produced it is, no doubt, best to ascribe their disappearance to a change in these conditions in the radiating material. In view, however, of the considerable amount of evidence of various kinds which tends to support the hypothesis that the phenomenon of a temporary star is due to the star's entering a nebula, the suggestion that the disappearance of the chief nebular lines is coincident with the emergence of the star from the nebula may perhaps deserve some slight consideration.

MOUNT WILSON SOLAR OBSERVATORY
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Contributions from the Mount Wilson Solar Observatory, No. 88
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ON THE DISTRIBUTION OF THE ELEMENTS IN THE SOLAR ATMOSPHERE AS GIVEN BY FLASH SPECTRA

By CHARLES E. ST. JOHN

The remarkable eclipse spectrum obtained by Professor Mitchell in 1905, the results of the reduction of which appeared in the *Astrophysical Journal* of December 1913, furnishes a body of observational data that lend themselves to the statistical method of discussion. In sharpness of definition this spectrum without doubt surpasses any other published, and probably no plate has been obtained more nearly at the proper epoch of the eclipse.

The general scheme of this discussion is to form numerous groups of lines on a simple plan. Each element has been considered by itself, and the lines of like solar intensity assigned to it have formed the ultimate group. This is necessary when the purpose is to bring into relief characteristics that are shown only by small differences.

The publication of a discussion of the data by Professor Mitchell in the *Astrophysical Journal* of March 1914 gives an opportunity for a consideration of the data from the point of view brought out in my papers on "Radial Motion in Sun-Spots."[1] The material used in that discussion was furnished by 506 lines, each line being measured, on an average, upon 14 plates of high dispersion, so that some 7000 differential measures were involved. The number of lines in Mitchell's Table I[2] is 2841. The present discussion is confined to the plate ending at λ 5879, and for the most part to the lines not appearing as blends in Mitchell's table, which reduces the number by approximately one-half; but what is lost in quantity is more than balanced by the precision obtained when the purity of the lines is assured. This is particularly true when characteristics of individual elements and lines are under investigation, as a treatment of the data *en masse* obscures the finer distinctions.

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[1] *Mt. Wilson Contr.*, Nos. 69 and 74; *Astrophysical Journal*, **37**, 377, and **38**, 157, 1913.

[2] *Ibid.*, **38**, 424, 1913.

LEVEL IN GENERAL

An indication of the low level reached by Mitchell can be obtained by comparing the relative numbers of weak Fraunhofer lines obtained by different observers as shown in Table I.

TABLE I

FLASH SPECTRA LINES CORRESPONDING TO SOLAR LINES OF INTENSITY<1

	0000	000	00	0	Total	Region	Percentage
*Frost 1900	......	......	......	5	5	350 A	......
†Mitchell 1901	......	......	......	5	5	1100	......
‡ Evershed 1900	......	......	5	20	25	1500	......
§Jewell 1900	......	1	12	26	39	2500	......
Mitchell 1905	20	87	220	278	605	2500	21
¶Hale and Adams without eclipse	34	46	17	8	105	180	80

* *Astrophysical Journal*, **12**, 307, 1900.
† *Ibid.*, **15**, 97, 1902.
‡ *Phil. Trans. Roy. Soc.*, **201** A, 457, 1903.
§ *Publications U.S. Naval Obs.*, **4**, Part 4, Appendix 1, 121, 1906.
¶ *Mt. Wilson Contr.*, No. 41; *Astrophysical Journal*, **30**, 222, 1909.

It is seen that 21 per cent of the lines observed by Mitchell are weaker than 1 on the Rowland scale of intensity and that the proportion reported by other eclipse observers is vanishingly small, while 80 per cent of those measured on the Hale and Adams plates are weaker than unity. It is especially noticeable that the ratio of the number observed by them to the number observed by Mitchell increases with decreased Fraunhofer intensity. In obtaining flash spectra without an eclipse, the slit of the spectrograph is held rigorously tangent to the edge of the photosphere by guiding upon a chromospheric line. This can be done with great precision and constancy by the method employed, and assures the possibility and the probability of the appearance upon the Hale-Adams plates of lines originating in the lowest levels of the solar atmosphere. From the clever method of Mitchell for fixing the critical instant of exposure, it is clear that the spectrum corresponding to practically the entire depth of the solar atmosphere could impress itself upon the plate, and the richness of his plate in weak lines and the paucity of such lines upon the previous eclipse plates are probably accounted for by their exposures being ended too soon—that is, before the advancing moon had uncovered the whole depth of the atmosphere; or perhaps too late—that is, after the continuous spectrum of the photosphere had obliterated the weaker lines in the flash spectrum.

The above comparison appears to sustain the assumption, not only that the weaker Fraunhofer lines originate at lower depths, but also that these depths increase regularly with the decreased intensity of the solar lines.

THE BEHAVIOR OF SOME TYPICAL ELEMENTS

Iron.—From the long list of lines, 356 were selected as due only to iron, the ground of selection being that no other identifications were suggested by Mitchell; that is, they were not blends nor were they bracketed with other lines. From these, which include all the pure iron lines, the enhanced lines were excluded and the remaining lines were classified according to the Rowland intensities with the result shown in Table II.

TABLE II

FE LINES AND LEVEL AS SHOWN BY FLASH SPECTRA

	Number of Lines									
	4	19	30	55	72	49	40	28	24	25
Solar intensities..	00	0	1	2	3	4	5	6	7–8–9 (7.8)	10–40 (16.4)
Flash intensities..	0.25	0.26	0.33	0.8	1.4	1.6	2.0	3.5	3.8	7.0
Heights in km...	275	279	288	344	369	397	425	488	590	806

The regular progression shown by the flash intensities and the mean of the heights assigned to iron lines corresponding to like Rowland intensities, as these intensities increase from 00 to 16.4, lead to the conclusion that in a quite definite sense one is justified in saying that the lines of iron of a given intensity in the Fraunhofer spectrum extend to quite definite elevations. The regular march of the elevations with solar intensities is more plainly shown when the intensities are plotted as abscissae and the heights as ordinates. As a convenient scale, in lieu of a better, the numerical intensity intervals are used as abscissae. Owing to the uncertainty in the Rowland intensity intervals the smoothness of the resulting graph (Fig. 1, *a*) cannot be interpreted, of course, as meaning that intensity and height are directly proportional, but only that the heights and the intensities progress together.

Titanium, scandium, and yttrium.—These related elements are interesting components of the solar atmosphere and may conveniently be considered together. A common characteristic is the high proportion of enhanced lines. As clear a separation as possible has been made between the normal or unenhanced lines and the enhanced lines. As there are several degrees of enhancement, it is probable that some enhanced lines still remain. For titanium, the table given by Reese[1] has been of service, and the information given by Mitchell's tables has rendered it possible to make a rather close separation for all elements. The large number of enhanced lines in the titanium spectrum shows so clearly the behavior of this class of lines that it has furnished criteria for separating the lines of the other elements.

TABLE III

TITANIUM, SCANDIUM, AND YTTRIUM

	Number of Lines								Element
	7	19	18	18	21	14	9	4	
Solar intensity......	000	00	1	0	2	3	4	5	Titanium Fig. 1, *c*
Flash intensity......	0.1	0.25	0.4	0.9	1.5	1.9	2.8	2.8	
Height.............	271	290	347	353	389	429	494	487	

	Number of Lines						Element
	5	9	5	10	3	3	
Solar intensity.......	000	00	0	1	2	3	Scandium and yttrium Weighted means Fig. 1, *e*
Flash intensity.......	0	0.25	1.0	1.6	1.3	1.0	
Height..............	290	295	360	385	483	433	

Lanthanum group = La, Ce, Dy, Er, Eu, Gd, Nd, Pr, Sa.—The elements La and Ce, according to the conclusions of Adams from his investigations on the spectrum of the limb and center of the sun and from his study of solar rotation, are at a low level in the solar atmosphere, and the same conclusion was reached in the writer's discussion of the data obtained from the displacements in the penumbrae of sun-spots.

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[1] *Astrophysical Journal*, **19**, 322, 1904.

Carbon.—It is probable that the lines referred to carbon are due to cyanogen[1] or to nitrogen, but for convenience of reference, the older designation will be used. Grotrian and Runge have recently obtained the cyanogen bands in the presence of nitrogen, when both cyanogen and carbon were evidently absent.[2]

The low level of carbon was remarked by Adams in his rotation work, and by the present writer in the second paper on radial motion where reference was made to the weakening of the carbon or cyanogen lines in the arc in the presence of metallic vapors.[3] The high proportion of the weakest carbon lines on the chromosphere plates of Hale and Adams and upon the eclipse plate of Mitchell is evidence of the low-level origin of these lines.

TABLE IV

LA GROUP AND CARBON

	Number of Lines							Element
	1	7	27	18	8	10	2	
Solar intensities	0000	000	00	0	1	2	3	La group
Flash intensities	0.0	0.7	0.9	1.1	1.8	0.8	2.0	Fig. 1, *g*
Heights	250	329	354	382	406	395	550	

	Number of Lines						Element
	6	13	10	7	7	4	
Solar intensities	0000	000	00	0	1	2	Carbon
Flash intensities	0.7	0.5	0.2	0.9	0.7	1.8	Fig. 1, *i*
Heights	342	342	335	443	479	738	

The curves showing the relation between height and the Rowland intensities for the elements Ti, Sc-Y, the La group, and CN are given under *c*, *e*, *g*, and *i* in Fig. 1. When one considers the number of observations upon which the points depend, the uncertainties that occur in estimating the lengths of arcs, and the

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[1] Kayser, *Handbuch der Spectroscopie*, **5**, 229.

[2] *Physikalische Zeitschrift*, **15**, 545, 1914.

[3] *Mt. Wilson Contr.*, No. 74, pp. 6–7; *Astrophysical Journal*, **38**, 162–163, 1913.

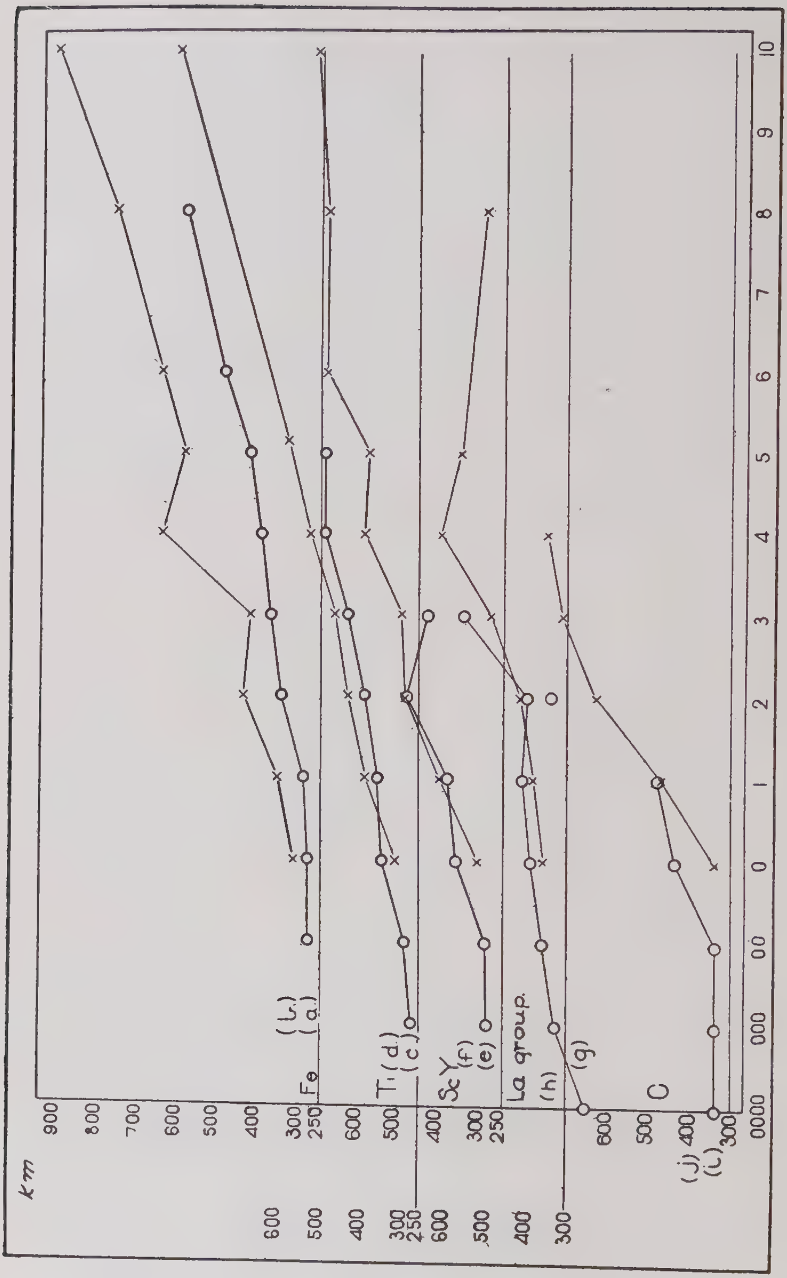

FIG. 1

O relation between solar intensity and height
× relation between flash intensity and height

increased regularity that accompanies increased weight, the connection between Rowland intensity and height is quite evident.

An estimation of the accuracy to be expected in the reduction of eclipse plates may be had from considering the heights and flash intensities. Mitchell says that "equality in height must entail an equal intensity in the flash spectrum."[1] In Fig. 1 are shown graphically the relations between heights and flash intensities, and though more observations are at our disposal, the regularity of the graphs is no greater than when Rowland intensities are used as abscissae. There are discordant observations in both cases, lines of equal flash intensity giving widely different heights, lines of very unequal flash intensities giving equal heights, and lines of the same solar intensity giving different heights—and this last is particularly true for elements with strong lines to the violet of λ 3850, in which region the apparatus was much less sensitive, owing to the low reflecting power of silver and speculum metal and to the decreased sensitiveness of the photographic plate. These discordant observations do not weaken the general statements that lines of equal mean intensity correspond to equal heights, and that lines of a given flash intensity mean equal heights, but rather tend to strengthen them, for if, in spite of these discordances, the means show that heights progress with both Rowland and flash intensities, it seems there must be a fundamental truth underneath the statements.

In saying that the Fraunhofer lines of intensity 4, for example, are at a definite elevation in the solar atmosphere, it is meant that when a sufficiently large mass of data is treated statistically it results that the lines of intensity 4, as a group, are of a higher level than those of an intensity less than 4, and at a lower level than groups of intensities greater than 4. The case is quite similar to that of stellar distances and proper motions. It cannot be said of a definite star with large proper motion that it is a near star, but out of a hundred stars with large proper motions it can be quite safely assumed that a very large majority are of the near-star group. There are undoubtedly differences between the effective levels of lines of equal solar intensity, just as the writer showed that there are differences in the radial displacements given by lines of like solar intensity, and that these differences are related to the classification of iron lines on other grounds. A thing that is generally true must first be established in its broad outline, and later, when some individual members of a group show themselves as definite exceptions, they offer a new means of obtaining added information and of pushing the analysis yet a step farther. It is yet too early to emphasize the exceptions, though data are accumulating that promise the added information. In the writer's opinion the criterion demanded by Mitchell in discussing this question is too exacting, when he requires that all lines of a given solar intensity must always be represented in the flash spectrum by lines of equal height.[1]

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[1] *Astrophysical Journal*, **39**, 128, 1914.

Some idea of the weight to be given to the means of the groups containing lines of like solar intensity may be had from an analysis of the group made up of solar intensity 3. This contains 72 lines and has a mean height of 369 km. Mitchell's estimates are made in 50 km steps so that heights of 350 and 400 km come nearest to this mean. A high degree of precision in any single measurement is an error of one unit of the scale, which in this instance is 50 km, so that heights of 300 and 450 km would still be within the range of error. Ninety per cent of the lines are within these limits. The one line below the lower limit is assigned a height much less than the average for lines of the same flash intensity, and the six above the upper limit are assigned heights greater than the average for lines of their flash intensities.

TABLE V

HEIGHTS AND RADIAL DISPLACEMENTS OF IRON LINES

Solar intensities....	00	0	1	2	3	4	5	6	7–9	10–40
Heights...........	275	279	288	344	369	397	425	488	590	806
Displacements.....	+0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.010	0.002

A comparison of the eclipse results for the iron lines with the radial displacements for lines of the same solar intensities is of interest, as the increasing displacements with decreased intensity were interpreted as consequences of greater depths.

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[1] *Astrophysical Journal*, **39**, 128, 1914.

The progressive decrease of displacements with increase of heights is a phenomenon that appears to find its straightforward interpretation in differences of effective level; that is, at the lowest height given by the iron lines, the vapors are flowing out of the spot tangential to the surface with the highest velocity, at greater heights the velocity is less, and the lines of greater solar intensity originate at these higher levels.

THE DISTRIBUTION OF THE ELEMENTS

When one considers the maximum height at which the different elements can be detected by flash spectra, a vertical distribution is shown that confirms the distribution obtained from the consideration of the sign and magnitude of the radial displacements; that is, the highest level is given by the H and K lines of calcium, the next by the hydrogen lines, while the heavy elements appear only in the lower portions of the solar envelope.[1] The vertical section of the solar atmosphere, in Fig. 2, is drawn to scale from data taken from Mitchell's first paper, the mean values being used where there are several lines of the same solar intensity. Some related radial displacements are also given. As in the terrestrial atmosphere, the percentage composition of the heavy elements is greatest in the lower portions, the lighter elements gaining in percentage, but decreasing in absolute density with height.

The distribution of the elements in accordance with their atomic weights, and the march of heights with intensities indicated by eclipse data are in satisfactory agreement with the interpretation of radial displacements as phenomena of level. In the case of the heavy elements, lanthanum and cerium, the displacements in the penumbrae of spots are greater than for iron lines of equal solar intensities, and an interpretation on the basis of level places the origin of a line due to the heavy metal at a lower level than a line of the same solar intensity due to iron.

The flash spectra heights given by lines due to the heavy metals are greater than for iron lines of the same solar intensity, as Mitchell has shown in his second paper. In discussing the matter he says:

> It therefore seems that St. John and the writer both use the differences in heights or differences in level to explain differences in the lines of the solar

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[1] *Mt. Wilson Contr.*, No. 74, pp. 3–8; *Astrophysical Journal*, 38, 159–164, 1913.

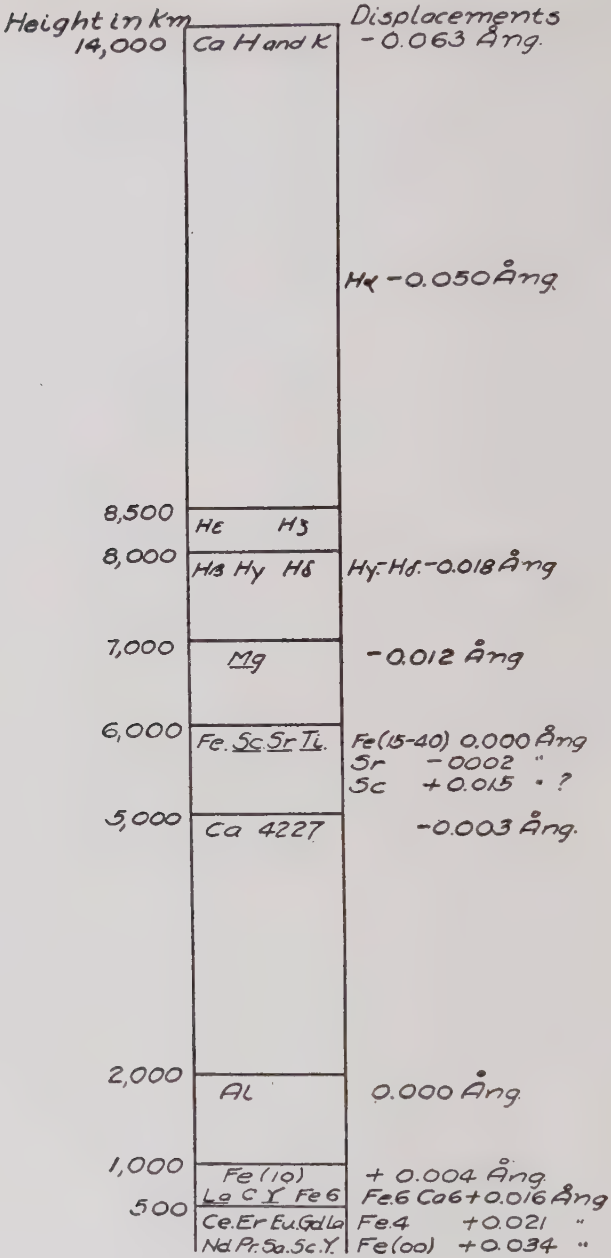

FIG. 2.—Absolute heights from flash lines; relative heights from displacements

spectrum. According to St. John's ideas, however, the rare earths are found in the low-lying regions of the reversing layer. Since they are found comparatively close to the photosphere, they are at a rather high temperature. As a result of this high temperature there is little absorption by the rare-earth vapors, and the Fraunhofer lines are not strong. These high temperatures, however, make more brilliant the lines of the flash spectrum, with the result that the intensities of the rare earths in the flash spectrum are much greater than in the Fraunhofer spectrum. Measures of the 1905 eclipse spectrum confirm the increased intensities in the flash spectrum demanded by St. John's theory, *but these measures do not show that the rare earths are found in shallow layers*, but exactly the reverse.[1]

The contradiction implied in the paragraph is only in appearance. Professor Mitchell informs the writer that he himself is not referring here to the absolute heights to which the rare-earth vapors extend, but to the fact that for lines of the same solar intensity the heights are greater in the case of the rare earths than for iron. My own conclusion was that, in the main, the heavy and rare-earth elements occur in detectable amounts only in the lower portions of the solar atmosphere, a conclusion that does not depend upon whether the lines of the heavy elements have their origin somewhat above or below the lines of iron of the same solar intensity, since none of the lines of the heavy elements reaches an elevation exceeding a few hundred kilometers, and with this conclusion Professor Mitchell writes that he is in entire agreement.

In Mitchell's second paper, the heavy elements (mean atomic weight, 148) are grouped with the light elements Sc (atomic weight, 44) and Y (atomic weight, 89) which are rich in enhanced lines, and it is possible that the heights reached by these elements have been mainly responsible for the high elevation of the group. In his Table III, p. 176, under solar intensity 3 to 5, there are 243 lines of the Fe group, with a mean height of 505 km, and 29 lines of the rare earths, with a mean height of 785 km. It is difficult to identify completely the 29 lines, but there are 22 lines attributed to Sc or Y only with a mean height of 864 km. There remain 7 lines with a mean height of 529 km to be accounted for. There are 4 lines of mean height 625 km, in which Sc or Y is the chief element, and 11 lines of mean height 491 km, in which one of the heavy-earth elements is the only or chief component. It is quite evident that the high elevation of 783 km is not due to the heavy elements. As to the higher elevation of the group when lines of solar intensity 6 to 10 are considered, it needs only to be said that no line of solar intensity 6 or more is known for the heavy elements, so that they have no influence here. The high elevation and flash intensity shown by the lines of the heavy elements relative to other lines of the same solar intensity appear prominently with decrease in solar intensity. The explanation seems to be found in the increased intensity of the flash lines of the heavy elements relative to the solar lines, which Mitchell says his measures confirm. An increased flash intensity due to the higher temperatures at the lower level carries with it an increase in height deduced from the length of the line. Hence it would seem that the heights given by flash spectra may be quite fictitious in the case of lines of very low level. The question is important in the interpretation of eclipse spectra and deserves further consideration.

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[1] *Astrophysical Journal*, **39**, 128, 1914.

FLASH SPECTRUM HEIGHTS FROM WEAK SOLAR LINES

The intensity assigned to a flash line is a complicated function of the length, width, and blackness of the line. If exposures of the same length could be given to the entire depth of the solar atmosphere and of sufficient duration to photograph the relatively cool H and K level, something like the following would be expected: a long, broad arc of weak intensity for the upper calcium, and short, narrow, and very black arcs for the lines of the La group, their blackness being due to the extremely high temperature of their vapors. For these lines the blackness would play an increasing rôle in the estimation of their intensities. A similar influence would make itself felt only in a less degree when determining the intensities on an ordinary eclipse plate; that is, the short arcs produced by the hot, low-lying vapors would be blacker in proportion to their length than the long arcs, and their intensities relative to that of the solar lines would increase with decrease of level. This increase of relative intensity with decrease of solar intensity, or increase of depth, is a marked feature of Mitchell's observations.

The exposure on the Mitchell eclipse plate was progressive, and hence the exposure at high levels was longer than at low levels. The highest level photographed is 14,000 km; the unenhanced lines of the heavy elements are all below 500 km. Had the exposure begun at the instant the projected limb of the moon was 14,000 km above the photosphere, the exposure for the outer envelope would have been nearly 30 times as long as for the region where the unenhanced lines of the La group originate. The actual case differed from this only in degree. Had the same time of exposure been given for the lowest levels as for the highest, the lines of very weak solar intensity would have been still stronger, and had the exposure for all levels been as short as for the very lowest, it is a question whether the relatively cool high-lying vapors would have photographed at all. In the case of eclipse plates, a strong emission of the low-lying vapors is shown by the presence of the flash reversals of the weakest Fraunhofer lines, though the exposure time for them is practically instantaneous.

To obtain an idea of the general case, the mean flash intensity was found for all lines of solar intensity 6 or less assigned to a single element and the difference F−S taken as shown in Table VI. The regularity of the increase of flash intensity relative to the Rowland intensity is very marked from a flash strength of 3 intensity intervals less to a strength of 3 intervals more than the Rowland intensities in passing from solar intensity 6 to 0000. A low level in the solar atmosphere appears to be favorable to a high flash intensity relative to solar intensities of the corresponding lines, carrying with it increased length of arc, and this high relative strength is characteristic of carbon and the La group of heavy elements, in which cases it reaches its highest value.

TABLE VI

FLASH INTENSITY RELATIVE TO SOLAR INTENSITY

	Number of Lines									
	18	69	164	166	189	185	166	109	71	44
Solar intensity....	0000	000	00	0	1	2	3	4	5	6
Flash intensity ...	0.44	0.3	0.38	0.55	1.1	1.3	1.9	1.9	2.3	2.8
F−S	+3.44	+2.3	+1.38	+0.55	+0.1	−0.7	−1.1	−2.1	−2.7	−3.2

Another interesting fact appears when the heights for the same flash intensities of different groups of elements are compared with the heights for the same flash intensities of iron. For 124 lines of the heavy elements and carbon, the heights average 76 km higher than for Fe lines of the same flash intensities; for 159 lines of Ti it is 5 km and for 73 lines of Sc and Y 2 km higher than for iron. It seems clear that some character of the flash lines of the heavy elements and carbon—the increased blackness, perhaps—has made it possible to follow the vanishing tips of the lines with less difficulty than in the case of the other elements and this has resulted in a greater length of arc for the flash lines of the heavy elements than for lines of the same flash intensity due to iron. When the heights for lines of the same solar intensities are compared, the average height for the heavy elements and C is 104 km, for Ti 54 km, and for Sc and Y 69 km greater than for the Fe lines. Since the flash lines of Ti, Sc, and Y yield the same heights as the flash lines of Fe of like intensity, the level of the lines of these elements relative to Fe lines of the same solar intensity is indicated with greater probability than for the heavy elements and carbon, in which case the high level relative to iron lines of like solar intensity is mainly accounted for by the greater height given by lines of the same flash intensity. An indication of this effect of low level appears in the graphs showing the relation between heights and solar intensities. For lines of medium intensity the slope of the curves is regular when the points depend upon a sufficient number of lines; but for lines of low intensity the heights decrease less rapidly or not at all. In the case of these lines, the actual differences in level are smaller than for stronger lines, and the increased emission due to the higher temperatures of the lower levels would tend to increase their blackness and to give, as shown above, a slightly anomalous height on eclipse plates, the effect becoming more apparent with decreasing intensity or increasing depth.

The relative intensity of the lines on a flash spectrum plate plainly depends upon the phase of the eclipse photographed, the exposure times, and the sensitiveness of the apparatus for different wave-lengths. Bearing upon this question the following examination was made of the probable sensitiveness of Professor Mitchell's apparatus: A Seed orthochromatic plate was exposed in a concave grating spectrograph of 1 m focus to sunlight reflected from a freshly burnished silver mirror, a combination similar to that used by Mitchell. The exposure and development were such as to end the spectrum between λ 5800 and λ 5900. The opacity of the plate was measured by a Hartmann microphotometer and the results plotted against wave-length. The maximum sensitiveness is near λ 4500, and at λ 3300 it is about one-third of the maximum. The combined reflecting power of silver and speculum metal was found from the data of Hagen and Rubens;[1] at λ 3300 the reflecting power is 0.23 of what it is in the visual region, so that the apparatus was approximately one-thirteenth as sensitive at λ 3300 as in the visual region.

This would give lower heights for lines in the ultra-violet. In a certain sense it would be like giving a shorter exposure to the ultra-violet lines of the high-level vapors, as the photographic effect of the radiation of the relatively cool high-level layers would be more decreased than that from the intensely hot low-lying layers, and the relative intensities of the high- and the low-lying lines would be greatly changed.

Evidence that the relative flash intensities are greatly altered in the ultra-violet region and change in such a way as to increase the intensities of the weak solar lines relative to the stronger one is found by comparing the values of F−S in the case of very weak medium, and very strong solar, lines for the two regions.

TABLE VII

		F−S	
		Violet	Ultra-Violet
Intensity..........	000–0	+1.1	+ 1.1
Mean intensity......	3	+1.52	− 1.82
Intensity..........	20	+8.4	−16.2

The intensity of the weak lines relative to the solar lines remains practically unchanged, but the relative intensity decreases progressively with increased solar intensity. The absolute flash intensity of ultra-violet lines of solar intensity 20 is a third of that for the violet lines of the same solar intensity, or the intensity of the

[1] Wood, *Physical Optics*, p. 466, 1911.

weak lines relative to that of the strong lines has increased threefold, entailing a great difference in their relative heights.

That the ultra-violet weak solar lines were photographed at all under these conditions is an indication of the enormous radiation from these low-lying layers, and the weakening of the flash lines which are due to the vapors extending to high elevations is what would be expected when the source of the flash line is a widely extended surface of relatively low intensity. In the one instance we have an intense emission concentrated in a small area; in the other, the source is a very extended surface of cooler vapor with a relatively feeble emission.

In view of the preceding considerations, the usual method, in discussing eclipse plates, of determining heights from length of arc appears of doubtful value in the case of low-level lines.

There are several facts established by observation, relative to the members of the group of heavy elements, all of which should be considered in an attempt to fix the relative level at which the lines of these vapors occur.

1. The intensity of the La and Ce lines at the sun's limb is greatly decreased in comparison with the intensity at the center, while that of most lines is increased. (Adams.)

2. The rotation value given by La and carbon is lower than for Fe lines of the same solar intensity. (Adams.)

3. The displacements given by the La and Ce lines and those of the rare-earth metals in the penumbrae of sun-spots are very large, larger than for the Fe lines of the same solar intensity. (St. John.)

4. The heights given by Mitchell are greater for the flash lines of the La group and carbon than for the Fe lines of the same flash intensity.

5. The heights found by Mitchell are greater for the lines of the La group and carbon than for iron lines of like solar intensities.

Mr. Adams has shown that the behavior of the La and Ce lines at the sun's limb finds an explanation in their low level, that the angular velocity of the solar vapors decreases from that of the high-level hydrogen to a minimum value for lanthanum and carbon, and that the low rotation value for these lines relative to that for

Fe lines of the same solar intensity finds its obvious cause in the low level of these lines relative to the Fe lines of the same solar intensity. The radial displacements of the Fraunhofer lines in the outer edge of the penumbrae of spots appear to be a Doppler effect and the regular increase with decrease of solar intensity, a phenomenon of level. Under this interpretation, the greater displacements given by the lines of the heavy elements than by Fe lines of the same solar intensity mean obviously a low level for these lines relative to those of Fe of like solar intensity.

The greater heights given by the flash spectrum for the lines of the heavy elements relative to those of iron of like solar intensity appear to Mitchell to be in contradiction to these three lines of observation. A large part of this excess in height comes, however, from the character of their flash lines, which has allowed their vanishing tips to be followed farther than in the case of the lines of iron of the same flash intensity. This characteristic of their flash lines, rather than their greater strength relative to the Fe lines of the same solar intensity, contributes probably the major part of the excess in height.

HEIGHT AND WAVE-LENGTH

In the discussion of the radial displacements, it was noticed that the displacements were greater in the red than in the violet for lines of the same solar intensity. This was attributed to our seeing into the sun to a greater depth in the red, owing to the greater scattering of the violet light—that is, the effective levels are lower for the longer wave-lengths. Inasmuch as it is the upper border of the effective layer, or the height to which the vapor emitting a given line extends, that determines the length of the arc, a similar effect might be expected from the flash spectrum. A large number of lines—666—of solar intensity 1 to 6, identified with single elements, were divided into groups of 100 A each. The mean heights for the groups were plotted as ordinates. The result is a curve, (Fig. 3, *a*), with a maximum at λ 3850, a slow drop to the red, and a rapid fall to the ultra-violet.

The maximum height given by the flash lines is at λ 3850, while the photographic maximum of the apparatus is at λ 4500. The curve, Fig. 3, *b*, represents approximately the effective sensitiveness of the apparatus for various wave-lengths as it is derived from the photographic sensitiveness and the reflecting powers of the surfaces.

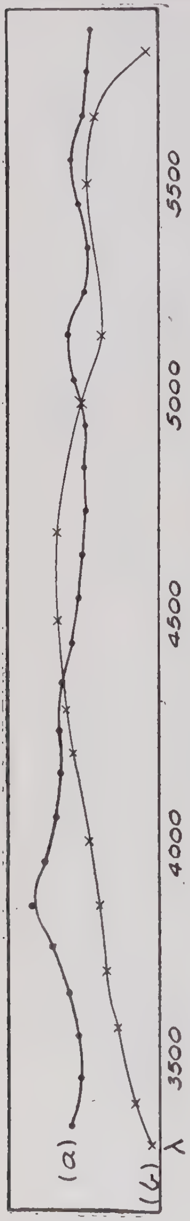

FIG. 3

(*a*), heights as ordinates, wave-lengths as abscissae. All single lines 1–6 solar intensity
(*b*), sensitiveness of apparatus derived from plate sensitiveness and reflecting power of the surfaces

It will be noticed that the heights do not follow the effective sensitiveness, that in the region λ 3700 to λ 4350 the heights are greater than in the region λ 4350 to λ 5000, though the sensitiveness is greater in the latter case, and that in the region λ 5000 to λ 5800 the heights are not low because of low sensitiveness of the apparatus, which for this region averages slightly higher than at λ 3850. Apparently the reported intensities and heights are fully as great as photographed, for Mitchell says that in estimating intensities allowance was partially made for the decrease in the sensitiveness of the plate in the green and yellow regions. With due allowance for plate sensitiveness and reflecting power of the surfaces, the fact still remains that the heights decrease with increasing wave-lengths.

EFFECTIVE LEVELS

The effective level of a line may be defined as that portion of the entire depth of a vapor that is mainly concerned in the production of the line. The light in a Fraunhofer line of intensity 4, for example, comes from a lower depth than in a line of intensity 10 because of the greater selective absorption in the latter. The light from the photosphere and from the lower, hotter layers of the vapor under consideration is selectively absorbed in both instances and fails to reach the surface, so that the emitted light of the respective wave-lengths comes from more or less sharply bounded shells of vapor having a larger radius for the line of intensity 10 than for the line of intensity 4.

The idea of effective levels has been tacitly employed, though not so definitely expressed in all spectroheliograph work. There is no question among solar physicists but that the light employed in H_3–K_3 photographs comes only from the highest level when a very narrow slit is used; that light forming the H_2–K_2 photographs comes from a low level, but not from the lowest levels; these are successively reached by setting the slit on the broad H_1–K_1 shading at increasing distances from the center of the line. It is clear that here we are dealing with successive shells of calcium vapor. It is probable that a similar succession of shells would obtain for the calcium lines of decreasing intensities, and that is what appears in the radial displacements and in the heights from the flash spectrum when a sufficient number of lines is present to obtain means of weight. For the calcium lines the succession is indicated as follows:

	H_3–K_3	H_2–K_2	4227	8–6	5–3	2–1
Radial displ.	−0.063	−0.044	−0.003	+0.017	+0.019	+0.028 A
Flash sp. hgt.	14,000		5,000	583	491	350 km

The negative signs indicate an inflow, the positive signs an outflow, from the spots, tangential to the solar surface.

What may be called the radiation centers of gravity of the effective layers are at not greatly different levels in the reversing layer. Lines of solar intensity 0000 to 10 occur below 1000 km, allowing an average distance of about 70 km for each layer. If such be the conditions in the solar atmosphere, it is clear that flash spectra measurements of heights made by 50 km steps could show the facts only by taking the means of a very large number of lines, that individual departures from the mean would be numerous, and that small differences between the short arcs would be difficult of determination. In point of view of precision there is a wide difference between the estimates of flash intensities and arc lengths upon a single flash spectrum plate and differential measurements of line displacements upon many plates of high dispersion.

GENERAL CONCLUSIONS

The facts determined by observations that bear upon the question of the distribution of the elements in the solar atmosphere are gradually increasing in number and are already sufficient for the deduction of some probable conclusions. The salient facts shown by flash spectra and displacements in the penumbrae of sun-spots, based upon mean values, are as follows:

From Flash Spectra	From Radial Displacement
The heights and flash intensities increase progressively with the solar intensity of the lines.	The displacements vary progressively with the solar intensity, from large negative values for the strongest lines to large positive values for the weakest lines.
The flash intensity of unenhanced lines relative to the solar intensity increases with decreased solar intensity as follows in the case of iron:	The displacements increase with decrease of solar intensity as follows in the case of iron:

Solar int. .	00	0	1	2	3	4	5	6	7–8–9	10–40
F−S.	+1.25	+0.26	−0.67	−1.2	−1.6	−2.4	−3.0	−2.5	−3.9	−9.4
Displ.	0.034	0.030	0.028	0.025	0.023	0.021	0.019	0.016	0.011	0.002

The vapors of the elements with the strongest solar lines extend to the greatest heights.	The elements with the strongest solar lines give small positive or negative displacements.
Enhanced lines are higher than the normal or unenhanced lines of the same solar intensity.	The displacements are smaller for enhanced lines than for normal or unenhanced lines of the same solar intensity.
The heights are greater for lines in the violet than in the yellow-green when lines of the same solar intensity are compared.	The displacements are less for lines in the violet than in the yellow-red when lines of the same solar intensity are compared.

These facts are all harmonized by the consideration that the vapors of the elements ascend in detectable amounts to different heights, that the lines of any one element originate at depths increasing with decrease of solar intensity, that the enhanced lines are higher than unenhanced lines of equal solar intensity, and that we see into the sun to greater depth at the red end of the spectrum than at the violet.

The general distribution of the elements in the solar atmosphere and the relative levels or heights for the lines of a given element, such as iron, were found by interpreting radial displacement in spots as Doppler effects varying with the depth. This interpretation finds strong confirmation in Mitchell's eclipse data, which give a similar distribution of the vapors of the elements and show that the heights of the lines of an element rich in lines, such as iron, increase progressively with their solar intensities. Thus we have added evidence that in the Evershed effect we have a means of sounding the solar atmosphere with a high degree of precision.

The writer finds no better expression for summing up this investigation than that used at the conclusion of the discussion of the displacements in the penumbrae of sun-spots, excluding the specific reference to the H_α line which does not appear in Mitchell's flash spectrum: "The resulting distribution shows that the H_3 and K_3 lines of calcium are the lines of highest level, followed by the H_α line of hydrogen, and that, in the main, the heavy and rare elements occur in detectable amounts only in the lower portions of the solar atmosphere."

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SOME SPECTRAL CRITERIA FOR THE DETERMINATION OF ABSOLUTE STELLAR MAGNITUDES

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In the course of a study of the spectral classification of stars whose spectra have been photographed for radial velocity determinations some interesting peculiarities have been observed. The stars investigated are of two kinds: first, those of large proper motion with measured parallaxes; second, those of very small proper motion, and hence, in general, of great distance. The apparent magnitudes of the large proper motion, or nearer stars, are somewhat less on the average than those of the small proper motion stars, so that the difference in absolute magnitude must be very great between the two groups. The spectral types range from A to M.

The principal differences in the spectra of the two groups of stars are:

1. The continuous spectrum of the small proper motion stars is relatively fainter in the violet as compared with the red than is the spectrum of the large proper motion stars. The magnitude of this effect appears to depend on the spectral type, and increases with advancing type between F0 and K0.

2. The hydrogen lines are abnormally strong in a considerable number of the small proper motion stars. Thus six stars which show the well developed titanium oxide bands characteristic of type M have hydrogen lines which would place them in types G4 to G6, and many others which show the bands strongly would be classified under type K from their hydrogen lines. That the spectra of these stars are not composite is shown by their radial velocities. The hydrogen lines in the spectra of the large proper motion stars which show the titanium oxide bands are without exception very weak.

3. Certain other spectrum lines are weak in the large proper motion stars, and strong in the small proper motion stars, and

conversely. It is with the possibility of applying this fact to the determination of absolute magnitudes that the results given in this communication mainly have to deal.

I. INTENSITY OF THE CONTINUOUS SPECTRUM

A comparison of the intensity of the continuous spectrum of several pairs of stars of small and of large proper motion photographed upon the same plate was made recently by one of us, and showed a marked weakening relatively in the violet region for a majority of the small proper motion stars.[1] With a view to supplementing these observations with the larger amount of material available in the radial velocity photographs we have calculated the densities at several points in the spectrum for a considerable number of these stars, and compared the resulting values for the stars of small with those of large proper motion.

The method employed, though by no means so accurate as a photometric measure of density, appeared to be as good as the character of the material would warrant. It is evident, of course, that in the case of an individual star the conditions of transparency under which the photograph was taken, the zenith distance, and some other factors might influence the result seriously. For the mean of a considerable number of photographs and stars, however, it would seem that these effects should counteract each other largely, or at least be similar for the two groups of stars under comparison. No photographs have been included which were taken at great zenith distances.

The plan adopted for the determination of the densities was as follows: A standard plate of α Tauri was first obtained, several spectra taken with different exposure times being placed side by side on the negative. The photograph of each star was then compared with this standard plate under a Hartmann spectrocomparator, and estimates were made of the intensity of the continuous spectrum relative to that of α Tauri at three selected points at the violet and four points at the red end of the spectrum. The points were selected in regions as free from lines as possible. The estimates were made in tenths of a unit between the α Tauri spectra. Thus

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[1] *Mt. Wilson Contr.*, No. 78, *Astrophysical Journal*, **39**, 89, 1913.

1.5 indicates an intensity half-way between the first and second of the standard spectra. After the comparisons had been finished the α Tauri photograph was measured under a microphotometer, and the densities were calculated at the points of comparison. The results for all of the stars were then reduced to densities.

The values for the groups of stars are given in Table I. The densities for the three violet wave-lengths have been combined to form a mean at λ 4220, and similarly for the four wave-lengths near λ 4955.

TABLE I

	Number of Stars	Average μ	Average Type	Density at λ 4220	Density at λ 4955
A0–A9	15	0″.020	A2	0.30	0.32
	16	.13	A3	.29	.32
F0–F9	10	.012	F4	.25	.37
	23	.66	F6	.32	.37
G0–G4	8	.009	G3	.22	.41
	30	.64	G2	.33	.41
G5–G9	14	.011	G7	.25	.48
	22	.64	G7	.40	.48
K0–K4	24	.011	K2	.16	.44
	22	0.70	K1	0.31	0.44

The features of note in these results are:

a) The small proper motion stars of types F to K are decidedly weaker in the violet part of the spectrum than the large proper motion stars.

b) The difference is inappreciable for two groups of A-type stars for which the ratio of proper motions is 1:6.5.

c) The difference increases with advancing type from F to K, being twice as great for the latter. The ratio of proper motions for the groups of small and of large proper motion stars is nearly the same for the stars between F and K. Hence if interpreted in terms of distance the ratio of distances should be nearly the same, and it would appear that at least a part of the absorption in the violet part of the spectrum of the distant stars must be ascribed, not to scattering of light in space, but to conditions in the stellar atmospheres. In the case of the A-type stars the results are inconclusive, since the ratio of the proper motions shows that the negative result found may be due to the fact that the difference of distance between the two groups of stars is insufficient to produce a measurable amount of scattering.

II. THE HYDROGEN LINES

The abnormal strength of the hydrogen lines in the spectra of certain of the small proper motion stars is of peculiar interest because of the possibility of selective absorption by hydrogen gas in interstellar space. The radial velocity affords a means of determining the origin of the additional absorption since it is highly improbable that the hydrogen in space would have the motion of the stars observed. Accordingly we have given especial attention to the determination of the radial velocities of these stars from the hydrogen lines as compared with other selected lines in the spectrum. The results obtained indicate that within the limits of error of measurement the hydrogen lines give essentially the same values as the other lines, and no differences have been found of an order to correspond to the abnormal intensity of the lines.

TABLE II

Star	Magnitude	Type	Metallic−H Lines	Computed Metallic−H Lines for Absorption	
				In Star	In Space
			km	km	km
Boss 539	5.6	G7p	−0.5	0.0	+13.4
707	5.6	G9p	+0.3	.0	+ 4.3
1309	5.8	G9p	+0.6	.0	−16.7
1560	5.3	G5p	+1.6	.0	+15.3
1846	5.7	G9p	+2.9	.0	−50.9
2020	5.8	G6p	+0.3	.0	−27.7
2144	6.2	G6p	+4.9	.0	+15.6
2378	6.3	G6p	+2.2	.0	+ 6.4
2612	5.6	G8p	−2.0	.0	+ 1.2
2915	6.0	G5p	+0.7	.0	−14.9
3867	6.0	G6p	−1.2	.0	−18.9
4159	5.6	G5p	−3.6	.0	−17.2
5125	5.6	G5p	+1.8	.0	+ 3.4
6089	5.4	G4p	−2.2	.0	−31.7
6145	6.0	F0p	−4.6	0.0	0.0

In Table II are collected the results for 15 stars which show abnormal strength of the hydrogen lines most prominently. All of the stars except Boss 6145 have the bands characteristic of type M. The classification given is based on the hydrogen lines. The column designated "Metallic—H Lines" gives the values in kilometers of the differences in the velocities derived from about 12 selected metallic lines and those from H_γ and H_β; a small systematic correction is applied to the latter, due probably to the effect of blended lines. These differences would, of course, be zero if all of the hydrogen absorption occurred in the stellar atmospheres. If it all occurred in space the differences would be those given in the final column on the assumption that the absorbing gas is at rest in space. The quantities are derived by applying to the velocities of the stars obtained from the metallic lines the corrections to these velocities for the motion of the sun in space.

If any appreciable hydrogen absorption occurred in space the differences, Metallic—H Lines, should, of course, be intermediate between the quantities in the last two columns. When, however, we combine the values for all of the stars, assigning weights according to the numbers in the last column, we find that 98 per cent of the hydrogen absorption must occur in the stellar atmospheres, and that but 2 per cent can possibly be due to hydrogen gas in space. This amount is far below the limits of accuracy of the observations.

III. THE RELATION OF LINE INTENSITY TO ABSOLUTE MAGNITUDE

Systematic differences of intensity for certain lines between stars of large and stars of small proper motion soon became evident in the course of the study of the spectral classification of these stars. In order to secure an accurate system of classification as well as to investigate these differences the following method was adopted. Pairs of lines were selected not far from one another in the spectrum and of as nearly as possible the same intensity and character, and estimations were made of their relative intensities. For classification purposes a line decreasing in intensity with advancing type, such as a hydrogen line, was combined with a line increasing in intensity with advancing type, such as an ordinary metallic line. In addition to these pairs used for classification purposes several pairs were selected which included all lines suspected of systematic deviations in certain stars.

The estimations were made on an arbitrary scale extending from 1 to about 12, 1 being the smallest difference in intensity which could be detected. The method, therefore, is analogous to the *Stufenmethode* of Argelander used in estimations of variable stars; hence, for physiological reasons, our scale will be approximately proportional to the logarithm of the intensity differences of the two lines. In general three plates were used for each star, and the photographs of the large and the small proper motion stars were intermingled in order that systematic effects on the estimation scale might be avoided.

After all of the estimations had been completed the material was reduced uniformly, and the results were examined with two objects in view: first, to investigate the changes of the estimated intensity differences with the spectral type, and on this basis to form a classification depending on certain well defined criteria; second, after correcting for changes with type to investigate changes with absolute magnitude.

An examination of the pairs of lines used for estimation indicated that the following pairs showed the largest and most definite changes with type. The Harvard scale of classification has been followed closely.

F8–G6 stars: $\frac{4227}{H_\gamma}$; $\frac{4326}{H_\gamma}$; $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$; $\frac{H_\gamma}{4384}$

G6–K9 stars: $\frac{4326}{H_\gamma}$ (wt. $\frac{1}{2}$); $\frac{H_\gamma}{4352}$; $\frac{H_\gamma}{4405}$ (wt. $\frac{1}{2}$); $\frac{H_\beta}{4872}$; $\frac{H_\beta}{4958}$

These lines, accordingly, have been used to determine the type of each individual star, and since no systematic differences for the different lines have been found, the mean of the determinations from the five pairs has been used as the final result for the spectral type. This method of classification has proved most satisfactory in use, and shows good internal agreement. The mean error of one determination depending on three plates is $+0.4$ subdivision of the Harvard scale, equal, for example, to the interval from G5.0 to G5.4.

As soon as the spectral type of each star had been obtained in this way, the results for the remaining pairs of lines were examined with a view to seeing whether all of them fell into agreement with the classification, or whether there were systematic differences for different groups of stars. For this purpose we constructed a normal curve for each pair of lines from the stars of rather low absolute luminosity, plotting as abscissae the spectral types, and as ordinates the estimations of intensity differences. Finally we formed for all of the stars the differences between our estimations of relative intensity and the values from the normal curve corresponding to the spectral type. These differences, combined into means for two separate groups, are shown in Table III.

At the head of each column of ratios is given the mean of the absolute magnitudes of the stars observed. Thus for the F8–G6 stars the mean of the absolute magnitudes of the small proper motion stars is -2.9, of the large proper motion stars, $+6.1$. Although the number of stars used in the estimate of the ratios of the different pairs of lines varies somewhat, the same mean magnitude, which was derived from all of the stars, is used throughout. The computation of the absolute magnitudes of the individual stars was made from the measured parallaxes where these were available. In the absence of such determinations, or when the parallax was very small or negative, the absolute magnitude was computed from the proper motion by aid of the parallax derived from the following formula:

$$\log \pi = -1.00 - 0.005m + 0.86 \log \mu$$

where m is the apparent magnitude and μ the proper motion. This formula is contained in an unpublished investigation by Kapteyn and Kohlschütter on the luminosity-curve of the K-type stars, and is based upon a discussion of the relation between proper motion and parallax for the K stars. The unit employed in the determination of absolute magnitudes is $0''.1$; that is, the absolute magnitude is the apparent magnitude of a star at a distance corresponding to a parallax of $0''.1$.

The number of stars used in each comparison in Table III is indicated by the figures in parentheses.

It is obvious from the method of derivation that the mean values in Table III for all the pairs of lines will be small in the case

TABLE III

Ratio	F8 to G6			G6 to K9		
	$M=-2.9$	$M=+6.1$	Remarks	$M=-2.7$	$M=+6.2$	Remarks
$\frac{4227}{H\gamma}$....	−0.2 (19)	−0.1 (47)	Class.	+1.9 (16)	+0.5 (19)	
$\frac{4326}{H\gamma}$....	0.0 (28)	0.0 (49)	Class.	−0.2 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H\gamma}{4352}$....	−0.3 (28)	+0.1 (49)	Class.	+0.9 (52)	0.0 (45)	Class.
$\frac{H\gamma}{4405}$....	+0.4 (28)	−0.1 (49)	Class.	+0.1 (52)	0.0 (45)	Class. (wt. ½)
$\frac{H\gamma}{4384}$....	+0.6 (28)	0.0 (48)	Class.	+0.2 (51)	+0.2 (43)	
$\frac{H\beta}{4852}$....	−0.7 (13)	+0.2 (29)		−0.4 (48)	−0.1 (43)	Class.
$\frac{H\beta}{4958}$....	+0.1 (16)	−0.2 (28)		−0.9 (46)	0.0 (40)	Class.
$\frac{4337}{H\gamma}$....	+3.2 (28)	−0.4 (49)		+1.9 (52)	+0.3 (45)	
$\frac{4415}{4435}$....	−1.2 (28)	−0.2 (49)		−1.1 (52)	+0.1 (45)	
$\frac{4435}{4444}$....	−1.0 (28)	−0.1 (49)		−0.7 (52)	−0.2 (45)	
$\frac{4415}{4456}$....	+1.7 (28)	−0.1 (49)		+3.9 (52)	+0.2 (45)	
$\frac{4456}{4462}$....	−4.8 (28)	+0.4 (49)	Abs. mag.	−6.6 (52)	−0.3 (45)	Abs. mag.
$\frac{4456}{4495}$....	−3.6 (26)	+0.5 (36)	Abs. mag.	−5.2 (51)	−0.3 (45)	Abs. mag.
$\frac{4216}{4250}$....	+8.3 (21)	−0.4 (44)	Abs. mag.	+5.8 (31)	+0.2 (33)	Abs. mag.
$\frac{4250}{4255}$....	−0.5 (20)	−0.1 (45)		−1.3 (42)	−0.1 (38)	
$\frac{4271}{4275}$....	−1.2 (27)	0.0 (46)		−0.5 (48)	−0.3 (43)	
$\frac{4368}{4376}$....	−0.8 (28)	−0.3 (41)		+0.1 (52)	0.0 (41)	
$\frac{4395}{4415}$....	+4.1 (28)	−0.2 (41)	Abs. mag.	+3.4 (52)	+0.4 (40)	Abs. mag.
$\frac{4408}{4415}$....	+5.7 (28)	+0.1 (48)	Abs. mag.	+4.9 (52)	+0.5 (45)	Abs. mag.
$\frac{4495}{4497}$....	−3.4 (15)	−1.0 (4)		−2.8 (46)	−0.9 (14)	
$\frac{4462}{4495}$....	+1.2 (25)	+0.1 (33)		+1.7 (51)	+0.2 (45)	
$\frac{4415}{4535}$....	−2.3 (27)	0.0 (40)		−1.5 (52)	0.0 (45)	
$\frac{4325}{4326}$....	−4.0 (21)	+0.1 (46)		−5.1 (27)	+0.2 (37)	

of the stars of small absolute magnitude, and that the values for the pairs used for classification purposes will be small for stars of both small and large absolute magnitude. The most prominent cases of lines where systematic differences are seen to exist between the stars of high and of low luminosity are the following:

STARS OF HIGH LUMINOSITY

Strong	Weak
	4325 Sc
4216 Sr	4435 Ca
4395 Ti, V, Zr	4456 Ca
4408 V, Fe	4535 Ti

The Sr line at λ 4216 is an extremely prominent chromospheric line, and the same is true in less degree of the enhanced Ti line at λ 4395. The line at λ 4408 is a blend, and as given by Rowland consists of V and Fe. Some other element may perhaps contribute to the stellar line. All four of the lines which are relatively weak in the high luminosity stars are well known sun-spot lines, being greatly strengthened in the umbrae of spots.

The following five pairs of lines were selected from Table III as the basis for an investigation of the individual stars:

$$\frac{4216}{4250} \qquad \frac{4395}{4415} \qquad \frac{4408}{4415} \qquad \frac{4456}{4462} \qquad \frac{4456}{4495}$$

The results given in Table III, estimated value—normal value, for these five pairs of lines were combined into means. By assuming a linear relationship between these mean values D, and the absolute magnitude M, we then derived the formulae:

F8–G6 stars: $M=+5.6-1.6\,D$
G6–K9 stars: $M=+6.8-1.8\,D$

The difference between the two constant terms shows merely that the average magnitude of the stars used for the normal curve is 5.6 for the first group, and 6.8 for the second group. The agreement for the two groups of the coefficient of D indicates how well the same relationship holds throughout the whole range of spectral type from F8 to K9. For the very faintest stars, below absolute magnitude 7, the linear relationship does not seem to hold strictly, but it has not seemed desirable for the present material to use a more complicated formula.

Tables IV and V show the absolute magnitudes computed from these formulae for 71 stars of types F8 to G6, and 91 stars of types G6 to K9. The spectral classification is that derived by the method already described and the parallax π is taken from *Groningen Publication*, No. 24. The first column of absolute magnitudes M contains the values calculated from the parallax or the proper motion, the latter being used wherever the measured parallax is less than $+0\overset{''}{.}05$. The second column of absolute magnitudes contains the values determined from the intensities of the spectrum lines.

The average difference between the two sets of absolute magnitudes is slightly less than 1.6 magnitudes for the F8–G6 stars, and 1.5 magnitudes for the G6–K9 stars. In view of the uncertainties attaching to the determination of absolute magnitudes from proper motions, this difference is not excessive. There appears, therefore, to be considerable promise in the application of spectrum line criteria to the determination of absolute magnitudes and parallaxes.

SUMMARY

Including the results described here, we have found as a product of our investigations of the spectra of large and of small proper motion stars three phenomena which appear to have a distinct bearing upon the problem of the determination of the absolute magnitudes of stars.

1. The continuous spectrum of the small proper motion stars is decidedly less intense in the violet region relative to the red than the spectrum of the nearer and smaller stars. This effect appears to be a function of the spectral type, and so must be ascribed in part, at least, to conditions in the stellar atmospheres.

2. A considerable number of the small proper motion stars show hydrogen lines of abnormally great intensity. Measures of the radial velocity show the source of the additional absorption to be mainly, if not wholly, in the stars themselves.

3. Certain lines are strong in the spectra of the small proper motion stars, and others in the spectra of the large proper motion stars. The use of the relative intensities of these lines gives results

TABLE IV

F8–G6

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Brad. 3212	6.2	G6	0″.419	+0″.15	+7	+7
Boss 9	6.3	G4	0.006	....	−3	−3
Boss 41	5.8	G2	0.002	....	−6	−1
Pi. $0^{h}130$	5.7	G5	1.390	+ .36	+8	+6
Pi. $0^{h}137$	7.4	G1	0.790	+ .02	+7	+6
Boss 138	5.4	G6	0.007	....	−4	−2
μ Cassiop.	5.3	G3	3.760	+ .11	+6	+10
Lal. 2450	8.1	G0	0.570	+ .02	+7	+8
Boss 349	5.5	G5	0.008	....	−4	−2
Boss 355	5.7	G6	0.004	....	−5	−5
Fed. 263	7.6	G4	0.750	+ .04	+7	+9
Lal. 3922–3	7.5	G0	0.510	− .04	+6	+4
Lal. 4141	6.9	G4	0.600	+ .03	+6	+6
Boss 619	5.6	F9	0.004	....	−5	−1
Lal. 5490–6	6.7	G3	1.000	+ .06	+5	+6
Boss 977	5.6	G6	0.020	....	−2	−6
Groom. 864	7.3	G0	0.730	+ .03	+6	+8
Boss 1131	6.8	G1	0.440	....	+5	+7
Groom. 884	7.1	F9	0.680	+ .09	+7	+7
Lal. 9091	7.0	F8	0.100	− .02	+3	+6
α Aurigae	0.2	G1	0.437	+ .07	−1	+1
λ Aurigae	4.8	G1	0.843	+0.11	+5	+7
Boss 1394	6.4	G5	0.002	....	−5	−1
Boss 1441	5.9	G4	0.014	....	−2	−1
Lal. 11196	6.5	G2	0.690	+0.06	+5	+5
Boss 1513	5.8	G5	0.007	....	−4	−4
Boss 1704	6.5	G1	0.017	....	−1	−1
W.B. $6^{h}1500$	7.7	G3	0.570	+ .01	+6	+7
Lal. 13849	6.5	G6	0.520	+ .11	+7	+6
Boss 1873	6.5	G4	0.019	....	−1	−2
28 Hev. Cam.	6.5	G6	0.478	.00	+5	+5
Lal. 15565	6.9	G7	1.190	+ .09	+7	+7
Boss 2150	5.5	G3	0.006	....	−4	−1
Pi. $7^{h}321$	7.0	G3	0.807	+ .05	+5	+6
Boss 2236	6.3	G1	0.362	....	+4	+7
Lal. 16904	8.1	G3	0.440	− .04	+6	+6
Boss 2338	6.1	G3	0.020	....	−1	−1
Lal. 19896	7.7	G0	0.470	− .05	+6	+5
Pi. $11^{h}32$	7.3	G6	0.808	+ .01	+7	+8
Groom. 1822	7.9	G0	0.670	+ .02	+7	+2
Lal. 22585	6.4	G6	0.470	+ .12	+7	+4
Groom. 1855	7.4	G6	0.340	+ .06	+6	+8
W.B. $12^{h}69$	7.3	G4	0.740	.00	+6	+5
Lal. 22908	7.5	G3	0.590	+ .09	+7	+6
Lal. 24414–6	6.9	G3	0.720	+ .01	+6	+5
Boss 3442	6.3	G6	0.008	....	−3	−1
Pi. $13^{h}114$	7.5	G6	0.940	+ .05	+6	+7
Lal. 27742 Br.	6.8	G4	0.680	.00	+6	+4
Lal. 27742 Ft.	7.6	G6	0.680	.00	+7	+7
Lal. 30694	6.8	G6	1.660	+ .07	+6	+7
Lal. 31132	6.7	G5	0.830	+0.14	+7	+7
Boss 4601	5.9	G6	0.009	....	−3	−2

TABLE IV—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 4763	6.2	G5	0″010	....	−3	−2
Groom. 2789N	6.6	G3	0.654	+0″02	+6	+7
Groom. 2789S	6.8	G2	0.630	+ .02	+6	+6
Boss 4910	6.3	G5	0.014	....	−2	+1
16 Cygni Pr.	6.3	G2	0.212	+ .16	+7	+9
16 Cygni Fol.	6.4	G2	0.220	+ .16	+7	+3
Lal. 38287	7.2	G6	0.600	+ .09	+7	+8
Pi. 20^h174	5.8	G2	0.317	+ .04	+3	+4
Groom. 3215	7.0	G6	0.480	+ .01	+5	+6
Fed. 3638	7.8	G4	0.700	− .02	+7	+6
Boss 5420	6.2	G6	0.012	....	−2	−1
Lac. 8777	6.5	G5	0.680	− .10	+6	+4
Lal. 43492	6.9	F9	0.840	+ .02	+6	+8
Boss 5769	6.1	G5	0.006	....	−4	0
Fed. 4371	7.5	G3	0.600	+ .06	+6	+8
Boss 5982	6.5	G0	0.015	....	−1	−1
B.D. 62°2244	7.3	G4	0.440	− .02	+6	+4
Pi. 23^h164	7.0	F8	0.620	− .05	+6	+7
Pi. 23^h267	6.2	G0	0.770	+0.01	+5	+1

for absolute magnitudes in satisfactory agreement with those derived from parallaxes and proper motions.

It seems very probable from physical considerations that the spectra of stars of quite different mass and size would differ considerably in certain respects even when the main spectral characteristics were the same. If the depth of the atmosphere for stars of similar spectral type is at all in proportion to the linear dimensions of the stars, we should expect the deeper reversing layers of the larger stars to produce certain modifications of the spectrum lines. Owing to the small scale of the stellar spectrum photographs, only the most marked changes could be distinguished, and among these the effect of the deep atmosphere upon the violet end of the spectrum should be especially prominent.

A case of somewhat similar nature is that found in observations of the center and the limb of the sun. The length of path through the solar atmosphere is much greater at the limb, and greater relatively for the lower and lower strata. On large-scale solar photographs the differences between the center and the limb spectra are very marked, but on very small-scale photographs, no doubt, only the most prominent differences could be observed.

TABLE V

G6–K9

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Boss 131	5.7	K0	0″.019	....	−2	−2
54 Piscium	6.1	K1	0.595	+0″.14	+7	+5
Lal. 1045	7.5	K3	0.810	+ .02	+7	+8
Boss 165	6.2	G6	0.016	....	−2	−3
Mayer 20	5.8	K1	1.366	+ .17	+7	+10
Boss 209	5.6	G6	0.017	....	−2	−6
Lal. 1799	8.0	K6	0.480	+ .06	+7	+6
Boss 267	6.2	K1	0.004	....	−4	−4
W.B. 1^h161	8.0	G8	0.490	− .01	+6	+8
Lal. 2682	7.8	K1	0.470	+ .01	+6	+5
Lal. 3022	7.8	G7	0.500	+ .05	+6	+9
Boss 410	5.9	G9	0.018	....	−2	−3
Boss 420	5.6	K1	0.009	....	−3	−4
Boss 430	6.1	K1	0.005	....	−4	−3
Boss 432	5.8	K0	0.180	....	+2	−1
Boss 434	6.0	G8	0.011	....	−3	−2
α Arietis	2.2	G8	0.239	+ .09	+2	−2
Boss 493	6.2	K4	0.011	....	−2	−2
Boss 539	5.6	G7	0.006	....	−4	−3
Boss 581	5.9	G9	0.010	....	−3	−3
Pi. 2^h123	5.9	K6	2.320	+ .14	+7	+7
W.B. 2^h927	8.2	G8	0.680	+ .10	+8	+8
W.B. 3^h113	7.8	K0	0.620	+ .08	+7	+1
Boss 768	5.6	G8	0.020	....	−2	−4
Boss 832	6.0	G7	0.002	....	−6	0
γ Tauri	3.9	G9	0.120	....	0	−1
Boss 1014	6.1	G7	0.011	....	−2	−2
ε Tauri	3.6	K0	0.120	....	−1	0
θ Tauri	4.0	G8	0.108	....	+1	+1
α Tauri	1.1	K3	0.203	+ .07	0	−1
Boss 1176	6.4	K2	0.012	....	−2	−2
W.B. 4^h1189	6.5	K9	1.250	+ .30	+9	+5
Boss 1247	5.8	K1	0.017	....	−2	−2
Boss 1348	6.0	K0	0.007	....	−4	−4
Groom. 990	8.1	K0	0.560	+ .03	+7	+11
Pi. 5^h146	6.4	K0	0.510	+ .12	+7	+7
Lal. 10797–8	7.3	K2	0.720	+ .08	+7	+7
Boss 1444	5.6	G8	0.009	....	−3	−4
Boss 1608	5.5	K2	0.011	....	−3	−2
6 Lyncis	5.9	G7	0.330	− .01	+4	+4
Boss 1632	6.0	K2	0.007	....	−4	−2
Boss 1643	5.9	K6	0.019	....	−2	−3
Lal. 13284–5	6.9	K9	0.570	+ .11	+7	+6
Lal. 13427	8.2	K1	0.710	+ .05	+7	+7
Boss 1846	5.7	K0	0.019	....	−2	−4
Boss 1868	5.9	K2	0.020	....	−2	−2
Boss 2148	6.4	K2	0.073	− .04	+1	−2
Boss 2220	6.2	G8	0.011	....	−2	−1
Pi. 8^h105	6.0	K3	0.040	−0.03	0	−1
55 ρ^1 Cancri	6.1	K0	0.541	+ .08	+6	+2
Boss 2449	5.2	G6	0.008	....	−4	+1
Boss 2455	6.1	G6	0.010	....	−3	−2

TABLE V—*Continued*

Star	m	Type	μ	π	M from π or μ	M from Spectrum
Lal. 18286	7.3	K3	0″.520	+0″.07	+6	+ 8
Lal. 19022	8.2	K5	0.800	+ .06	+7	+ 6
Groom. 1596	8.2	G6	0.480	+ .07	+7	+ 6
Boss 2795	6.0	G7	0.016	....	−2	− 2
Boss 2910	5.2	K2	0.113	+ .02	+1	− 2
83 Leonis Br.	6.2	G8	0.743	+ .02	+6	+ 6
83 Leonis Ft.	7.6	K8	0.736	+ .02	+7	+ 5
Boss 3125	5.9	K1	0.012	....	−3	− 2
Pi. $11^{\rm h}218$	6.8	G6	0.670	− .03	+6	+ 2
Boss 3406	6.0	G9	0.020	....	−2	− 3
Lal. 26196	7.6	K1	0.680	+ .14	+8	+ 7
Boss 3810	5.6	G9	0.002	....	−6	− 2
Lal. 27744	6.7	G7	1.380	+ .13	+7	+ 8
OΣ 298	7.9	K6	0.480	+ .05	+6	+ 7
W.B. $15^{\rm h}720$	6.8	K1	0.480	+ .05	+5	+ 6
Lal. 30024–6	7.0	K1	0.490	+ .09	+7	+ 9
Boss 4228	6.0	G8	0.020	....	−2	− 6
Lal. 30699	7.8	G7	0.470	+ .05	+6	+ 4
Lal. 33439	6.7	K2	0.650	+ .04	+6	+ 8
Boss 4724	5.8	G8	0.006	....	−4	0
Groom. 2875	6.7	K1	0.660	− .05	+6	+ 8
Lal. 38383	7.2	K1	1.390	+ .03	+8	+10
Pi. $20^{\rm h}23$	7.3	G9	0.550	+ .08	+7	+ 9
Boss 5177	5.8	G8	0.020	....	−2	− 3
Pi. $20^{\rm h}29$	5.7	G9	1.256	+ .01	+6	+ 6
Boss 5317	6.2	K1	0.012	....	−2	− 2
η Cephei	3.6	K0	0.836	+ .10	+4	+ 3
Boss 5397	5.6	K4	0.005	....	−4	− 4
Boss 5486	6.2	K0	0.020	....	−1	− 2
Boss 5655	5.8	K1	0.006	....	−4	− 2
Groom. 3689	8.1	G8	0.610	+ .04	+7	+ 6
Boss 5868	6.1	K4	0.016	....	−2	− 4
Pi. $22^{\rm h}214$	6.5	G9	0.470	− .02	+5	+ 3
Lal. 45028	7.8	K2	0.500	+ .05	+6	+ 7
Brad. 3077	5.6	K7	2.105	+ .16	+7	+ 5
Lal. 45755	7.6	G8	0.670	+ .04	+7	+10
Boss 6016	5.9	G8	0.300	....	−5	− 3
B.D. +58°2605	7.5	K1	1.080	+0.07	+7	+ 8
Boss 6123	5.8	K0	0.020	....	−2	− 2
Boss 6176	5.8	G7	0.018	....	−2	− 2

The difference, however, in the relative intensity of the violet portion of the continuous spectrum at center and limb as compared with the red portion, which is so marked a feature of the observations, would appear equally well on photographs taken with high and low dispersion.

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THE SPECTROSCOPIC ORBIT OF RX HERCULIS DETERMINED FROM THREE PLATES WITH A NEW PHOTOMETRIC ORBIT AND ABSOLUTE DIMENSIONS

By HARLOW SHAPLEY

The ordinary solution for the orbit of a spectroscopic binary involves six independent unknowns. These elements may be the period, an epoch, the longitude of periastron, the eccentricity of the orbit, the maximum apparent orbital velocity, and the radial velocity of the system with respect to the sun. Frequently the number of unknowns is reduced to four by the assumption of a circular orbit.

The complete solution for the elements of an eclipsing binary from its light-curve may involve as many as thirteen independent unknowns, but the precision of the photometric observations is seldom sufficient to permit the derivation of more than seven or eight of the quantities theoretically possible. The elements most generally obtained are the period, an epoch, the inclination, the relative light-emissions and dimensions of the two stars, the radius of the orbit, and occasionally the eccentricity and longitude of periastron.[1]

Solutions for orbits of double stars based upon variations in radial velocity, therefore, give four elements in common with solutions based on variations in apparent magnitude, and it is obvious that for any star which is both an eclipsing and a spectroscopic binary the existence of either solution greatly simplifies the derivation of the other, both from an observational and from a computational point of view. There is accordingly a particular interest and advantage in making photometric observations of certain spectroscopic binaries whose orbits are known,[1] and likewise in making spectroscopic observations of eclipsing binaries for which good light-curves exist or are possible; for not only is the problem of the second orbit in either case much simplified, but also the combination of the two sets of computed data affords knowledge of the absolute radii, masses, and densities of the component stars, and occasionally of the surface luminosities in terms of the sun.

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[1] For some systems it is possible to determine also the ellipticity of the stars, the reflection effect, and the periastron effect, and to estimate the degree of darkening at the limb (*Astrophysical Journal*, **39**, 405, 1914). The apparent stellar magnitude at maximum is a thirteenth unknown, but except in systems of elliptical stars its determination is usually independent of the study of the light-curve.

Suppose the complete solution of a light-curve has given, among other quantities, four of the six unknowns generally derived from spectroscopic observations: P, the period; T, the epoch of principal minimum (or of node or periastron passage); e, the eccentricity; and ω, the longitude of periastron counted from the ascending node of the primary component. Then each determination of the radial velocity of the brighter star gives the linear relation

$$V_1=\gamma+K_1[\cos u+e\cos\omega] \tag{1}$$

between the two remaining unknowns—γ, the constant radial velocity of the center of mass of the system, and K_1, the semi-amplitude of the velocity variation. (u, the argument of the latitude, is readily obtained from tables when P, e, and ω are known.) If the lines in the spectrum of the second component are measurable, then

$$V_2=\gamma+K_2\,[\cos u+e\cos(\omega+\pi)] \tag{2}$$

and in both cases the last term in the brackets drops out for orbits circular or nearly so. We have then simply

$$V=\gamma+K\sin\theta \tag{3}$$

where θ is the phase angle from minimum. It is evident that a very few spectrograms made near greatest elongation, giving a large coefficient of K, will suffice, whether the orbit is circular or eccentric, to obtain γ and K with average precision, hence to determine completely the spectroscopic orbit. The following discussion shows what use we can make of the measures of a few lines on three plates of RX Herculis, an eclipsing binary the solution of whose light-curve eliminates four unknowns from the spectroscopic problem.

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[1] See discussion of this phase of the combination by Stebbins, *Astrophysical Journal*, 34, 105, 1911.

THE PHOTOMETRIC OBSERVATIONS

A new light-curve of RX Herculis is derived below from two series of observations, one made at Harvard and one at Princeton, neither of which has as yet been published. A long series of visual estimates of this star was obtained by Luizet during the years 1899–1904.[1] Especial care had been taken in making the observations, with the result that the light-elements derived from them are very accurate. The light-scale is not photometric, however, and it is not advisable to put great reliance upon the range of variation or upon the shape of the curve. The period given by Luizet is $0^{d}.889288$, and the range of variation is $0^{m}.63$, with the maximum at the seventh magnitude, approximately. The maximum light had been observed more than 300 times and there was no evidence of ellipticity or of a secondary minimum.[2]

The preliminary solution of Luizet's light-curve by the present writer was indeterminate. It was found that either the period must be double the given value with primary and secondary eclipses indistinguishable, or the period must be as given and the system supposed to consist of a small bright star with a faintly luminous, larger companion. The shape of the light-curve as derived from Luizet's measures contradicted the first supposition and the absence of a shallow secondary minimum contradicted the second. As the star is bright enough for spectroscopic work, it was considered of sufficient importance to justify a thorough photometric study, and accordingly it was put on the list of stars to be observed by the writer with the polarizing photometer of the Princeton University Observatory. The observations, which are published in detail elsewhere, are discussed briefly in the following paragraphs.

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[1] The observations and light-curve are published in *Astronomische Nachrichten*, **168**, 283, 1905, and *Bulletin astronomique*, **22**, 232, 1905; see also *Astronomische Nachrichten*, **165**, 183, 1904; *Astronomical Journal*, **22**, 162, 1902; *Popular Astronomy*, **21**, 142, 1913.

[2] In conspicuous disagreement are the results deduced by Yendell from visual estimates, *Astronomical Journal*, **22**, 162, 1902.

The comparison star used in observing RX Herculis is a=B.D. +12°3546 ($7^{m}.7$). The observations were unusually difficult on account of the brightness of the two stars, and because many of the measures were made when the field was very low. Of the total of 133 sets (each of 16 comparisons) 63 were made during changing light. The 70 sets outside of eclipse, in agreement with Luizet's results, show no deviation from constant light. Hence the shorter period, which demands a secondary at the phase of 0.445 days, becomes doubtful. Moreover, an examination of the observations during changing light shows that the branches are not so steep as measured by Luizet and, therefore, that the double-period solution for the orbit is now suitable.

An investigation of the depths of alternate minima did not reveal definitely, so far as my observations were concerned, any difference larger than may be accounted for by errors of observation, but the data bearing on the point were rather weak. There was an indication that the minima at even epochs were slightly deeper, but it was not until the Harvard observations were examined that the difference was established beyond doubt. Consequently the photometric orbit already published is based upon an assumption of eclipses equally deep.[1] The existence of unequal minima shows at once that only one-half a revolution period elapses between successive eclipses. A third point in favor of the double value of the period is the spectroscopic evidence obtained at the Yerkes Observatory. The lines due to both components are visible on the plates made in 1905.[2] The recent measurement of these plates has furnished the data upon which the present spectroscopic orbit is based. They will be discussed on a later page.

A new mean epoch of heliocentric minimum, derived from the curve of the several partially observed eclipses, is J.D. 2419658.5882, G.M.T. This time is eight minutes in advance of that predicted and, assuming that no secondary oscillations exist, furnishes a small correction to Luizet's period. The new light-elements are:

$$\text{Primary Minimum} = \text{J.D. } 2419658.5882 + 1^{d}.7785740\ E.$$

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[1] *Astrophysical Journal*, **38**, 163, 1913.

[2] *Ibid.*, **22**, 215, 1905.

The weighted mean of the maximum light measures is $a-v=0^m.877 \pm 0^m.004$, the probable error of a single observation being $\pm 0^m.033$.

Through the kindness of Professor Pickering the proof sheets of a series of 127 observations of RX Herculis, made at Harvard by Professor Wendell during the years 1906–1910, have been furnished me in advance of publication in *Harvard Annals*, **69**, Part II. The measures were made with a polarizing photometer identical with the one used at Princeton, and as each set consists also of sixteen comparisons, the combination with equal weight of the observations of the two series is entirely justifiable. The maximum light, according to 23 observations by Wendell, is $7^m.07 \pm 0^m.006$, and the probable error of a single observation is slightly less than $\pm 0^m.03$. For combination with the Harvard results all the Princeton magnitudes were reduced so that the maximum magnitude should have the above value.

The preliminary plot of Wendell's observations confirmed the correction to Luizet's light-elements, but in addition showed an outstanding displacement of the minimum. This latter difficulty was found traceable to the chance adoption without alteration of the originally published initial epoch, which was referred to the Paris meridian, while the observation times are all referred to Greenwich. Two positive corrections have been applied, therefore, to the phases published at Harvard: one of ten minutes to reduce to Greenwich time, and the other increasing with the time from four to seven minutes to allow for the revision of the period.

With these adjustments the two sets of observations are found to be in extremely good agreement. The difference in the depths of successive minima of nearly a tenth of a magnitude is shown distinctly. In primary minimum there are 38 observations from Harvard and 41 from Princeton; in secondary, 66 observations from Harvard and 25 from Princeton. In the following tables of normal magnitudes the observations in the two minima are collected in order of phase into groups of five.[1] The phases refer to the middle epochs of the two minima. The third and fourth columns give the number of observations contributed from Harvard and Princeton, respectively, to each normal. The last two columns contain the residuals from theoretical light-curves computed on the two hypotheses of uniformly luminous disks and of disks completely darkened at the edge. In Fig. 1 are given the normal points and the computed uniform light-curves, with the primary minimum at the top of the diagram.

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[1] Half-weight is given to two Princeton observations made under very unfavorable conditions. The phase of one Harvard observation is arbitrarily altered because of the evident impossibility of the recorded time of observation.

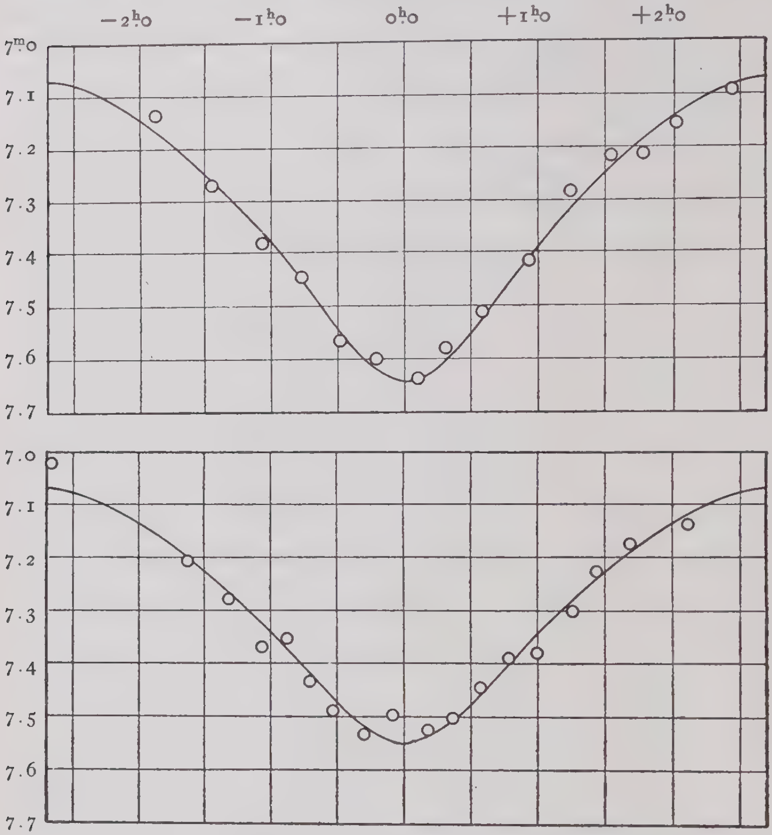

Fig. 1.—The mean light-curve of RX Herculis; primary minimum above, secondary minimum below.

THE SPECTROSCOPIC OBSERVATIONS

The total available spectroscopic data for RX Herculis consist of four spectrograms, made with the Bruce spectrograph at the

Yerkes Observatory. Because of the faintness of the star and the nature of its spectrum, the investigation of its orbit has not hitherto

TABLE I

NORMAL MAGNITUDES OF RX HERCULIS NEAR PRIMARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	$-1^{h}53^{m}.0$	3	2	7.136	$-0^{m}.03$	$-0^{m}.01$
2	−1 27.1	0	5	7.266	+ .01	+ .02
3	−1 4.6	2	3	7.380	+ .01	+ .01
4	−0 47.4	2	3	7.442	− .01	− .02
5	−0 29.8	3	2	7.564	+ .02	+ .01
6	−0 13.2	2	3	7.600	− .02	.00
7	+0 5.0	3	2	7.636	.00	+ .02
8	+0 18.0	2	3	7.578	− .03	− .02
9	+0 34.8	2	3	7.512	− .01	− .02
10	+0 55.4	3	2	7.414	.00	.00
11	+1 14.4	3	2	7.284	− .03	− .04
12	+1 32.0	4	1	7.216	− .03	− .01
13	+1 46.8	3	2	7.212	+ .03	+ .04
14	+2 0.6	2	3	7.154	+ .01	+ .03
15	+2 26.2	3	2	7.096	+ .01	+ .02
16	+2 58.5	1	3	7.058	−0.03	−0.03

TABLE II

NORMAL MAGNITUDES OF RX HERCULIS NEAR SECONDARY MINIMUM

No.	Phase	No. Observations		Magnitude	O−C	
		Harvard	Princeton		Uniform	Darkened
1	$-2^{h}40^{m}.0$	0	5	7.022	$-0^{m}.05$	$-0^{m}.05$
2	−1 37.8	3	2	7.206	+ .01	+ .03
3	−1 18.4	4	1	7.280	+ .01	+ .02
4	−1 3.8	4	1	7.370	+ .04	+ .03
5	−0 53.6	5	0	7.352	− .02	− .03
6	−0 42.8	3	2	7.436	+ .02	+ .01
7	−0 31.8	4	1	7.490	+ .02	+ .01
8	−0 18.0	4	1	7.536	+ .02	.00
9	−0 5.6	4	1	7.498	− .05	− .06
10	+0 10.8	4	1	7.522	− .01	− .03
11	+0 21.8	4	1	7.502	.00	− .02
12	+0 34.4	3	2	7.444	− .02	− .03
13	+0 47.2	4	1	7.394	− .01	− .02
14	+0 59.8	3	2	7.380	+ .03	+ .03
15	+1 14.8	4	1	7.302	+ .02	+ .03
16	+1 26.6	3	2	7.226	− .01	.00
17	+1 41.2	4	1	7.174	− .01	.00
18	+2 6.3	6	0	7.138	+0.02	+0.03

been attempted, and only a short note has been published in general description of the spectral lines.[1] At my request Professor Frost very kindly consented to measure the plates, as far as might be possible, and has recently communicated to me his results. Upon one of the plates the lines were too indefinite to permit a quantitative estimate of the velocity. For the other three the results are as tabulated in Table III. Concerning these measures Professor Frost writes:

> For the first two of the plates the results are clear, and have some fair degree of reliability, but I would not affirm that they were nearer than 10 km of the truth. . . . . I have re-examined the plates with a view of distinguishing between the components in intensity. The slightly less intensity of the broader component makes estimation difficult. On Plate 558 at λ 4481, broader component (toward red) showed slightly fainter than narrower component. The same remark refers to Plate 562. On Plate 839 the width and intensity of the components at λ 4481 were thought to be equal and the same for the other lines on that plate.

TABLE III

Plate No.	Date	G.M.T.	Primary Component		Secondary Component	
			Velocity	No. Lines	Velocity	No. Lines
558......	1905, July 16	16^h28^m	+ 84 km	2	−130 km	2
562......	1905, July 21	17 1	+ 42	3	− 74	4
839......	1906, Sept. 8	15 25	−108	3	+ 75	2

On plate 839 the measure of one component of the line at λ 4481 gave a velocity of +188 km. The note is added:

> I cannot well account for the discrepancies in the positive values, but the very large positive displacement at λ 4481 is evident. The two other positive lines gave values of +69 and +82 km.

As this large positive value is in glaring disagreement with all the other measures on all the plates, there is nothing to do but neglect it completely.

THE PHOTOMETRIC ORBIT

Although a fairly good photometric orbit of RX Herculis is already available,[2] it was thought best to revise the solution because of the additional photometric material from Harvard. The computations have followed the usual lines,[1] except that in certain details the spectroscopic data have influenced the choice between possible, ambiguous interpretations of the light-variations. The revised light-curve has shown definitely that the minimum at the even epoch is the deeper. The star approaching after that eclipse, according to the estimates of Professor Frost, is probably somewhat the fainter. At first glance there may appear to be a contradiction here. In general at the deeper minimum the star of greater surface intensity is eclipsed,[2] and almost invariably in practice the component with greater surface brightness has the greater total brightness.[3] If, however, the more intense star is small enough relative to its companion, it may have less than half the light and accordingly show fainter spectral lines. It is found from a study of the light-curve that RX Herculis is one of these rare exceptions and that the condition—fainter component smaller and with greater surface intensity—is not only possible but is required by the most probable solution.

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[1] *Astrophysical Journal*, **22**, 215, 1905.

[2] *Ibid.*, **38**, 158, 1913.

Let us consider first the orbit from the standpoint of uniformly luminous disks. The range of variation at primary minimum is $0^{m}.57$ and at secondary $0^{m}.48$, corresponding to losses of light in terms of the total of $1-\lambda_p=0.408$ and $1-\lambda_s=0.357$, respectively.[4] The solution of the light-curve at the two minima gave $\chi\,(k, \alpha_0, \frac{1}{4}) = 2.01$, with an uncertainty of ± 0.01. With the above value of the losses of light this allows an indeterminateness of small range in the values of k and α_0 (the ratio of radii and fraction of eclipse) when computed from the relation $\alpha_0 = 1-\lambda_1+\frac{1-\lambda_2}{k^2}$. If we assume that the large star is in front at primary minimum, k must lie between unity and 0.87; if we assume the small star to be in front at primary minimum, k must lie between unity and 0.98 or between 0.87 and 0.80, and for the last mentioned value the secondary minimum would be grazing total. The foregoing values of k are all possible without misrepresenting the curve; the best value ,however, appears to be $k=0.90$ (small star eclipsed at primary) to which corresponds $\alpha_0=0.85$ and $\chi(k, \alpha_0, \frac{1}{4})=2.010$.

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[1] A detailed discussion of the method of solving for the orbital elements on the basis of two partial eclipses is to be given in *Contributions from the Princeton University Observatory*, No. 3, and an application is there made to the light-curve of RX Herculis.

[2] This does not necessarily hold if the orbit is eccentric and the inclination differs from 90°. For RX Herculis, however, we assume a circular orbit; later this assumption is proved.

[3] There are only three or four exceptions out of a hundred eclipsing binaries.

[4] The notation and formulae used in the present paper are essentially the same as those in former discussions of eclipsing binaries published in the *Astrophysical Journal* during the last two years.

Suppose that from among the several possible values of the ratio of radii we choose $k=1.00$; then $\alpha_0=1-\lambda_p+1-\lambda_s=0.765$. The light of the star eclipsed at primary is then 53.3 per cent of the total, and the component approaching after primary eclipse would be fifteen-hundredths of a magnitude brighter than its companion, rather than fainter as indicated by the spectrograms. Obviously, then, the star in front at the primary eclipse must be the larger. If $k=r_2/r_1=0.936$, the components would be just equal in brightness. For the best values of the last paragraph ($k=0.90$, $\alpha_0=0.85$), however, the condition is attained that is in complete accordance with the light-curve and the spectrograms, for then the light of the star eclipsed at primary minimum is 48 per cent of the total; that is, the more intense surface, being the smaller, is approximately a tenth of a magnitude fainter. It is possible also that the difference in breadth of the spectral lines may be attributed to the inequalities of the components.

The solution for the elements on the darkened hypothesis gives results differing but little from those derived on the foregoing assumption. The computations have been much more difficult, however, because for unequal stars the theoretical light-curves at the two minima differ considerably from the uniform, and perceptibly from each other. The ranges of variation at the two minima were altered for this reason to $0^m.55$ and $0^m.49$, respectively. There was again a small indeterminateness in the solution for k and α_0', and therefore the same division of light was assumed as was derived before. For $L_2=0.48$, $L_1=0.52$, the ratio of the stars by chance came out the same. Their dimensions also are but little changed from the uniform values, chiefly because the computed duration of darkened eclipse is only three minutes longer than for the uniform orbit. There is much less uncertainty in the photometric orbit of this star because of the indecision between the darkened and uniform hypotheses than is usually the case.

Light-curves have been computed for the two eclipses in both cases and the residuals from them are given in the last two columns of Tables I and II. The average deviation during changing light is $\pm 0^{\rm m}.018$ for the uniform solution and $\pm 0^{\rm m}.020$ for the darkened. The residuals are slightly smaller in primary than in secondary minimum. The representation of the observations in the minima is approximately of the same accuracy as during maximum light, and no attempt further to improve the accordance of theory and observation is justified. A shift of the epoch of secondary by a minute or two would reduce the average deviation slightly and would break up the apparently systematic nature of the run of residuals, but the adjustment would be within the possible errors of the observations and would not affect the computed elements.

TABLE IV

ELEMENTS OF THE PHOTOMETRIC ORBIT OF RX HERCULIS

	Uniform	Darkened
Ratio of radii, k	0.90	0.90
Fraction of eclipse, α_0, α_0',	0.85	0.828
$p(k, \alpha_0)$, $p(k, \alpha_0')$	−0.728	−0.691
$\sin^2 \theta'$	0.1411	0.1476
Semi-duration of eclipse, t'	$2^{\rm h}37^{\rm m}$	$2^{\rm h}40^{\rm m}$
Radius of brighter star, r_b	0.201	0.206
Radius of fainter star, r_f	0.181	0.185
Least apparent distance of centers, $\cos i$	0.069	0.078
Inclination of orbit plane, i	$86^\circ.0$	$85^\circ.5$
Light of brighter star, L_b	0.52	(0.52)
Light of fainter star, L_f	0.48	(0.48)
Ratio of surface intensities, $\frac{J_f}{J_b}$	1.14	1.14
"Equal-mass" density		
Bright star, $\bar{\rho}_b$	0.26	0.25
Faintstar, $\bar{\rho}_f$	0.36	0.34

The two sets of elements are given in Table IV. The epoch and period have been given on a preceding page. The ellipticity of the stars is negligible according to the maximum light-measures. The eccentricity of the orbit is discussed in the following paragraphs.

DISCUSSION OF ECCENTRICITY

In the solution for the elements of the photometric orbit the apparent equidistance, equal widths, and symmetry of the two minima have been accepted as sufficient proof that the orbit is circular. It may be of interest to see how great an uncertainty there is in this assumption. The two components of the eccentricity, $e \cos \omega$ and $e \sin \omega$, are determined independently of each other from the light-curve, the former with high precision from the displacement of the secondary minimum from the midpoint between the primaries, and the latter indirectly, and with less ease and precision, from the relative durations of the various phases of the two eclipses. Considering first the transverse component,

$$e \cos \omega = \frac{\pi}{P(1+\operatorname{cosec}^2 i)}\left(t_1 - t_2 - \frac{P}{2}\right) = 0.88 \times (\text{disp. in days}) \qquad (4)$$

Inspecting the light-curve, we find that the combined uncertainty in the epochs of the two minima cannot exceed five minutes and is probably less than three. A change of epoch by as much as five minutes would nearly double the average value of the residuals. Hence $e \cos \omega$ is certainly less than ± 0.003.

To determine strictly the uncertainty in assuming $e \sin \omega = 0$, we should make independent solutions of the light-curve of each minimum, using only the depth of the other to make the derivation of the elements complete. Then, with close approximation,

$$e \sin \omega = \frac{r_1'' - r_1'}{r_1'(1.33 + 0.67 \cos \theta')} \qquad (5)$$

where r_1' and r_1'' are the radii of the larger star derived from the solution of the primary and secondary minimum, respectively, and θ' is the phase angle of the end of eclipse. It is sufficient for the present purpose, however, to notice that when the inclination is 90° and $k=1$ (both of which conditions are approximately fulfilled in this and similar cases), the uncertainty in the radii is one-half that in θ'. Again referring to the light-curve, we find that the uncertainty in the adopted values for the semi-duration of eclipses is less than ± 4 minutes at the primary, and less than ± 5 minutes at the secondary; that is, since $t = P\theta/2\pi$, θ' must lie within

± 0.010 for one eclipse and ± 0.012 for the other of the values adopted. Hence $r_1' - r_1'' < \pm 0.011$, and since the radius of the larger star is approximately 0.20 we conclude from equation (5) that $e \sin \omega < \pm 0.03$. Combining this result with that obtained for $e \cos \omega$, the eccentricity is found to be not larger than 0.03. Its adopted value is zero and the probable error is estimated to be less than ± 0.02.

THE SPECTROSCOPIC ORBIT

The discussion up to this point has disposed of the elements P, T, e, and ω. It remains only to derive γ and K from the velocity variations by means of equation (3), and the spectroscopic orbit is complete. Using the light-elements given on a previous page, the phases of the middle of the exposure times have been computed. They are given in Table V, column 4, expressed in mean longitudes and referred to the preceding principal minimum. The primary star is taken to be the one eclipsed at secondary minimum.

TABLE V

SPECTROSCOPIC OBSERVATIONS OF RX HERCULIS

PLATE NO.	JULIAN DAY AND G.M.T.	EPOCH	PHASE	PRIMARY COMPONENT		SECONDARY COMPONENT	
				Observed Velocity	O−C	Observed Velocity	O−C
558......	2417043.686	−1473	279°.2	−130 km	−7 km	+ 84 km	−2 km
562......	2417048.709	−1468	215.8	− 74	+6.5	+ 42	−1.5
839......	2417462.642	−1235	120.0	+ 75	+1.5	−108	+2.5

Plotting the observed velocities against the phases, we have Fig. 2. A preliminary solution shows that it is possible to assume the components equal in mass and be well within the probable accuracy of the observations. Then $K_1 = K_2$ and we have six equations for the determination of γ and K. Their solution gives:

$$\gamma = -18.5 \pm 1.4 \text{ km}$$
$$K = 106 \pm 1.7 \text{ km}$$

The residuals from the velocities computed with these elements are given in Table V, and, so far as they go, give ± 3.5 km for the probable error of the mean of the measures on one component on one plate. This accuracy is even more than was hoped for, considering the faintness of the star and the nature of its spectrum. It is to be noted, however, that the plates were purposely made near times of greatest separation of the lines and troubles from blends were thereby avoided.

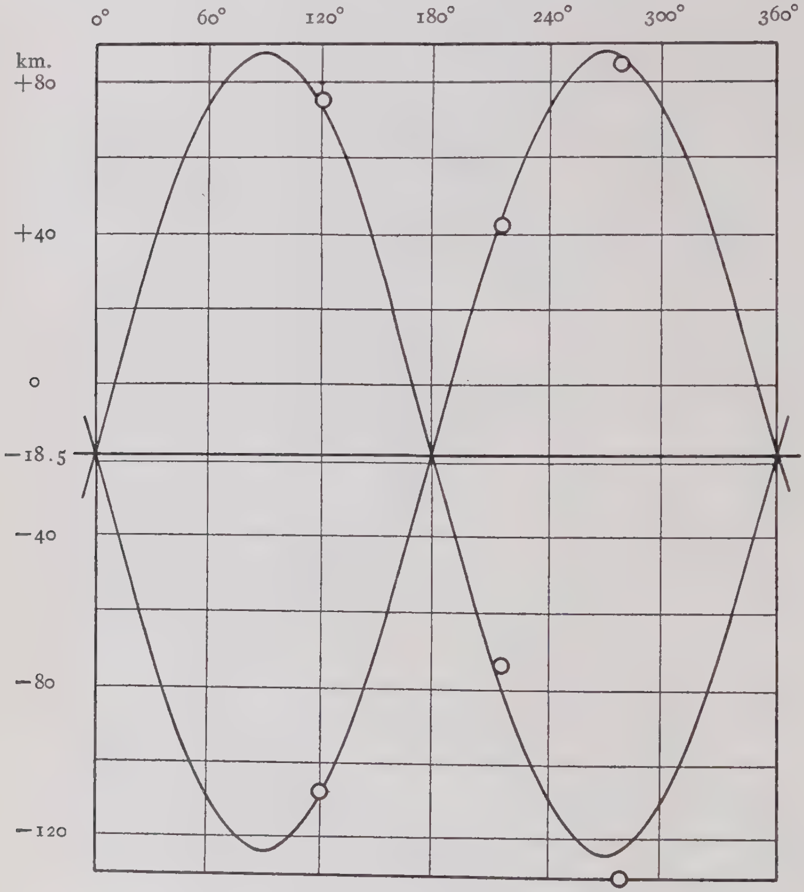

Fig. 2.—The velocity-curve of RX Herculis

It is evident from the velocity-curve that the residuals can still be reduced slightly by assuming the masses not equal. There is an indication that the secondary star, that is, the one with 48 per cent of the light but with the greater surface intensity, is a trifle the more massive. But as is well known, the fainter companions of all spectroscopic doubles have smaller relative masses. This is without exception, as far as the writer knows. The above condition, then, would be in disagreement with the general rule. But it is at the same time true that, in all cases where such information is available, the star of least surface brightness is the one of smaller mass (for small total light and small surface luminosity go together in general). That would be in agreement with the suggested difference in mass of the components of RX Herculis, but the point is of little importance in this case of nearly equal stars. The masses are without doubt sensibly equivalent and will be so considered.

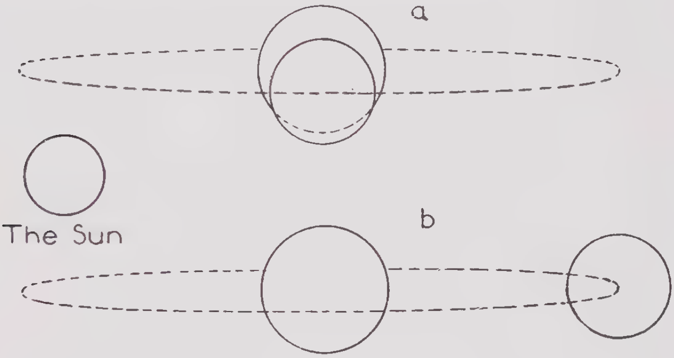

FIG. 3.—The system of RX Herculis compared with the sun
a, at secondary minimum
b, at greatest elongation

The following are the adopted elements of the spectroscopic orbits:

$P = 1.7785740$ days
$\mu = 202^\circ.4093$
$T =$ J.D. 2419658.5882 G.M.T. (primary minimum)
$e = 0.00 \pm 0.02$ (estimated)
$K_1 = K_2 = 106 \pm 1.7$ km
$\gamma = -18.5 \pm 1.4$ km
$a_1 \sin i = a_2 \sin i = 2{,}590{,}000$ km
$m_1 \sin^3 i = m_2 \sin^3 i = 0.88$

The probable errors of the apparent orbital radii and apparent masses are probably less than 2 per cent, but exact computation is not possible since the probable error of the adopted period has not been computed. The star is near the solar apex, and therefore the apparent motion of the center of mass of the system is evidently to be ascribed almost entirely to the motion of the sun. The mass of the system is smaller than we usually find for binaries of early spectral types.

ABSOLUTE DIMENSIONS

The spectroscopic and photometric elements may now be combined and we get the following comparison between RX Herculis and the sun (see Fig. 3):

	Uniform	Darkened
Distance of centers in solar radii	7.47	7.47
Radius of brighter star in solar radii	1.50	1.54
Radius of fainter star in solar radii	1.35	1.38
Apparent distance of centers at middle of eclipse in solar radii	0.52	0.58
Mass of each star	0.89☉	0.89☉
Density of brighter star	0.26☉	0.25☉
Density of fainter star	0.36☉	0.34☉

From the description of the spectrum of RX Herculis in *Astrophysical Journal*, **22**, 215, it should evidently be classed as B9p, the anomalous character being the simultaneous presence of λ 4481 and the faint helium lines. If we knew the relation between the luminosity of its surface and that of a G-type star, we could at once deduce its parallax, since we know the stellar magnitude and the relative surface area of the star with respect to the sun. Or, vice versa, if the parallax were measured directly we could find the relation between the surface luminosities.

Assuming various values of the relative brightness of unit surface areas and expressing this quantity in stellar magnitudes (taking the solar surface as the standard), the computation for the parallax has been made, using the data derived for the brighter component (magnitude 7.78), with the results given in the following table (p. 97):

Surface luminosity with respect to the sun	$-3^{\mathrm{m}}.0$	$-2^{\mathrm{m}}.0$	$-1^{\mathrm{m}}.0$	$0^{\mathrm{m}}.0$
Corresponding brightness per unit surface area	$15.8\odot$	$6.3\odot$	$2.5\odot$	$1.0\odot$
Absolute magnitude	$0^{\mathrm{m}}.82$	$1^{\mathrm{m}}.82$	$2^{\mathrm{m}}.82$	$3^{\mathrm{m}}.82$
Parallax	$0''.0041$	$0''.0064$	$0''.0102$	$0''.0162$
Total light of brighter star	$37.3\odot$	$14.9\odot$	$5.9\odot$	$2.4\odot$

The four assumptions with regard to the surface luminosity correspond roughly to what we judge from other sources to be the relative intensities for spectral types B, A, F, and G, respectively. We should expect, then, that the parallax of RX Herculis is of the order of $0''.006$ and that the total light-emission of the system is thirty times that of the sun.

Among the faint eclipsing binaries similar to RX Herculis are the following, for which in a corresponding manner spectroscopic material of considerable value might be obtained without great difficulty:

	Magnitude	Spectrum
U Ophiuchi	5.7	B8
Y Cygni	6.9	A
Z Herculis	7.1	F
RR Centauri	7.4	F
W Ursae Majoris	7.9	G

SUMMARY

1. The solution for the elements of the spectroscopic orbit of a faint star is shown to be easily possible when only a few measures of the radial velocity have been obtained, provided that the system is also an eclipsing binary and provided that the period, the epoch of minimum, the eccentricity, and the longitude of periastron have been derived from the light-curve.

2. A new photometric orbit of RX Herculis has been computed from unpublished observations obtained at Harvard and Princeton (Table IV). Alternate minima are found to differ in depth by nearly a tenth of a magnitude. The stars are nearly equal in size and are sensibly spherical. Their surfaces are separated by three times the radius of the larger star.

3. From measures of the lines on three plates made with the Bruce spectrograph at the Yerkes Observatory, it has been possible to derive very satisfactory spectroscopic orbits of both components.

4. The combination of elements from the photometric and spectroscopic orbits gives the actual dimensions of the stars. (Such information is reliably known for only two other binary systems.) The mass is unusually low for an early-type star. The density is slightly above normal. The volume of both components is six times that of the sun.

5. With a reasonable assumption regarding the surface luminosity, the parallax of the system is found to be 0″.006. The total light of the system is likely more than thirty times that of the sun.

Mount Wilson Solar Observatory
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NEW VARIABLES IN THE CENTER OF MESSIER 3

By HARLOW SHAPLEY

In the investigations of the variable stars in the globular cluster Messier 3 by Professor Bailey at Harvard,[1] the central portion could not be studied successfully on his plates because the great density of stars made it impossible to distinguish individual images. Thus practically all of the region within one minute of arc of the center of the cluster has remained unexplored. Outside the central burned-out nucleus Bailey found 137 variables,[2] and for 110 of them has determined the periods and light-curves, finding that in all cases the periods are shorter than eighteen hours and that the variation is of the typical cluster type. Judging from the relation between the number of variables and the total number of stars at different distances from the center, Bailey estimated that there should be about twelve more variables in the central part of the cluster.

Photographs of the cluster made with the 80-foot-focus Cassegrain combination of the 60-inch reflector have a scale sufficiently large to permit a relatively easy examination of the individual brighter stars of the central region. Six plates have been obtained on three different nights, three with exposures of an hour each and three with exposures of fifteen minutes. The intercomparison of the plates in the Zeiss stereocomparator resulted in the discovery of more than 30 new variable stars. For several of these it is not possible to be absolutely certain of the variation, because either the ranges are small or the variation is observed on only two of the plates available. For 23 of the stars, however, there can be no doubt of the variability. It may be possible later to add a few more to this list of variables from among those suspected. Only two of the 23 are more than one minute of arc from the center of the cluster. The outer portions of the cluster were not systematically examined, for Bailey's thorough study no doubt makes that unnecessary.

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[1] *Harvard Annals*, **38**; *ibid.*, **78**, Part I.

[2] The later measurements of the plates at Harvard indicate that about five stars of the original list are probably not variable and that a few others are somewhat doubtful.

In Table I the new variables are listed in order of right ascension. All but two of them are contained in von Zeipel's catalogue of Messier 3, which gives the positions of 1571 stars.[1] The numbers in the second column of Table I, and the right ascensions and declinations in the third and fourth columns are taken from that work. For No. 1 and No. 14 the positions were determined by reference to neighboring stars. In a large number of cases the catalogue position refers to the mean place of a double, triple, or multiple star which could not be resolved on the plates measured by von Zeipel and usually was not even suspected of being other than a single star. In such instances the notes following the table indicate which component is the variable.

The last two columns of Table I give, respectively, the maximum and minimum magnitudes observed on my plates. These numbers should not be taken as in any way indicating the total range. For many of the variables the length of the exposure no doubt minimizes the brightness at the sharp-pointed maxima; and many of them may be very close composite stars with one component varying through a large range, but without having conspicuous effect on the image of the composite mass. The magnitudes are based on estimates referred to Bailey's series of comparison stars as standards.

NOTES ON THE INDIVIDUAL STARS IN TABLE I

No. 1.—The new variable is the preceding companion of von Zeipel's No. 486, which is also Bailey's variable No. 29. Bailey partially resolved the double and considered at least one of the pair variable. Both components vary through a wide range.

No. 3.—Von Zeipel thought this was perhaps a double and made measures on the two ends of a nebulous streak as well as upon the middle. The star appears single on the Mount Wilson plates. (No star is seen in the neighboring catalogued position of No. 655.)

No. 4.—This is apparently a composite star, but it is the central nucleus that undergoes variation.

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[1] *Annales de l'Observatoire de Paris*, 25, F1–F101, 1908.

No. 5.—Von Zeipel's No. 676 is composed of five distinct stars, of which the variable is the brightest and most centrally located.

No. 7.—The variable is the brighter and southern component of a close double.

No. 9.—Von Zeipel's position refers to the center of mass of the image of a double. The variable precedes its faint companion and at minimum is equal to it in brightness.

TABLE I

NEW VARIABLE STARS IN MESSIER 3

No.	Von Zeipel's Number	α 1900	δ 1900	Magnitude	
				Maximum	Minimum
		13^h37^m	28°		
1	....	$30^s.12$	$51'\,46''.5$	15.4	16.9
2	620	32.785	52 1.31	15.6	17.0
3	638	33.005	53 16.59	16.0	16.8
4	654	33.151	52 39.27	16.2	16.7
5	676	33.440	52 31.47	15.2	16.8
6	678	33.479	52 18.38	15.5	16.8
7	698	33.810	53 36.42	15.3	16.7
8	703	33.870	52 19.52	16.6	17.1
9	714	33.970	53 15.94	15.4	16.9
10	742	34.393	52 16.45	15.6	17.0
11	749	34.469	53 28.57	16.0	17.4
12	769	34.707	53 21.37	16.3	17.0
13	783	34.905	53 3.42	14.3	14.9
14	....	35.35	52 50.8	15.9	17.2
15	881	36.140	52 52.59	16.0	16.9
16	892	36.344	52 47.33	15.3	15.7
17	901	36.408	52 2.74	15.0	16.8
18	944	37.116	53 8.76	15.7	16.8
19	950	37.178	52 28.92	15.5	17.1
20	985	37.660	53 27.22	15.1	16.5
21	1052	38.665	53 1.13	16.5	17.4
22	1092	39.538	53 9.70	16.3	16.9
23	1193	42.194	51 59.27	15.7	16.8

No. 12.—The variable is the following component of a close double.

No. 16.—The variable is one of a group of stars whose images are partly superposed. The variation is very definite, though small; it would probably be measured much larger if the variable could be isolated.

No. 17.—The variable is the most southern member of a triplet.

No. 19.—Two or three faint companions precede the variable. Von Zeipel thought the image irregular in form.

No. 21.—The variable is the northern, following component of a double.

No. 23.—Von Zeipel's No. 1193 is a close double, two minutes of arc distant from the center of the cluster. The variable, which is the preceding component, is brighter than its companion at maximum and fainter at minimum. It is not certain that the companion does not also undergo an appreciable variation.

It is to be noted that one-third of the new variables are equal to or fainter than $17^{m}.0$ at minimum. The faintest minima observed by Bailey were $16^{m}.9$, and he has suggested that the apparent absence of variables among the thousands of fainter stars is probably real. It is not possible from the data now available, beyond the evidence of the above table, either to support or to oppose this hypothesis, but in the next observing season it is proposed to secure plates that will definitely decide the matter. The light-curves and periods have of course not been determined for the new variables, but they are presumably analogous to those derived by Bailey for stars more distant from the center.

Table II contains the data relative to the stars suspected of light-variation. The arrangement is the same as in Table I, and the notes explain some of the individual cases.

TABLE II

STARS SUSPECTED OF VARIATION

No.	Von Zeipel's Number	α 1900	δ 1900	Magnitude	
				Maximum	Minimum
		$13^{h}37^{m}$	28°		
1..........	604	$32^{s}.497$	$53' \; 16''.29$	16.5	16.8
2..........	624	32.806	52 42.25	....	....
3..........	....	32.97	52 52.1	16.0	16.5
4..........	666	33.323	53 14.79	16.3	16.6
5..........	702	33.866	52 28.34	15.5	15.8
6..........	705	33.878	53 30.93	15.5	16.0
7..........	712	33.952	53 4.89	15.7	16.5
8..........	720	34.119	52 30.95	....	....
9..........	894	36.362	53 52.23	....	....
10..........	1007	38.036	53 49.59	16.5	16.9
11..........	....	38.58	53 32.8	....	....
12..........	1106	39.988	52 38.22	17.2	17.6
13..........	1142	40.867	53 18.31	....	....

NOTES ON THE INDIVIDUAL STARS IN TABLE II

No. 2.—Von Zeipel's 624 is a triplet. It is difficult to tell which of the stars varies, but it is probably the following component.

No. 7.—The suspected variable is one or other of the two brightest stars in the composite mass catalogued as No. 712.

No. 8.—This star is apparently somewhat out of the position given by von Zeipel. It was at the limit of measurement on his plates. The variability is doubtful.

No. 9.—This extremely close double is Bailey's variable No. 2. It is almost certain that both components are variable. The preceding component varies through a range of a magnitude at least.

No. 10.—The suspected star is the faint preceding companion to von Zeipel's No. 1007, for which the position is given in the table.

No. 12.—A defective image may be the cause of the suspected variability.

No. 13.—The suspected star is the closest of the two faint preceding companions to von Zeipel's No. 1142, for which the position is given in the table.

Several variables of Bailey's list, which were found by him to be difficult of separation from surrounding stars or which for other reasons could not be studied or definitely proved variable, were examined on the Mount Wilson plates. The results are collected in the following notes. The numbers are those assigned in *Harvard Annals*, 78.

No. 2.—See the reference to this star in the note on No. 9 in Table II above. Only a very small range was observed at Harvard and no period was derived.

No. 4.—This relatively bright star was suspected of a small variation at Harvard. Mount Wilson plates show a variation from $14^{m}.0$ to $15^{m}.3$.

Nos. 8, 30, 73, 95, 98, 127, 129, and 130, all of which are stars concerning whose light-variability there is some question, show no appreciable variation on the Mount Wilson plates.

The suspected variability of No. 99 is definitely confirmed.

The variability of No. 122, doubted by Bailey, is established. The star is a close triplet and the variable is apparently the following, southern component.

Nos. 135 and 136.—Bailey writes concerning both of these stars, which are near the center of the cluster and difficult, that the variation, if genuine, is very small, though when first examined the stars seemed to vary. For the former the variation measured at Mount Wilson is from $15^{m}.3$ to $16^{m}.8$, and for the latter from $15^{m}.7$ to $16^{m}.6$.

Mount Wilson Solar Observatory
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ON THE NATURE AND CAUSE OF CEPHEID VARIATION[1]

By HARLOW SHAPLEY

The purpose of the present discussion is an attempt to investigate the question of whether or not we should abandon the usually accepted double-star interpretation of Cepheid variation. In addition to the brief statement of some general considerations and correlations of the many well known characteristics of Cepheid and cluster variables, certain recently discovered properties of these stars are discussed in greater detail, because chiefly upon them are based the conclusions reached in this study.

It seems a misfortune, perhaps, for the progress of research on the causes of light-variation of the Cepheid type, that the oscillations of the spectral lines in nearly every case can be so readily attributed, by means of the Doppler principle, to elliptical motion in a binary system. The natural conclusion that all Cepheid variables are spectroscopic binaries has been the controlling and fundamental assumption in all the recently attempted interpretations of their light-variability, and the possibility of intrinsic light-fluctuations of a single star has received little attention.

From the very first there have been serious troubles with each new theory. Considered from the spectroscopic side alone, the Cepheids stand out as unexplainable anomalies. There are persistent peculiarities in the spectroscopic elements, such as the low value of the mass function, the universal absence of a secondary spectrum, and the minute apparent orbits. Practically the only thing they have in common with ordinary spectroscopic binaries is the definitely periodic oscillation of the spectral lines, which permits, with some well known conspicuous exceptions,[2] of interpretation as periodic orbital motion. Adding, then, to the spectroscopic abnormalities the curious relations between light-variation and radial motion, the difficulties in the way of all the proposed simple solutions seem insurmountable. Geometrical explanations of the light-variation fail completely, and little better can be said of the hypotheses that involve partly meteorological and partly orbital assumptions.

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[1] Read at the seventeenth meeting of the American Astronomical Society, August 1914.

[2] The irregularities in the velocity-curve of ζ Geminorum have been discussed by Campbell (*Astrophysical Journal*, **13**, 94, 1901), Russell (*Astrophysical Journal*, **15**, 260, 1902) and Plummer (*Monthly Notices*, **73**, 661, 1913). The deviations from purely elliptical motion in the case of W Sagittarii have been considered by Curtiss (*Lick Observatory Bulletins*, **3**, 36, 1904; *Astrophysical Journal*, **20**, 149, 1904); in the

The writer can offer no complete explanation of Cepheid variability as a substitute for the existing theories that are shown to be more and more inadequate. At most, only the direction in which the real interpretation seems to lie can be pointed out, and an indication given of the strength of the observational data that would support the theory developed along the lines suggested. The principal results of a rather extensive investigation, further details of which it is hoped can be published in subsequent papers in the near future, are outlined in the following paragraphs. The main conclusion is that the Cepheid and cluster variables are not binary systems, and that the explanation of their light-changes can much more likely be found in a consideration of internal or surface pulsations of isolated stellar bodies.

THE ESSENTIAL IDENTITY OF CEPHEID AND CLUSTER VARIABLES

The subdivision of the short-period variables into the cluster type and the Cepheid type is an artificial one. This proposition scarcely needs proof, although the assumption of the essential similarity of the two groups is important in the following discussion. Practically all writers on the subject are more or less inclined to accept this view.[1] The definition of the cluster-type variable is,

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case of Y Ophiuchi by Albrecht (*Lick Observatory Bulletins*, **4**, 134, 1907) and later by Zurhellen (*Astronomische Nachrichten*, **177**, 329, 1908) and Miss Udick (*Publications of the Allegheny Observatory*, **2**, 151, 1912); in the case of RT Aurigae by Duncan (*Lick Observatory Bulletins*, **5**, 120, 1909). For many Cepheids the total range of velocity variation is so small that secondary oscillations and other irregularities of considerable relative importance may easily be lost in the accidental errors (Curtiss, *op. cit.*, p. 39).

[1] See, for instance, Nijland, *Hemel en Dampkring*, April 1913; Williams, *Journal of the British Astronomical Association*, **23**, 134, 1912; Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 191, 1912, and **25**, 121, 1913; Townley, *Publications of the Astronomical Society of the Pacific*, **25**, 239, 1913.

in fact, by some merely "short-period Cepheid." Others, including Hartwig[1] and Kron,[2] have considered only those with rapidly decreasing brightness and constant light at minimum as "antalgol" or cluster-type variables. Kron calls the shortest-period variable known a Cepheid,[3] and Hertzsprung[4] designates as Cepheids only those variables whose periods are greater than a day. The writer proposes to adopt, merely as a convenience, the latter practice, arbitrarily calling the Cepheids of periods less than a day cluster-type variables; for there is at present no evidence of real difference between the two classes in the nature or probable causes of the light and velocity variations.[5] Hertzsprung[6] calls attention to the maxima in the frequency-curve of the periods at twelve hours and at seven days, and notes also that the longer-period Cepheids are in the galaxy, while the shorter-period Cepheids or cluster variables are apparently distributed more at random over the sky. Making the reasonable assumption that the data, though rather meager, are sufficient, nevertheless, to establish the reality of both phenomena, these conditions do not impeach the hypothesis that the light and velocity variations of the long- and short-period Cepheids are attributable to the same causes, and that the only modifications necessary in an explanation of one, to make it applicable to the other, are those depending on the length of the periods and other gradative characteristics, such as differences of spectral type and relative speed of light-change at corresponding phases. Among the several arguments that tend to prove the inherent similarity of the two groups of Cepheids, the following are the most important.

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[1] *Vierteljahrsschrift der Astronomischen Gessellschaft*, **37**, 284, 1902.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 53, 1912. See also Newcomb-Engelmann, *Populäre Astronomie*, 5th ed., p. 623, Leipzig, 1914.

[3] XX Cygni, period $3^h 14^m$.

[4] *Astronomische Nachrichten*, **192**, 262, 1912.

[5] It is hardly necessary to remark that Cepheids and Geminids are physically identical. The latter term merely signifies that the rise and decline of brightness require approximately equal intervals of time. There are numerous types of variation intermediate between the chosen typical curves of ζ Geminorum and δ Cephei. A significant feature that has not been pointed out explicitly heretofore is that the smaller the ratio of interval of increasing light to interval of decreasing light the greater the eccentricity. Orbits of Geminids therefore have smaller eccentricities. This really amounts to observing that the light-curves and velocity-curves of all classes of Cepheids are generally identical in form. See the study by Luizet, "Les Céphéides considérées comme étoiles doubles," *Annales de l'Université de Lyon*, Nouvelle Série, I, Fascicule 33, 67–148, 1912.

Some Geminid curves, however, permit of hypothetical interpretations that cannot be applied to the more typical Cepheid. Russell has found (*Popular Astronomy*, **22**, 142, 1914) that, if the time of rise to maximum is greater than one-fourth the period, the light-variations may be interpreted as the rotation of a spotted body, but such an explanation is otherwise untenable. The writer has shown (*Laws Observatory Bulletin*, **2**, 71, 1913; *Astronomische Nachrichten*, **194**, 353, 1913) that for certain symmetrical curves of the Geminid type the light-variations may be due entirely to the rotation of a single ellipsoidal star. This explanation is a possible and plausible one, but for SZ Tauri, one of the stars suitable for such an interpretation, Haynes has found a typical Cepheid velocity variation (*Lick Observatory Bulletins*, **8**, 85, 1914).

[6] *Astronomische Nachrichten*, **179**, 376, 1909; **192**, 262, 1912; **196**, 205, 1913. Chandler reached some of the same conclusions twenty-five years ago (*Astronomical Journal*, **9**, 1, 1889).

a) For RR Lyrae, period 13.6 hours, which is commonly classified as a cluster-type variable, the spectroscopic orbit by Kiess[1] resembles in all details the peculiar orbits characteristic of the longer-period Cepheids. The light-curve is typical of cluster variables in all its properties.[2]

b) From the photometric standpoint, Graff and Bottlinger[3] have found no essential differences between light-curves of cluster and Cepheid types, and insist on the artificiality of the division into two classes. Very few, if any, of the cluster-type variables have rigorously constant light at minimum phase, as Plummer,[4] among others, has shown. In fact, it was partly for this reason that Hartwig abandoned, in the *Vierteljahrsschrift* catalogue, the former term "antalgol" and the former distinction between cluster and Cepheid variables.[5]

c) Russell's harmonic analyses of the mean light-curves of typical cluster variables and typical Cepheids indicate the necessity of analogous interpretations of the two.[6]

d) An unpublished investigation by the writer of the relation between the periods and spectral types of all variables shows the existence of a continuous property from the longest-period Cepheids to the shortest-period cluster variables.

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[1] *Lick Observatory Bulletins*, **7**, 140, 1913.

[2] Kiess, *Publications of the Astronomical Society of the Pacific*, **24**, 189, 1912.

[3] *Astronomische Nachrichten*, **196**, 113, 1913.

[4] *Monthly Notices*, **73**, 657, 1913.

[5] *Vierteljahrsschrift der Astronomischen Gessellschaft*, **48**, 287, 1913.

[6] An abstract is printed in *Popular Astronomy*, **22**, 142, 1914.

e) The shift of the maximum intensity in the spectra toward the violet with increasing light is a property common to both classes.[1]

IRREGULARITIES IN THE LIGHT-ELEMENTS

Starting, then, with the apparently well-grounded assumption that the reasoning relative to the nature of the cluster-type variation applies equally well to Cepheid variation, the first argument presented against the binary character of Cepheids deals with the irregular oscillations in the photometric period. In a paper presented to the American Astronomical Society at its last meeting, the writer reported on the oscillations in the periods of several cluster-type stars.[2] The study of such irregularities must necessarily, for the present at least, be confined to photometric observations, for the faintness of the stars, and the consequent length of spectroscopic exposure would conceal irregular oscillations in the velocity measurements.[3] The investigation, as is also obvious, must succeed first with the stars of shortest period, for with them the light-change is of sufficient rapidity to permit the determination of points on the steep ascending branch of the light-curve with high precision.

Further observations of SW Andromedae, made since the last report, have confirmed the previous results, showing that the time of the rise to maximum light varies from the mean predicted time by ten or fifteen minutes within the short interval of two or three days, but evidently without exhibiting regular periodicity. The uncertainty of the determination does not exceed three or four minutes. The similar oscillations in the light-curve of RR Lyrae were apparently periodic throughout the interval of two or three years covered by the earlier series of Harvard observations,[1] but the later work at Harvard[2] and at the Lick Observatory[3] shows a different amplitude and perhaps no periodicity at all. The Lick and Harvard observations were made with visual photometers. In a recent letter Professor Hertzsprung writes that he also finds irregularities from night to night in photographic observations of the star.

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[1] For the Cepheids see the work of Albrecht (*Lick Observatory Bulletins*, **4**, 131, 1907) and of other Lick observers. For the cluster-type variables see the work of Kiess, referred to above, and the indirect determinations of the maximum intensity shifts presented in a later section of the present paper.

[2] An abstract is printed in *Popular Astronomy*, **22**, 144, 1914.

[3] The average length of exposure on RR Lyrae with a one-prism spectrograph attached to the 36-inch refractor of the Lick Observatory was more than two hours—nearly one-sixth of the entire period. RR Lyrae is the brightest cluster variable known (excepting β Cephei).

The oscillation in the time of brightening to maximum seems to be a pretty general characteristic, though possibly not universal. It is shown definitely for several other stars besides those mentioned above,[4] and perhaps most noticeably in the case of XX Cygni, which is discussed in the next section. The most remarkable feature of the oscillation, which from night to night is conspicuously large in some cases, is that a change in the mean period of the light-variation has been recorded for only two stars, and these changes are relatively minute.[5] If the observed oscillations were definitely periodic, it would perhaps be possible to attribute them in some kind of a binary system to orbital changes, such as the rotation of the line of apsides. But the sudden and unpredictable changes in the light-variation, very likely accompanied by analogous oscillations in the velocity-curve, introduce another difficulty into the binary system theory.

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[1] *Harvard Annals*, **69**, Pt. I, 45, 1909.

[2] The manuscript of these observations was kindly sent to the writer by Professor Pickering; more recently the work has been published in *Harvard Annals*, **69**, Pt. II, 124, 1913.

[3] *Lick Observatory Bulletins*, **7**, 141, 142, 1913.

[4] *Popular Astronomy*, **22**, 144, 1914.

[5] According to Kron the mean period of XX Cygni is decreasing by about a tenth of a second a year (*op. cit.*, p. 47). Roberts finds that the mean period of S Arae is decreasing by four-hundredths of a second a year (*Astrophysical Journal*, **33**, 200, 1911). The long-accepted secular change in the period of δ Cephei, first established with some uncertainty by Chandler (*Astronomical Journal*, **13**, 101, 1893) and later maintained by Nijland (*Astronomische Nachrichten*, **161**, 229, 1903), has recently been completely rejected by Luizet in his monograph on the light-variations (*Annales de l'Université de Lyon*, Nouvelle Série, Fascicule 33). Belopolsky, however, finds an oscillation in the spectroscopic period (*Mitt. Pulk.*, **3**, 63, 1909). W. J. S. Lockyer has found (Dissertation, Göttingen, 1896) that, while the mean period of η Aquilae is constant, there is an oscillation in the epoch of maximum through an amplitude of ten hours. Hellerich (Dissertation, Berlin, 1913) has studied the periods of ten Cepheids and finds no necessity of second-order terms.

IRREGULAR CHANGES IN THE LIGHT-CURVES

The second argument against the double-star explanation of Cepheid variation lies in the continually changing form of the light-curves from one maximum to the next. Again we will consider mainly the cluster-type stars. For the Cepheids the length of the period prohibits, in general, continuous observations throughout successive epochs, and in the long run the irregularities smooth out in a mean light-curve. Curtiss[1] has noticed, however, that the light-curve of W Sagittarii, period 7.6 days, changes shape with the time, and similar results are suggested for other Cepheids by various observers.[2] For the stars with periods less than a day, however, many long series of observations have been made, and although the work was rarely if ever undertaken for the purpose of seeking short-period changes in the form of the light-curve, nevertheless irregularities have often been found. Many observers have noticed that the errors are larger in observing short-period variables than in any other class. Roberts[3] and Innes[4] were suspicious of the large deviations in their measures of S Arae. Sperra[5] concluded from an extensive treatment of his visual observations on SW Draconis and SU Draconis that the shapes and durations of both maxima and minima varied from night to night. This was, I believe, the first and only serious attempt that has been made to question the supposed clocklike precision of short-period variation. Plummer and Martin[6] are disinclined to accept Sperra's results without further proof, for the photographic observations at Dunsink (exposure times from thirty minutes to an hour) do not confirm such irregularities (though they do show remarkable irregularities, supposedly permanent, in the mean curves). The proof is now at hand, however, and with amazing clearness in the observations of XX Cygni, published at Potsdam.[1] The three-hour period of this star is obviously a great advantage in the study of the changing form of the light-curve. If the variations of SW Andromedae and RR Lyrae referred to above could have been followed regularly throughout their entire periods, there is little doubt that the oscillations in the time of the rise to maximum would have been found to constitute only a part of the irregularities.

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[1] *Lick Observatory Bulletins*, **3**, 168, 1905.

[2] For instance, the light-curve of δ Cephei, as pointed out by Luizet (*op. cit.*, pp. 58–60).

[3] *Astrophysical Journal*, **33**, 201, 1911.

[4] *Annals of the Cape Observatory*, **9**, 126B, 1903.

[5] *Astronomische Nachrichten*, **184**, 241–252, 1910.

[6] *Monthly Notices*, **73**, 440, 1913.

In his discussion of nearly three thousand observations of XX Cygni, Kron finds a small secular change in the mean period, but does not consider real the deviations from the mean curves. The observations of ten observers are included. Both Schwab and Guthnick considered the irregularity of form a real phenomenon, and the former notes that, while the maximum and minimum magnitudes remain sensibly constant, the times between the ascent and descent past magnitude 11.2 vary from 35 minutes to an hour.[2] A study of the observations shows the same phenomenon in the work of all the observers.[3] The various forms of light-curve cannot be attributed to night errors. The differences between the curves at different epochs is distinctly larger than the errors of the observations. This is particularly true for Kron's photometric work. The average deviation of his measures from a normal curve, based on his own observations, is much larger than that of measures on the comparison stars, and probably more than twice as large as the average deviation of the observations from separate nightly curves. There appears, however, to be no definite periodicity in the changing shape of the curve; as a rule sharp maxima follow each other for a few days, to be succeeded by an intermediate type and then by a series of relatively wide, flat-topped maxima. Sometimes the extreme change in form occurs on successive nights. In the variations of the light-curve, as in the oscillations of the time of the rise to maximum light, the regularity and continuity of the phenomena that would be demanded by an orbital explanation is apparently lacking.

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[1] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Pt. III, 1912.

[2] *Astronomische Nachrichten*, **170**, 369, 1906.

[3] This and other points relative to the anomalies of the light-variation will be discussed more fully in a later communication. Contrary to all other experience with Cepheid variables the photographic range measured by Parkhurst and Jordan (*Astrophysical Journal*, **23**, 84, 1906) is less than the visual range. To examine this question more closely a series of simultaneous photographic and photovisual observations has recently been made with the 60-inch reflector. This will furnish a definitive color-curve, as well as serve as a control on the secular change in the period. (Parkhurst and Jordan also suspected oscillations in the period; *op. cit.*, p. 86.)

CHANGES IN COLOR AND SPECTRAL TYPE

A third argument against the binary interpretation of Cepheids is the difficulty such theories would have in explaining the periodic change of the spectral type, though it must be admitted that to a certain extent Duncan's hypothesis,[1] if otherwise acceptable, could account for spectral changes through the medium of atmospheric absorption. The evidences of the change of spectral type with changing light, though not well known nor generally recognized, are decisive and important. Schwarzschild,[2] Wirtz,[3] and more particularly Wilkens[4] have demonstrated for Cepheids of longer period that the range of light-variation is greater in the photographic than in the visual part of the spectrum. The photographic work of Martin and Plummer[5] suggests similar results for cluster-type variables, while the recent simultaneous photographic and photovisual observations by Mr. Seares and the writer at

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[1] *Lick Observatory Bulletins*, **5**, 91, 1909; *Publications of the Astronomical Society of the Pacific*, **21**, 123, 1909.

[2] *Publikationen der v. Kuffnerschen Sternwarte*, **5**, C100, 1900. The photographic range of η Aquilae is found to be double the visual range. More recently Kohlschütter has repeated the photographic work and finds that the color-curve has an amplitude of four-tenths of a magnitude (*Astronomische Nachrichten*, **183**, 265, 1910).

[3] *Astronomische Nachrichten*, **154**, 327, 1901. Wirtz measures the photographic ranges of δ Cephei and ζ Geminorum.

[4] *Astronomische Nachrichten*, **172**, 316, 1906. An average value of 1.6 is found by Wilkens for the ratio of photographic to visual range for the Cepheid variables SU Cygni, X Cygni, T Vulpeculae, S Sagittae, and U Vulpeculae. The visual ranges, it should be remarked, are collected by him from various sources and can hardly be considered homogeneous or reliable. The results, however, are qualitatively dependable, but more work along this line is desired. The Cepheids mentioned in the three last notes are of spectral types F to K5, with an average very close to the solar type.

[5] SU Draconis, *Monthly Notices*, **73**, 166, 1912; SW Draconis, *ibid.*, **73**, 440, 1913; XZ Cygni, *ibid.*, **74**, 225, 1914.

Mount Wilson establish the fact definitely.[1] The shift of the maximum intensity in the spectra of Cepheids, discovered by Albrecht,[2] has been confirmed by Kiess[3] and other Lick observers. These two factors—the greater photographic range and the shift of the maximum intensity—would suggest as an underlying and common cause a change in the spectral type. Albrecht and Duncan[4] have observed that Wright's spectrograms of η Aquilae suggest a later type of spectrum at minimum than at maximum. The Harvard classification of TT Aquilae[5] at maximum is G, at minimum, K.

For the cluster-type variables there is more direct evidence of distinct and continuous change. At the writer's request Miss Cannon has examined some of the Harvard spectrograms of certain cluster variables. For RR Lyrae no definite change was recorded on the plates examined, and similarly for XZ Cygni, but the spectrum, when faint, was extremely uncertain. For SW Andromedae the spectrum was of type A at maximum and clearly of a redder type at minimum. The most conclusive results, however, are obtained from the series of spectrograms taken by Mr. Pease[6] in July of this year with the 60-inch reflector of the Mount Wilson Observatory. The variable RS Boötis, period $9^{h}.1$, shows a continuous change of spectral type from F0 at minimum to B8 at maximum. One consequence of this result is that hereafter the classification of all Cepheid and cluster-type spectra must be made with due specification of the corresponding phase of light-variation.[7]

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[1] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[2] *Lick Observatory Bulletins*, **4**, 131, 1907.

[3] *Ibid.*, **7**, 140, 1913.

[4] *Ibid.*, **5**, 93, 1909.

[5] *Harvard Annals*, **55**, 285, 1909.

[6] A report on this work was presented at the meeting of the American Astronomical Society at Evanston, Illinois, August 25–28, 1914.

[7] Miss Clerke writes: "The spectrum [of δ Cephei] is of the solar type, and does not change with the brightness" (*Problems in Astrophysics*, p. 320, London, 1903). This, however, should not discourage new attempts to classify the spectrum of the type-star and of other longer-period Cepheids at various phases of their light-changes. It is possible, of course, that the changes in color index and shifts of maximum intensity are not generally accompanied in the longer-period Cepheids by those changes in the absorption lines that are necessary to give a different spectral classification under the present system, in which the absorption lines receive much attention and the background intensities but little. But in the early study of the spectrum of δ Cephei, published by Belopolsky in *Bulletin*, No. 3 of the Imperial Academy of Sciences of

Another difficulty in the spectral changes, that must not be overlooked in attempting a complete explanation of the Cepheid phenomena, is a peculiarity observed by Albrecht[1] on his plates of Y Ophiuchi and T Vulpeculae. Various lines showed large irregular shifts, which are not progressive with the phase of the star in its light-period.

CONCERNING EXISTING HYPOTHESES

The fourth principal argument against the binary interpretation of Cepheids is the inadequacy of all the existing double-star hypotheses. To many this is not only the best argument but is sufficient in itself.[2] A detailed criticism of these attempted explanations is unnecessary, for this has been generously provided by the proposers of the theory themselves, as well as by others, including Campbell,[3] Plummer,[4] Brunt,[5] Kiess,[6] and Ludendorff.[7] There is

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St. Petersburg, 1894, the variations in the relative intensities of several lines are suggested and certain deviations from the solar spectrum are explicitly pointed out on many spectrograms. The question is one to be answered definitely by future researches. (A very recent study of the changes in the spectra of δ Cephei and ζ Geminorum has been made at St. Petersburg by Lohmann, but the paper is not yet available to the writer.)

[1] *Lick Observatory Bulletins*, **4**, 131–132, 1907.

[2] There has been a growing and but half-concealed discontent with the double-star explanations of Cepheids. Ludendorff writes (*Astronomische Nachrichten*, **193**, 304, 1912): "Freilich kann man sich aus verschiedenen Gründen des Eindrucks kaum erwehren, dass die in den Spektren der δ Cephei-Sterne beobachteten periodischen Linienverschiebungen nicht durch Radialbewegungen der Sterne, sondern durch irgendwelche andere Ursachen hervorgerufen werden." On the other hand, Paddock interprets Ludendorff's data in a manner favorable to Duncan's double-star hypothesis (*Publications of the Astronomical Society of the Pacific*, **25**, 180, 1913).

Plummer considers, in a recent paper on the nature of the Doppler principle when based on the Ritz theory of light, the possibility of getting around the difficulties presented by the velocity variations of certain variables (obviously Cepheids are meant) by abandoning the binary interpretation altogether, but he is led rather to abandon the Ritz theory (*Monthly Notices*, **74**, 660, 1914). Until the velocity variation of RR Lyrae was discovered he was inclined to suggest that cluster-type variation might be 'a prominence effect on a large scale" (*ibid.*, **73**, 658, 1913).

[3] *Stellar Motions*, pp. 305 ff., New Haven, 1913; *Lick Observatory Bulletins*, **6**, 51, 1910.

[4] See various papers cited above.

[5] *Observatory*, **36**, 59, 1913.

[6] *Publications of the Astronomical Society of the Pacific*, **24**, 186, 1912; see also other papers cited above.

[7] *Astronomische Nachrichten*, **184**, 384, 1910.

one point, however, that has not been considered, which is of prime importance in the discussion of Cepheid phenomena. Russell[1] and Hertzsprung[2] have independently shown that the Cepheids are stars of small peculiar motions and small parallaxes, and hence of great absolute brightness. The former finds a mean absolute magnitude of -2.4 and the latter of -2.3, that is, the average Cepheid (the spectrum is of solar type) is nearly 700 times as bright as the sun. It is reasonable to assume that the Cepheids and the sun have a comparable surface brightness. The average Cepheid, then, has a volume between fifteen and twenty thousand times as great as that of the sun.

Interpreted as spectroscopic binaries these giant stars move in orbits whose apparent radii average less than one-tenth the radii of the stars themselves.[3] In order that the radii of the real orbits may greatly exceed those of the apparent orbits, the inclinations must be very small, a condition which cannot be supposed to exist generally for Cepheid orbits. The difficulty in applying the hypotheses of Eddie,[4] Loud,[5] Duncan,[6] and Roberts[7] is therefore immediately apparent. Moreover, if the mass of the average Cepheid is admitted to be as much as five times the solar mass, the density is still astonishingly low—hardly three ten-thousandths that of the sun. Considering the low average value of the mass function[8] derived from the orbits of the Cepheids, and taking a random distribution of the orbital inclinations, the non-luminous second body, to which Duncan's theory assigns the extensive atmosphere that must envelope the giant primary, has about one-tenth of the mass and therefore must move with an average apparent orbital velocity of about 200 km a second. Remembering the size of the primary star compared with its orbit, we know that the mass of the secondary must be still smaller and the velocity higher to separate the stars.

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[1] *Science*, N.S., **37**, 652, 1913.

[2] *Zeitschrift für wissenschaftliche Photographie*, **5**, 107, 1907; *Astronomische Nachrichten*, **196**, 201, 1913.

[3] The average value of $a \sin i$ for 15 Cepheids is 1,116,000 km. The greatest value is 2,000,000 km, and the least is 45,000 km.

[4] *Astrophysical Journal*, **3**, 227, 1896.

[5] *Ibid.*, **26**, 369, 1907.

[6] *Lick Observatory Bulletins*, **5**, 91, 1909.

[7] *Astrophysical Journal*, **33**, 197, 1911; *Monthly Notices*, **66**, 329, 1906.

[8] The average value of $\frac{m_1 \sin^3 i}{(m_1+m)^2}$ for 15 Cepheids is 0.0025. The greatest value is 0.0058, and the least is 0.00001 for Polaris.

A SUGGESTED EXPLANATION OF CEPHEID VARIATION

In the face of all these difficulties, it seems appropriate to abandon completely the attempts to interpret Cepheids on the basis of a binary-star assumption. It has been shown by Russell[1] that the light-variations cannot be explained satisfactorily by the uniform rotation of a single spotted star; the light-change must be intrinsic, and not just apparent. The explanation that appears to promise the simplest solution of most, if not all, of the Cepheid phenomena is founded on the rather vague conception of periodic pulsations in the masses of isolated stars. The vagueness of the hypothesis lies chiefly in our lack of knowledge of the internal structure of stellar bodies, and not in the difficulty of explaining the observed facts if once we assume the stars to be ideally gaseous figures of equilibrium. Moulton[2] has considered the matter of explaining certain types of stellar variation from this point of view, but his conclusions are scarcely applicable to Cepheid variables in the light of our present knowledge of their peculiar properties.[3] According to him, the light-change should be due to the heat generated by the oscillation of a spherical star from an oblate to a prolate form, there being a maximum of light-emission every time the star passes through its mean spherical figure. The period of velocity variation, then, should be double that of the light-change,[4] and this, of course, does not conform with known conditions.[1] It is to this phenomenon of pulsating stellar masses, however, that the writer would ascribe the light and velocity variation of Cepheid and cluster variables, and the theoretical work of Moulton,[2] Jeans,[3] Emden,[4] and others[5] on the properties of gaseous spheres already justifies the conclusion that such oscillations are both possible and probable. They might arise, as Moulton suggests, from the collision with masses of only planetary dimensions, from the near approach of two stars, or in other ways.[6]

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[1] *Popular Astronomy*, **22**, 142, 1914. See the footnote on a preceding page relative to this work. Russell's investigation, then, opposes the explanation suggested by Hellerich (Dissertation, Berlin, 1913).

[2] *Astrophysical Journal*, **29**, 257, 1909.

[3] The hypothesis also is not obviously applicable as a complete or even partial explanation of the variation of elliptical eclipsing binaries, considering the present state of the orbital theory. The presence of the secondary spectrum, the very large periodic shift of the spectrum lines, and many other factors are almost unimpeachable proofs of the binary character of eclipsing variables. Very little if any effect on the light-variation can be attributed in any of the systems studied to pulsations, or even to tidal disturbances.

[4] That is, if the shift of the spectrum lines is to be attributed to a radial motion of the source.

Without any pretense of explaining clearly or fully on this hypothesis all the properties of Cepheid variation that have given the double-star theories such hopeless difficulty, a few points favoring the pulsation suggestion will be summarily stated. There will exist originally, as the result of the initial disturbance, a great number of oscillations with different periods. The character of these various vibrations will depend on the nature of the stellar structure. For the ideal homogeneous fluid mass investigated by Kelvin,[7] and for the polytropic gaseous sphere defined and studied by Emden,[8] the period of vibration of each type is independent of the volume and mass and depends only on the mean density and the order of the harmonic term defining the oscillation.[9] For any given

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[1] It is well to keep in mind, however, the secondary maxima in the light-curves of η Aquilae and similar variables. Therein perhaps is a visible trace of the secondary heating of each oscillation period.

[2] *Op. cit.*

[3] *Philosophical Transactions of the Royal Society of London*, **199 A**, 1, 1902; *ibid.*, **201 A**, 157, 1903; *ibid.*, **213 A**, 457, 1914.

[4] *Gaskugeln*, Leipzig, 1907.

[5] The numerous papers by Ritter, *Wiedemanns Annalen*, **5–20**, 1878–1883, are of fundamental importance in this problem. His consideration of vibrational variable stars has been briefly discussed by Moulton (*op. cit.*).

[6] Perhaps then we should expect to find great numbers of these variables in condensed regions, such as the globular clusters and the Magellanic clouds.

[7] *Mathematical and Physical Papers*, **3**, 384, 1890.

[8] *Op. cit.*, pp. 13, 37, 448 ff.

[9] The vibration period in seconds is for the former

$$\tau = 2\pi\left(\frac{2m+1}{2m(m-1)}\right)^{\frac{1}{2}}\left(\frac{R}{G}\right)^{\frac{1}{2}}$$

and for the latter

$$\tau = 2\pi\left(\frac{R}{mg}\right)^{\frac{1}{2}}$$

where m is the order of the spherical harmonic defining a given oscillation, R is the radius, and g, the surface gravity, is proportional to radius times mean density.

mean density the most important oscillation is that corresponding to the second-order harmonic. Its period is the longest, its amplitude the greatest, and it may persist with inappreciable change in period almost indefinitely, while the oscillations of higher order are more rapidly destroyed by friction.

If, then, we attribute the principal light-change in a Cepheid variable to this principal oscillation, and if we are willing to adopt Emden's polytropic gaseous sphere as a stellar model, we can compute at once the density of each individual variable. Obtained in this way the densities are probably of the right order of magnitude, whatever function of the radius, within reasonable limits, the density is assumed to be; and for an incompressible homogeneous fluid they would be only 2.5 times as large. The densities in terms of the sun for all the Cepheids of known periods and spectra[1] have been derived in this manner, with the results given in Table I.[2]

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[1] It is impossible to say, of course, whether the spectra listed were obtained near maximum or near minimum. Errors in the grouping, for the brighter Cepheids at least, will probably balance in the means.

[2] The data for this computation have been derived mainly from *Harvard Annals*, **56**, 191–195, 1912. In the table of "short-period variables" given in that publication are included many stars that are now known to be eclipsing binaries. It is very likely that further study will show that others are not Cepheids, but in the means in Table I are included all variables not known to be eclipsing stars.

The densities in the table are computed for the mean periods of each class. If the means of the individual densities were taken one badly discordant period would greatly distort the result.

A striking fact shown by Table I is the progressive relation between spectrum and period, and hence, between spectrum and density. "The redder the tint, the longer the period" was observed by Chandler in 1888, but referred mainly to long-period variables (*Astronomical Journal*, **8**, 137, 1888). He called attention to the importance of this correlation to variable star hypotheses. Campbell noticed that "the length of periods seems to increase with the spectral types, but the relationship is not strongly marked" (*Lick Observatory Bulletins*, **6**, 51, 1910).

Russell's theory of the order of stellar evolution is supported by the results of Table I when we remember that the Cepheids are giant stars. For it is obviously fair to conclude from the progression of the densities that, if the present Cepheid hypothesis approximates the truth, the order of evolution is in the direction from type M to type B. The Cepheids then are young stars. Perhaps the long-period redder variables of the Mira Ceti type, with spectra of types M*a*,....M*d*1, M*d*2,... M*d*10, N, etc., are still earlier in their evolution. Are they merely Cepheids of very long period? In many respects they appear to be—the light-curves are similar, the shape and time of successive maxima often oscillate around probably rigorously constant mean values, the spectra change progressively throughout the light-period, and finally

The extremely low densities for the Cepheids of the redder spectral types until recently might have thrown serious doubt on the hypothesis that demands such abnormally low values. But now for two reasons we are ready to accept as possible these supposedly impossible densities. In the first place, as Hertzsprung has pointed out[1] in his proof that the Cepheids of solar spectral type are giant stars, the average mean density must be of the order of 6×10^{-5}, if the masses are comparable with that of the sun. They may be larger, but from our knowledge of stellar masses in general we are inclined to believe that they are not more than ten times that of the sun,[2] which is sufficient to prove the point. In the second place, the densities of several long-period eclipsing binaries of types G and K are now available for comparison.[3] For instance: RX Cassiopeiae, type K0, mean density 5×10^{-4}; W Crucis, type Gp, mean density 3×10^{-6}; SX Cassiopeiae, type G3, mean density 5×10^{-4}; RZ Ophiuchi,[4] type F8, density of one component 10^{-3}.

TABLE I

Type	No. Stars	Mean Period	Mean Density
M	3	33	0.000006
K	9	18	0.000020
G	31	11	0.000056
F	31	6	0.000200
A	9	0.4	0.04
B*	1	0.19	0.2

* β Cephei is properly to be assigned to the Cepheid type. Notwithstanding the very short period ($4^{h}5$), the preliminary orbit by Frost (*Astrophysical Journal*, **24**, 259, 1906) is typical of the Cepheids. Guthnick's discovery (*Astronomische Nachrichten*, **196**, 357, 1913) of the light-variation and its nature (a typical cluster-variable curve of small amplitude) further supports this classification, and the harmony of the hypothetical density with that for the other groups of Cepheids is probably a third favorable argument. The presence of a second spectrum, however, is suspected on certain plates, according to the preliminary announcement (*Astrophysical Journal*, **24**, 261, 1906).

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they are apparently stars of great absolute luminosity. Long ago, however, Chandler gave several good reasons for distinctly separating "long period" and "short period" variables (*Astronomical Journal*, **9**, 1, 1889), and to a certain extent these distinctions are still to be maintained.

[1] *Astronomische Nachrichten*, **196**, 203, 1913 (footnote).

[2] Russell, *Nature*, **93**, 283, 1914; *Popular Astronomy*, **22**, 294, 1914. Ludendorff, *Astronomische Nachrichten*, **189**, 151, 1911.

[3] *Contributions from the Princeton University Observatory*, No. 3, 1914.

[4] The spectrum has been reclassified recently by Miss Cannon at the writer's request. The components are of nearly equal brightness (visually), but it is very likely that the photographic spectrum classified is that of the smaller component which has high relative surface brightness, and density as given above. The density of the fainter component is 2×10^{-5} (*Astronomische Nachrichten*, **194**, 225, 1913).

For the A- and B-type spectra, the Cepheid densities above are, of course, entirely normal compared with eclipsing star densities.[1]

As previously stated, the Cepheids without doubt are enormously large. Their small observed velocity variations, even if attributed altogether to motion in the line of sight and not at all to pressure-shifts, are not larger than might arise from a radial oscillation through but a small fraction of their mean diameters.[2] In the central mass of the star the period of the supposed pulsation should, of course, be perfectly regular, but its effect need by no means be regular on the radiating surface.

We may suppose that, because of the internal vibration, the photosphere of the star is periodically scattered or broken through by the rush of hotter gases from the interior. Maximum light and maximum velocity of approach would obviously be approximately synchronous, and their coincidence would naturally be independent of the direction of the observer in space. Ludendorff's correlation of range of light and range of velocity is highly significant in this connection.[3] The essentially harmonic nature of the oscillation at the surface of the star would easily lend itself to interpretation as elliptic motion, though non-elliptic motion need not be unexpected, nor the anomalous behavior of certain spectral lines. In stars in which the initial disturbance is of recent origin, the presence of secondary oscillations could be expected, which would affect the light as well as the velocity.[4]

It should be noted as an important factor in the explanation of Cepheid variation, that a change in the spectrum of a given radiating surface from one type to the next will change the visual brightness of that surface by approximately one stellar magnitude, and the color range by four-tenths of a magnitude. These quantities correspond very closely to what is observed in cluster variables, and suggest that, if desired, it is unnecessary to go farther for the explanation of the light-variation than to suppose that a surface of approximately constant area progressively changes its spectral type as the result of a periodic flow and ebb of heat.[1] That the light-change should be of a more explosive character for the cluster variables than for the longer-period Cepheids would be expected because of their higher mean densities.

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[1] *Astrophysical Journal*, **38**, 173, 1913.

[2] The radial motions observed in sun-spots are suggestive in this connection.

[3] *Astronomische Nachrichten*, **193**, 301, 1913. He finds that $2K=47.3A$ with close approximation, where A is the amplitude of magnitude variation and $2K$ is the total range of velocity variation in kilometers.

[4] By means of a detailed periodogram analysis of the light-curve of the famous irregular variable SS Cygni, Gibb has recently found a prominent underlying periodicity of 40.86 days (*Monthly Notices*, **74**, 678, 1914). The spectrum of SS Cygni is extremely peculiar in that it varies through a number of different types, occasionally showing bright lines and at other times an apparently continuous spectrum (*Harvard Annals*, **56**, 211, 1912).

Various other details suggesting the possibility of the above interpretation could be cited, but this sketch of the pulsation argument will suffice for the present, since the purpose of the paper is not so much to advance an alternative theory as to question the validity of the spectroscopic binary hypothesis.

MOUNT WILSON SOLAR OBSERVATORY
August 13, 1914

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[1] Cf. the hypothesis proposed by Schwarzschild as an alternative explanation of the change by six-tenths of a magnitude in the color index of η Aquilae (*Publikationen der v. Kuffnerschen Sternwarte*, **5**, C125, 1900).

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ANOMALOUS DISPERSION IN THE SUN IN THE LIGHT OF OBSERVATIONS

By CHARLES E. ST. JOHN

In a series of recent articles[1] Professor Julius has considered the displacements of the Fraunhofer lines at the center and limb edges of eccentrically located sun-spots from the point of view of his theory of anomalous dispersion. For the subject-matter he has used the data published in my paper on "Radial Motion in Sun-Spots."[2] In these articles he sets forth a new deduction from the theory of anomalous dispersion—the "mutual influence" of the Fraunhofer lines upon each other: in particular, that a weak line on the violet side of, and near to, a stronger line is displaced less, but if on the red side more, than the average amount. This deduction offers a means of making a quantitative test of the rôle played by anomalous dispersion in the solar atmosphere. These displacements are well suited for a definitive test of the theory, for they are purely differential, and numerous other lines upon the same plates are available for standards of reference.

Professor Julius selected 82 lines from my published list of displacements at the outer edge of the penumbrae of spots, and he finds that those to the violet of stronger lines are displaced less, and those to the red more, than the average. There are, however, many other lines in my list that fulfil the conditions of selection proposed by Professor Julius. These give results opposite to those obtained from the lines selected by him. Moreover, his treatment of the data in obtaining the normal displacements appears to have involved an error, the effect of which is to introduce residuals of the sign required by his theory. These points will now be given consideration in detail. In such a discussion three things demand attention in order that no systematic errors may be introduced and that proper weight may be given to crucial cases whose bearing upon the question is of special importance: (1) the determination of the standard displacements; (2) the inclusion of all lines falling within the adopted conditions of selection; (3) consideration of the effects in the immediate neighborhood of strong lines showing in the laboratory marked anomalous dispersion phenomena.

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[1] *Versl. kon. akad. v. wetensch. Amsterdam*, **22**, 1243, 1914; *Observatory*, **37**, 252, 1914; *Astrophysical Journal*, **40**, 1, 1914.

[2] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 327, 1913.

STANDARD DISPLACEMENTS

The 506 lines of Table I in my first paper[1] do not form a homogeneous series of observations. Attention was called to this point in my discussion of the data. From this it results that the normal displacements for different spectral regions cannot be determined with high precision by deducing them for each region by any smoothing-out process involving the results for all the segregated regions. The following regions were covered by the observations:

a) From λ 3624 to λ 3724. Here a few plates were taken with the purpose of obtaining the line λ 3694 assigned by Jewell to ytterbium, as it began to appear probable that the heavy elements would be of particular interest.

b) From λ 3879 to λ 4410. This is a homogeneous region, as overlapping plates were obtained by which the observations were interconnected and hence are comparable.

c) From λ 4634 to λ 4829. The earlier observations were upon this region, as it was desired first to repeat Evershed's observations. The mean result for all lines in this region is high in comparison with regions on either side; and, of the six regions, it is the one in which the necessity of separate consideration is the greatest, when the behavior of individual lines is to be studied.

d) From λ 5123 to λ 5349. This region includes the great magnesium lines of the *b* group.

e) From λ 5598 to λ 6065, in order to obtain the D lines of sodium.

f) From λ 6393 to λ 6643, to include the H_α line of hydrogen.

To obtain the normal displacements, Professor Julius used the mean displacements for the iron lines grouped into three regions, namely: the two violet series, mean λ 4017; the blue-green series, mean λ 4992; and the yellow-red series, mean λ 6121, reproduced in Table I.

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[1] *Mt. Wilson Contr.*, No. 69, p. 6; *Astrophysical Journal*, **37**, 333, 1913.

TABLE I

DISPLACEMENTS OF IRON LINES

Region	Intensity							
	1	2	3	4	5	6	7	8
λ 4017	0.022	0.020	0.014	0.014	0.013	0.011	0.008	0.006
λ 4992	0.030	0.026	0.026	0.025	0.018	0.016	0.006	0.007
λ 6121	0.032	0.030	0.032	0.024	0.028	0.024	0.019	0.016

Using these data, he says:

> . . . . On the basis of the two rules found by St. John that connect the displacements with line intensity and with wave-length, it was possible to indicate a "normal" displacement peculiar to the spectral region and the intensity of each measured line. With these normal values the observed values had to be compared.[1]

The method of derivation of the normal displacements is not explicitly stated, but supposedly they were obtained either graphically or analytically from the data given. In either case the derived values would depend upon, and be influenced by, the displacements in the separate regions and would not refer with high precision to any one of the homogeneous regions. The mean displacements for the iron and nickel lines of intensity 2–3 for each of the six spectral regions are plotted in Fig. 1, and the curve drawn with consideration of the weights of the points. Owing to the very large displacements given by the blue region, λ 4634–λ 4829, the normals for the adjoining regions, deduced from the data for all regions, are too large, and for the blue region itself too small. The use of normal displacements interpolated from such a graph might be justifiable, if the lines whose displacements are to be compared with these normals were well distributed over the regions concerned in their derivation. It is evident that if the lines whose displacements are to be compared with such normals are not evenly distributed, but are confined to one or two regions, systematic

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[1] *Astrophysical Journal*, **40**, 13, 1914.

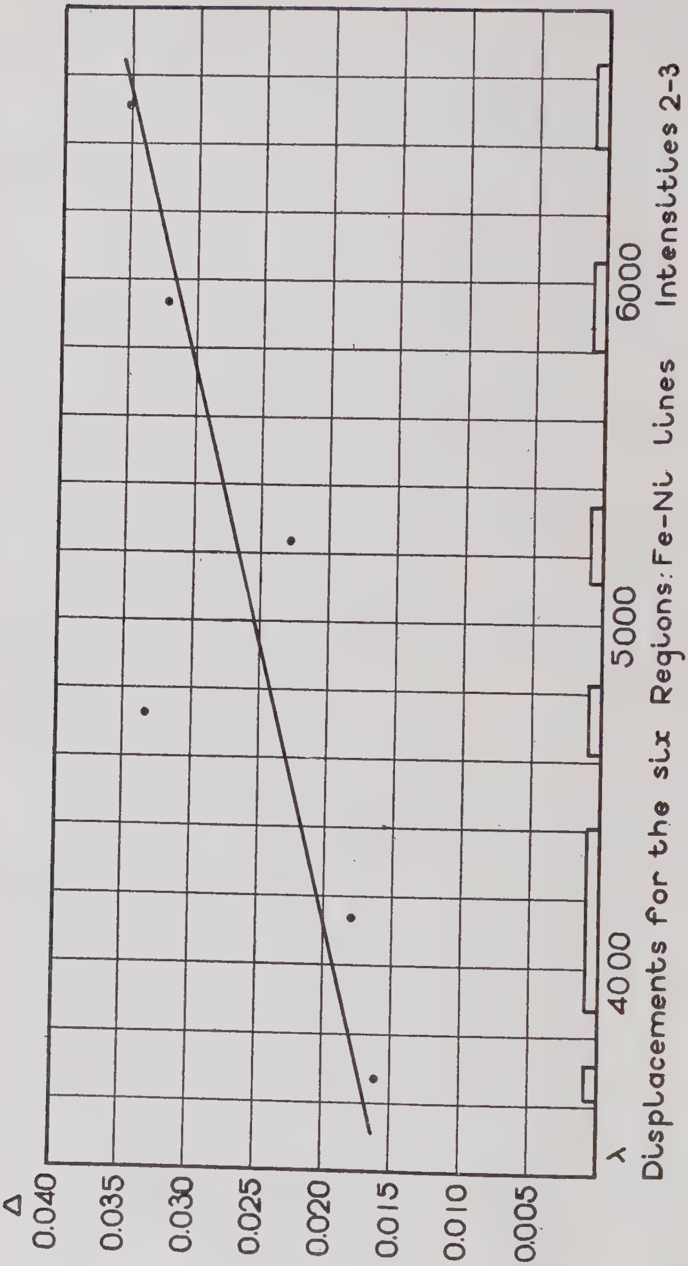
Δ
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
λ
4000
5000
6000
Displacements for the six Regions: Fe-Ni Lines Intensities 2-3

FIG. 1

residuals will result. Moreover, the normal displacements used by Professor Julius are derived from the displacements of the iron lines only. This neglects any differences depending upon individual elements, differences that are real and systematic.

To obtain residuals capable of representing any systematic behavior of lines within the refractive influence of stronger lines, the standard displacements should be derived from the limited homogeneous series of observations in which they are to be used, and, as far as the data permit, they should refer to the particular elements under consideration. Standard displacements of this character have been obtained by determining the mean displacements for the lines of each element of a given intensity for each of the six homogeneous series. In the rare cases where fewer than three lines are available, the mean displacement for all lines of the same intensity in the respective series has been used.

In the first section of Tables II and III are reproduced the data given by Professor Julius for lines on the violet and red sides of stronger lines, respectively. In the eighth column are shown the mean displacements for the lines of the elements of the intensity and region under consideration, and in the ninth the residuals obtained by using these means as standard displacements. In the last column are remarks relative to the character and properties of the influencing lines that bear upon their power to produce the assumed effects.

ADDITIONAL LINES

Attention has been called to the necessary condition that all lines within the definition of the category be included. Professor Julius says:

> . . . . I selected *all* cases in which a measured line *A* of intensity 3 or lower was on the *violet* side of a stronger line *B* (generally of intensity 4 or higher) at a distance of about 0.5 A or less. A few cases in which the line *A* had another strong companion *B′* equally near but on the other side were of course discarded. Forty-three pairs answering the conditions were found. . . . .[1]

After a definition has been adopted, it is always hazardous to disregard it. The lanthanum line, λ 3995, of intensity 1, is between

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[1] *Astrophysical Journal*, 40, 13, 1914.

TABLE II A

FRAUNHOFER LINES TO THE VIOLET OF STRONGER LINES AS SELECTED BY PROFESSOR JULIUS

LINES	DIFFERENCE	ELEMENT	INTENSITY	OBSERVED DISPLACEMENT	JULIUS		ST. JOHN		
					Normal Displacement	Residual	Mean	Residual	Remarks
3649.137 3649.438	.301	Cr	1 5	0.014	0.022	−0.008	0.018	−0.004	
3662.096 3662.378	.282	Ni (Ti)	3 5	.015	.015	.000	.015	.000	
3686.926 3687.234	.308	Cr	1 3	.016	.022	− .006	.018	− .002	
3687.234 3687.610	.376	Fe Fe	3 6	.010	.015	− .005	.015	− .005	
3688.210 3688.558	.348	V	1 4	.018	.022	− .004	.017	+ .001	
3690.599 3690.870	.271	Fe	2 3	.017	.020	− .003	.016	+ .001	
3707.702 3708.07	.366	Ti Fe	2 6	.013	.020	− .007	.016	− .003	
3708.964 3709.389	.425	Co Fe	1 6	.015	.022	− .007	.018	− .003	
3895.119 Blend		Co	3 4?	.012	.015	− .003	.013	− .001	Blend of weak lines
3895.583 3895.803	.220	Mn Fe	3 7	0.008	0.015	−0.007	0.017	−0.009	Anomalous dispersion very weak

3898.032		Fe	3	0.007	0.015	−0.008	0.015	−0.008	
3898.2			4						Perhaps V5 at 3898.151
3899.171		Fe ·	3	.013	.015	− .002	.015	− .002	A line displaced by its own action as it forms the major portion of the influencing line
3899.21		(V-Fe)	3						
3906.438		Co	2	.010	.020	− .010	.016	− .006	
3906.628	.190	Fe	10						Anomalous dispersion very weak
3913.123		Ni	2	.016	.020	− .004	.019	− .003	
3913.609	.486	Ti	5						
3916.879		Fe	5	.009	.014	− .005	.014	− .005	
3917.324	.445	(Fe)	6						Intensity 5
3947.522		?	2	.014	.021	− .007	.019	− .005	
3947.675	.153	Fe	4						
3956.603		Fe	4	.010	.014	− .004	.014	− .004	
3956.879	.276	Fe	6						
3958.073		Co	2	.018	.022	− .004	.016	+ .002	
3958.355	.282	(Zr-Ti)	4						
3962.995		Ti	3	.012	.016	− .004	.012	.000	
3963.281	.286		4						Blend
3995.899		La	1	.014	.023	− .009	.022	− .008	On the other side is a line of intensity 5, 0.436 A distant, whose effect is neglected
3996.140	.241		3						
3997.115		Fe	2	.019	.022	− .003	.019	.000	
3997.547	.432	Fe	4						
4035.752		Co	2	0.016	0.022	−0.006	0.016	0.000	
4035.883	.131	Mn	4						Anomalous dispersion strong

TABLE II A—*Continued*

LINES	DIFFERENCE	ELEMENT	INTENSITY	OBSERVED DISPLACEMENT	JULIUS		ST. JOHN		
					Normal Displacement	Residual	Mean	Residual	Remarks
4109.609		Nd	1	0.016	0.024	−0.008	0.022	−0.006	
4109.953	.344		4						Intensity 3
4132.100		V	2	.011	.022	− .011	.021	− .010	
4132.235	.135	Fe	10						
4133.755		Fe	2	.022	.022	.000	.019	+ .003	
4133.95	.255	(Fe-Ce)	4						Fe 3 at 4134.010
4149.360		Zr	2	.016	.022	− .006	.018	− .002	
4149.533	.173	Fe	4						
4216.136		CN	1	.022	.024	− .002	.018	+ .004	
4216.351	.215	(Fe)	4						Intensity 3
4233.328		(Mn-Fe)	4	.017	.017	.000	.014	+ .003	
4233.772	.444	Fe	6						
4271.325		Fe	6	.009	.014	− .005	.011	− .002	
4271.934	.609	Fe	15						Anomalous dispersion very weak
4274.746		Ti	2	.016	.023	− .007	.017	− .001	
4274.958	.212	Cr	7						Anomalous dispersion very strong
4289.525		Ca	4	.015	.019	− .004	.016	− .001	
4289.885	.360	Cr	5						Anomalous dispersion very strong
4294.936		Zr	2	.018	.023	− .005	.018	.000	
4295.29	.354	Dy	6						Dy? Blend
4302.353		Fe	2	0.019	0.023	−0.004	0.019	0.000	
4302.692	.339	Ca	4						Anomalous dispersion very weak

4315.138..... 4315.262.....	.124	Ti Fe	3 4	0.010	0.021	−0.011	0.012	−0.002	
4408.364..... 4408.54......	.218	V (V)	2 4	.027	.024	+ .003	.021	+ .006	Fe 3 at 4408.582
5168.832..... 5169.16......	.323	Ni (Fe)	1 7	.026	.028	− .002	.028	− .002	Blend
5188.863..... 5189.018.....	.155	Ti Ca	2 3	.015	.026	− .011	.019	− .004	
5226.707..... 5227.043.....	.336	Ti Fe-Cr	2 4	.020	.026	− .006	.019	+ .001	Intensity 3
5250.385..... 5250.817.....	.432	Fe (Fe)	2 3	.028	.027	+ .001	.023	+ .005	
5298.194..... 5298.455.....	.261	Cr Cr	1 4	.021	.029	− .008	.024	− .003	Anomalous dispersion weak
5598.524..... 5598.711.....	.187	Fe Ca	1 4	.019	.030	− .011	.025	− .006	Anomalous dispersion very weak
5615.520..... 5615.877.....	.357	Fe Fe	2 6	.025	.027	− .002	.023	+ .002	
5624.245..... 5624.769.....	.524	Fe (Fe)	1 4	0.027	0.031	−0.004	0.025	+0.002	
Mean....	....	....	....	....	....	−0.0051A	....	−0.0018A	

TABLE II B

Additional Lines to the Violet of Stronger Lines

Lines	Difference	Element	Intensity	Observed Displacement	Julius		St. John		
					Normal Displacement	Residual	Mean	Residual	Remarks
3630.045.....		Ni	1	0.024	0.022	+0.002	0.018	+0.006	
3630.492.....	.447	Fe	4	....	....	....	....	....	
3692.364.....		V	1	.017	.022	− .005	.017	.000	
3692.790.....	.426	Fe	2	....	....	....	....	....	
3692.790.....		Fe	2	.020	.020	.000	.016	+ .004	
3692.170.....	.380	Fe	3	....	....	....	....	....	
3895.377.....		Ti	2	.020	.020	.000	.017	+ .003	
3895.803.....	.426	Fe	7	....	....	....	....	....	
3897.596.....		Fe	2	.009	.020	− .011	.019	− .010	
3898.032.....	.436	Fe	3	....	....	....	....	....	
4006.304.....		Ni	1	.024	.023	+ .001	.024	.000	
4006.464.....	.160	Fe	2	....	....	....	....	....	
4006.464.....		Fe	2	.020	.022	− .002	.019	+ .001	
4006.776.....	.312	Fe	3	....	....	....	....	....	
4024.726.....		Ti	3	.016	.015	+ .001	.012	+ .004	
4024.881.....	.155	Fe	4	....	....	....	....	....	
4070.431.....		Mn	3	.023	.016	+ .007	.017	+ .006	
4070.930.....	.499	Fe	4	....	....	....	....	....	
4111.509.....		Ce	1	.028	.024	+ .004	.022	+ .006	
4111.940.....	.431	V	4	....	....	....	....	....	
4136.678.....		Fe	4	0.023	0.014	+0.009	0.014	+0.009	
4137.156.....	.478	Mn	6	....	....	....	....	....	

4183.480.....		Zr	1	0.024	0.024	0.000	0.021	+0.003	
4183.619.....	.139	V	2N	....	....	....	....	....	
4195.006.....		Cr	1	.020	.024	− .004	.021	− .001	
4195.492.....	.486	Fe	5	....	....	....	....	....	
4277.544.....		Zr	0	.026	.025	+ .001	.020	+ .006	
4277.692.....	.148	....	2d?	....	....	....	....	....	
4284.838.....		Ni	1	.030	.024	+ .006	.024	+ .006	
4285.164.....	.326	Ti	2	....	....	....	....	....	
4289.237.....		Ti	2	.022	.023	− .001	.017	+ .005	
4289.525.....	.288	Ca	4	....	....	....	....	....	Anomalous dispersion very weak
4314.964.....		Ti	1	.025	.024	+ .001	.023	+ .002	
4315.138.....	.174	Ti	3	....	....	....	....	....	
4377.948.....		Fe	1	.030	.024	+ .006	.022	+ .008	
4378.419.....	.471		2Nd?	....	....	....	....	....	
4708.846.....		Ti	2	.031	.024	+ .007	.032	− .001	
4709.271.....	.425	Fe	3	....	....	....	....	....	
4752.289.....		Ni	2	.040	.024	+ .016	.037	+ .003	
4752.613.....	.324	Ni	3	....	....	....	....	....	
4756.300.....		Cr	2	.026	.024	+ .002	.032	− .006	
4756.705.....	.405	Ni	3	....	....	....	....	....	Anomalous dispersion ?
4789.528.....		Cr	2	.034	.024	+ .010	.032	+ .002	
4789.849.....	.321	Fe	3	....	....	....	....	....	
5148.222.....		Fe	2	.023	.024	− .001	.023	.000	
5148.410.....	.188	Fe	3	....	....	....	....	....	
6496.688.....		Fe	2	0.037	0.031	+0.006	0.033	+0.004	
6497.128.....	.440	Fe	4	....	....	....	....	....	
Mean....	....	....	....	....	....	+0.0023A	....	+0.0025A	

TABLE III A

FRAUNHOFER LINES TO THE RED OF STRONGER LINES AS SELECTED BY PROFESSOR JULIUS

Lines	Difference	Element	Intensity	Observed Displacement	Julius		St. John		
					Normal Displacement	Residual	Mean	Residual	Remarks
3694.24.... 3694.344...	.209	(Fe-Ni) Yt	8 3	0.020	0.015	+0.005	0.015	+0.005	Blend of intensity 6 at 3694.135
3694.344... 3694.576...	.232	Yt La	3 1	.027	.022	+ .005	.018	+ .009	
3704.603... 3704.842...	.239	Fe V	4 1	.016	.022	− .006	.018	− .002	
3706.24.... 3706.363...	.188	(Mn-Ti-Ca) Fe	7 3	.017	.015	+ .002	.015	+ .002	Intensity 6 at 3706.175
3711.364... 3711.552...	.188	Fe Fe	4 3	.015	.015	.000	.015	.000	
3898.2..... 3898.531...	.380	.... Mn	4 2	.014	.020	− .006	.020	− .006	V5 at 3898.151
3947.675... 3947.918...	.243	Fe Ti	4 2	.013	.021	− .008	.017	− .004	
3948.925... 3949.039...	.114	Fe Ca	4 1	.018	.022	− .004	.022	− .004	
3950.102... 3950.497...	.395	Fe Y	15? 5	.013	.014	− .001	.014	− .001	Intensity 5
3984.17.... 3984.294...	.203	(Fe-Mn) Mn	6 2	.021	.020	+ .001	.020	+ .001	The influenced line is a part of the blend
3989.912... 3990.011...	.099	Ti Fe	4 3	0.012	0.015	−0.003	0.015	−0.003	

4018.25 4018.420	.164	(Mn) Fe	7 3	0.022	0.015	+0.007	0.015	+0.007	Blend at 4018.256
4078.49 4078.631	.116	? Ti	4 3	.017	.016	+ .001	.012	+ .005	Fe 4 at 4078.515
4079.4 4079.570	.200	 Mn	5 3	.018	.016	+ .002	.017	+ .001	Blend of Mn 3 and Fe 2 at 4079.370
4134.54 4134.840	.300	(V-Fe) Fe	6 5	.014	.014	.000	.014	.000	Blend of two lines of intensity 3
4161.682 4161.961	.279	(Ti) Sr	5 1	.025	.024	+ .001	.022	+ .003	Intensity 4
4184.32 4184.472		(Ti-Gd) Ti	5 2	.022	.022	.000	.017	+ .005	The influenced line is a part of the blend
4196.35 4196.699	.327	? La	4 2	.024	.022	+ .002	.019	+ .005	Fe 4 at 4196.372
4236.112 4236.429	.317	Fe Ni	8 1	.024	.024	.000	.024	.000	Anomalous dispersion very weak
4240.64 4240.872	.332	(Zr-Ce-Fe) Cr	4 1	.022	.024	− .002	.021	+ .001	Intensity 2 at 7240.540
4338.084 4338.430	.346	Ti Fe	4 1	.025	.024	+ .001	.022	+ .003	
4637.685 4638.193	.508	Fe Fe	5 4	.027	.021	+ .006	.029	− .002	
4667.626 4667.768	.142	Fe Ti	4 3	.027	.023	+ .004	.030	− .003	
4679.027 4679.409	.382	Fe Ni	6 2	.037	.024	+ .013	.037	.000	
4703.177 4703.994	.817	Mg Ni	10 3	.035	.023	+ .012	.034	+ .001	
4731.651 4731.984	.333	(Fe) Ni	4 1	0.030	0.026	+0.004	0.036	−0.006	

TABLE III A—*Continued*

LINES	DIFFERENCE	ELEMENT	INTENSITY	OBSERVED DISPLACEMENT	JULIUS		ST. JOHN		
					Normal Displacement	Residual	Mean	Residual	Remarks
4736.963... 4737.540...	.577	(Fe) Cr	6 2	0.034	0.024	+0.010	0.032	+0.002	
4762.567... 4762.820...	.253	Mn Ni	5 1	.039	.026	+ .013	.036	+ .003	Anomalous dispersion?
5129.42.... 5129.546...		(Ti-Ni) Ni	5 2	.026	.024	+ .002	.024	+ .002	The influenced line is a part of the blend
5129.546... 5129.805...	.259	Ni Fe	2 1	.033	.028	+ .005	.028	+ .005	
5131.642... 5131.942...	.300	(Fe-C) Ni	3 1	.029	.028	+ .001	.028	+ .001	Intensity 2
5152.087... 5152.361...	.274	(Fe-C) Ti	3 0	.031	.031	.000	.027	+ .004	
5192.523... 5193.139...	.616	(Fe-Nd) Ti	5 2	.021	.026	− .005	.019	+ .002	
5283.802... 5284.281...	.479	(Fe) Ti	6 1	.026	.028	− .002	.028	− .002	
5298.455... 5298.672...	.217	Cr Ti	4 1	.022	.028	− .006	.028	− .006	Anomalous dispersion weak
5349.652... 5349.928...	.276	Ca Fe	4 1	.027	.028	− .001	.028	− .001	
5857.674... 5857.976...	.302	Ca Ni	8 3	.030	.027	+ .003	.032	− .002	
5953.0..... 5953.386...		(Ti-Fe) Ti	5 1	.038	.032	+ .006	.038	.000	The influenced line is a part of the blend
6400.217... 6400.528...	.311	Fe Fe	8 2	0.026	0.031	−0.005	0.035	−0.009	
Mean...	....	....	......	....	....	+0.0014A	....	+0.0004A	

TABLE III B

ADDITIONAL LINES TO THE RED OF STRONGER LINES

LINES	DIFFERENCE	ELEMENT	INTENSITY	OBSERVED DISPLACEMENT	JULIUS		ST. JOHN		
					Normal Displacement	Residual	Mean	Residual	Remarks
3623.925....	.333	Mn-Fe	4						
3624.258....		Ca	3	0.012	0.015	−0.003	0.015	−0.003	
3645.967....	.368	Fe	4						
3646.335....		Ti	1	.012	.022	− .010	.018	− .006	
3693.170....	.446	Fe	3						
3693.616....		Co	1	.021	.022	− .001	.018	+ .003	
3724.526....	.444	Fe	6						
3724.970....		Ni	1	.018	.022	− .004	.018	.000	
3900.681....	.433	Ti-Fe-Zr	5						
3901.114....		Ti	1	.017	.023	− .006	.023	− .006	
3968.625....	.788	Ca	700						Anomalous dispersion strong
3969.413....		Fe	10	.004	.003	+ .001	.003	+ .001	
3984.091....	.388	Cr-Fe	7						
3984.479....		Cr	2	.024	.022	+ .002	.024	.000	
3991.333....	.247	Cr-Zr	3						
3991.580....		Fe	1	.020	.023	− .003	.022	− .002	
3992.971....		V-Cr	3						
3993.246....	.275	Fe	2	.025	.022	+ .003	.019	+ .006	
4020.424....	.123		2						
4020.547....		Sc	1	.014	.023	− .009	.022	− .008	
4023.533....	.301	Co	3						
4023.834....		Sc	2	.026	.022	+ .004	.020	+ .006	
4118.708....	.226	Fe	5						
4118.934....		Co	4	0.013	0.015	−0.002	0.015	−0.002	

TABLE III B—*Continued*

LINES	DIFFERENCE	ELEMENT	INTENSITY	OBSERVED DISPLACEMENT	JULIUS		ST. JOHN		
					Normal Displacement	Residual	Mean	Residual	Remarks
4137.567....	.242	....	2	....	....	....	....	....	
4137.809....		Fe, Ce	1	0.014	0.024	−0.010	0.022	−0.008	
4150.411....	.197	....	4	....	....	....	....	....	
4150.608....		Co	1	.017	.024	− .007	.022	− .005	
4196.699....	.138	La	2	....	....	....	....	....	
4196.837....		Fe	1	.020	.022	− .002	.022	− .002	
4224.673....	.347	Cr-Fe	3	....	....	....	....	....	
4225.020....		Cr	2	.021	.023	− .002	.022	− .001	
4226.904....	.702	Ca	20	....	....	....	....	....	Anomalous dispersion very strong
4227.606....		Fe	4	.010	.017	− .007	.014	− .004	
4261.891....	.251	....	2	....	....	....	....	....	
4262.142....		Cr	1	.024	.024	.000	.022	+ .002	
4273.482....	.161	Fe	3N	....	....	....	....	....	
4273.643....		Zr	2N	.021	.023	− .002	.020	+ .001	
4289.885....	.492	Cr	5	....	....	....	....	....	Anomalous dispersion very strong
4290.377....		Ti	2	.013	.023	− .010	.017	− .004	
4295.914....	.130	Cr-Ti	2	....	....	....	....	....	
4296.044....		Ni	1	.020	.024	− .004	.024	− .004	
4304.729....	.153	....	2	....	....	....	....	....	
4304.882....		Zr	0	.020	.025	− .005	.020	.000	
4359.784....	.123	Cr	3	....	....	....	....	....	Anomalous dispersion very weak
4359.907....		Zr	0	.019	.025	− .006	.020	− .001	
4400.555....	.183	Sc	3	....	....	....	....	....	
4400.738....		V	1	.021	.024	− .003	.022	− .001	
4786.727....	.276	Ni	3	....	....	....	....	....	
4787.003....		Fe	2	0.025	0.024	+0.001	0.030	−0.005	
Mean....	....	....	......	....	....	−0.0034A	....	−0.0017A	

two lines of intensities 3 and 5, each within the limiting distance of 0.5 A. It is a delicate question to decide whether the line of intensity 3, which is nearer, or the line of intensity 5 is the more influential. The measured displacement of this lanthanum line is small, 0.014 A. It is on the violet side of the weaker line and shows a large negative residual. The decision was in favor of the weaker. The lanthanum line λ4123 shows a large displacement, 0.023 A. It is 0.523 A to the violet of a much stronger line. It is omitted, though favorable cases exceeding the limit by much larger amounts are included. Of the 14 lines included by Professor Julius in which either the distance exceeds 0.5 A, or some one or more of the other limitations are exceeded, two only are unfavorable to the thesis under consideration.

It is a question whether the mutual influence of a blend is the same as that of a single line whose intensity is the sum of the intensities of the constituents; for example, it is at least doubtful whether a line of intensity 5 should be considered susceptible to the influence of a blend consisting of 2 lines each of intensity 3 to the same degree as of a single line of intensity 6, as Professor Julius has apparently done. However this may be, it seems clearly inadmissible to include the influenced line itself in the blend and particularly inadmissible when the influenced line forms the major part of the blend, for then we have the remarkable case of a line displaced by its own influence, as λ 3899.171.

An examination of Rowland's Table shows 49 lines not included by Professor Julius, 24 on the violet and 25 on the red side of stronger lines. Of these additional lines those on the violet are all within the limiting distance of 0.5 A; of those on the red, two exceed it. These two lines are now included, mainly because of their important bearing on the theory of anomalous dispersion: they are a line of intensity 4 at 0.7 A to the red of a line of intensity 20, and a line of intensity 10 at 0.79 A from a line of solar intensity 700. In both cases the anomalous dispersion of the controlling lines is strong and the lines are within the limits used by Professor Julius, as he included the Ni line λ 4703 of intensity 3 at 0.817 A from a line of intensity 10. For the additional lines the normal displacements given by Professor Julius are used for their respective intensities and spectral regions. In a few cases in which a direct transfer could not be made, the normals are interpolated from those used by Professor Julius.

The negative residual of −0.0051 A found by Professor Julius for the 43 lines to the violet is reduced to −0.0018 A when the displacement for each line is compared with the mean for the element, intensity, and region; but if all lines to the violet are taken into account, the corresponding mean residuals are −0.0024 A and −0.0003 A. When all lines to the red of stronger lines are considered, the mean residual is practically zero whichever standard is used. The results are assembled in Table IV.

TABLE IV

RESIDUALS GIVEN BY WEAK SOLAR LINES NEAR STRONGER LINES

Lines to Violet			Lines to Red		
	According to			According to	
	Julius	St. John		Julius	St. John
43 original........	−0.0051	−0.0018	39 original........	+0.0014	+0.0004 A
24 additional......	+ .0023	+ .0025	25 additional.....	− .0034	− .0017
Weighted mean	−0.0024	−0.0003	Weighted mean	−0.0005	−0.0004

A comprehensive impression may be obtained by combining the residuals derived in this paper as in Table V.

TABLE V

	Lines to Violet	Lines to Red	Total
Sum of favorable residuals..............	−0.125	+0.087	0.212 A
Sum of unfavorable residuals............	+0.105	−0.113	.218
Unfavorable excess from 131 lines........	....	....	.006
Mean excess per line...............	....	....	0.00005

This confirms the conclusion expressed in my second paper on "Radial Motion in Sun-Spots" that until we are able to depend upon solar wave-lengths to the fourth decimal place, at least, the contribution of anomalous dispersion to the relative positions of Fraunhofer lines, if any, will be masked by phenomena due to other causes.

SYSTEMATIC ERRORS

The results obtained by Professor Julius will now be discussed in the light of the systematic errors that appear to have been introduced by the method used by him in obtaining the normal displacements.

By referring to the graph in Fig. 1, or to a somewhat similar graph in the paper by Professor Julius,[1] it will be seen that normal displacements derived by combining the observations of the six series are too high for the violet and green regions because of the very large displacements given by the blue region, λ 4634–λ 4829.

In Table VI are given the residuals obtained by comparing the normal displacements used by Professor Julius with the mean displacements of all lines of the corresponding intensities in the respective regions.

TABLE VI

COMPARISON OF THE JULIUS NORMALS WITH THE MEANS FOR ALL LINES OF LIKE INTENSITY

Region	Intensity	Means	Normals	Residuals	Mean
Violet..........	1	0.0205	0.0220	−0.0015 A	−0.0022 A
	2	.0182	.0218	− .0036	
	3	.0147	.0159	− .0012	
	4	.0140	.0167	− .0027	
Blue..........	1	.0376	.026	+ .0116	+ .0093
	2	.0322	.024	+ .0082	
	3	.0334	.023	+ .0104	
	4	.0281	.021	+ .0071	
Green..........	1	.0275	.0285	− .0010	−0.0024
	2	0.0225	0.0263	−0.0038	

Of the 43 lines on the violet side of strong lines, 35 are in the violet region and 8 in the green, so that for these lines the residuals must of necessity be systematically negative, and this accounts in great part for the mean negative residual of −0.0051 A. Of the 24 additional lines, 19 are in the violet and green regions, so that of the 67 lines on the violet side of stronger lines 62 are in the regions where the normals derived from a consideration of the displacements for all regions are too high; and the systematic errors introduced account for the final negative value −0.0024 A, when such normals are used, for it is evident from Table VI that any large group of lines selected at random from the violet and green regions must yield a mean negative residual of about −0.0023 A.

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[1] *Astrophysical Journal*, 40, 23, Fig. 6, 1914.

On the other hand, of the 39 lines on the red side of stronger lines some occur in each spectral region and the systematic residuals are less in evidence because of the more even distribution of the lines. It is illuminating, however, to consider them somewhat in detail. Of the 39 lines 7 are in the blue region, λ 4634–λ 4829, where the derived normals are too small and hence the residuals are necessarily positive. In fact, for these 7 lines the sum of the residuals is +0.063 A, while for the other 32 lines the sum is −0.005 A. It is evident from Table VI that a few lines from the blue region will furnish positive residuals sufficient to overbalance the negative residuals that the other 32 lines yield. It is equally evident that no weight can be given to these seven positive residuals of mean magnitude +0.009 A when the 22 other lines of the same elements and intensities in this region that are not on the red side of stronger lines give a mean residual of +0.009 A, precisely the same as that given by the lines within the limiting distance of stronger lines. In fact, it is impossible to select 7 lines from the 29 of the same elements and intensities that would not give a mean positive residual irrespective of their distance from stronger lines. When all the lines to the red are taken into the reckoning, they are fairly well distributed, and when so large a number is involved, the errors introduced by the normal displacements used by Professor Julius cease to be systematic and the positive residual disappears.

MUTUAL INFLUENCE AND LABORATORY RESULTS

The third point requiring attention is whether the mutual influence exerted by a strong line on a weak one in the solar atmosphere varies with the amount of anomalous dispersion manifested by the strong line under terrestrial conditions. The exaggerated importance ascribed to the possible effects of anomalous dispersion in the solar atmosphere apparently arose from the fact that anomalous dispersion was first studied in the laboratory with the D lines of sodium which show it to an enormous degree, and the suggestion that such a phenomenon must be of influence in the solar atmosphere seemed to foreshadow great possibilities. Later investigation has shown that the anomalous dispersion possessed by the two sodium lines is exceptional and that few lines show it to a marked degree. Geisler has investigated some 300 lines belonging to 25 elements and found the following results:[1]

	Anomalous Dispersion					
	Very Strong	Strong	Medium	Weak	Very Weak	?
No. of lines........	9	17	34	80	123	36

The effect in the case of the sodium lines is so large that they cannot be included in a scale adjusted to the other lines. In view of the rarity of strong anomalous dispersion, it seems probable that the effect, in the few cases where weak lines are under the influence of strong lines showing anomalous dispersion to a high degree, would stand out clearly in contrast to the instances where the anomalous dispersion of the influencing line is unknown or very weak. It was with such a comparison in view that care was taken to include among the additions the line of intensity 4 at 0.7 A to the red of the line λ 4226.9 of calcium, intensity 20, which shows very strong anomalous dispersion, and the iron line λ 3969 of intensity 10 which is 0.788 A to the red of the H line of calcium, intensity 700. This calcium line has strong anomalous dispersion, and its "dispersion band" is some 10 or 12 A broad. Professor Julius has given an explanation of the calcium flocculi, of the progressive changes in the wave-lengths of the H and K lines in passing from the center to the limb, and of the great wings of these lines, based upon their anomalous dispersion. It would seem that the neighborhood of such lines is precisely the place where the mutual influence would manifest itself, and any failure here bears heavily against the theory. There are 5 Fe lines of intensity 10 measured in the violet region, namely:

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[1] *Zeitschrift für wissenschaftliche Photographie*, **7**, 89, 1909.

λ	Radial Displacement	
3906	0.003 A	
3969	0.004	0.79 A to red of calcium H, intensity 700
4132	0.002	
4260	0.003	
4404	0.003	
	0.003 A	

The remarkable thing is not the divergence in the displacements, but the extraordinary agreement. The iron line λ 4227 of intensity 4 is 0.7 A to the red of the calcium line 4226.9 of intensity 20. The calcium line shows very strong anomalous dispersion, but the residual for the iron line is −0.007 A or −0.004 A, according to whether the normals of Professor Julius or the means for Fe lines of intensity 4 in this region are used as standards of reference.

The comparative behavior of weak lines under the influence of strong lines showing great anomalous dispersion, and of those under the influence of strong lines showing weak and very weak anomalous dispersion, would seem to be of great importance and to deserve particular attention in the effort to establish a theory of mutual influence depending upon anomalous dispersion.

The data for such lines appear in Table VII, where the means show no systematic variation in the residuals depending upon the extreme differences in the anomalous dispersion exhibited by the influencing lines under laboratory conditions; and as there is no systematic difference between these residuals and those involving all the lines, it appears that the influencing lines showing either great or small anomalous dispersion produce neither more nor less effect than those manifesting it in a moderate degree.

ADDITIONAL MEASUREMENTS

So far the criterion of mutual influence has been applied only to the data given by the 506 lines of my original table. Since the announcement of the deduction of mutual influence new measures have been made upon the plates. In making the original series of measurements, lines in the near neighborhood of very strong lines were in general omitted from the observing program, but in the case of elements represented by few lines, these were measured wherever found. In this new series the measures have been extended to all lines in the near neighborhood of strong lines showing great anomalous dispersion. The measurements have been made only upon a few plates of the best quality and include many lines extremely difficult to measure with precision, so that large accidental variations may be expected and the statistical method must be relied upon to determine probable results.

TABLE VII

Anomalous Dispersion of Influencing Lines

λ	Very Strong and Strong		λ	Weak, Very Weak, and ?		
	Julius	St. John		Julius	St. John	
4035...	−0.006	0.000	3895..	−0.007	−0.009	To the violet of stronger lines
4274...	− .007	− .001	3906..	− .010	− .006	
4289...	−0.004	−0.001	4271..	− .005	− .002	
			4289..	− .001	+ .005	
			4302..	− .004	.000	
			4756..	+ .002	− .006	
			5298..	− .008	− .003	
			5598..	−0.011	−0.006	
Mean	−0.0057	−0.0007	....	−0.0055	−0.0034	
3969...	+0.001	+0.001	4236..	0.000	0.000	To the red of stronger lines
4227...	−0.007	−0.004	4290..	− .010	− .004	
			4359..	− .006	− .001	
			4762..	+ .013	+ .003	
			5298..	− .006	− .006	
			5857..	+0.003	−0.003	
Mean	−0.0030	−0.0015	....	−0.0010	−0.0018	

A few observations have also been made upon a large symmetrical spot that appeared on the eastern limb of the sun August 13, 1914. Measures of these plates are combined with those from the first series and are given in Tables VIII and IX. The displacements of 144 lines have been measured, of which 16 are within 1 A of the

TABLE VIII

Residuals for Lines to the Violet of Much Stronger Lines Showing Large Anomalous Dispersion

Influencing Line	Element	Intensity	Distance 0–1 A	Residual	Distance 1–2 A	Residual	Distance 2–3 A	Residual	Distance 3–5 A	Residual
3933.825	Ca	1000	....	....	1.040	−0.006	2.556	−0.008	3.375	−0.001
					1.664	− .016	....	....	3.803	− .004
									4.328	− .004
									4.462	− .004
									4.565	− .004
3944.160	Al	15	0.439	+0.001						
			.671	+ .002						
			.838	+ .016						
			.922	+ .012						
3961.674	Al	20	.393	− .001	1.252	+ .002				
3968.625	Ca	700	....	....	1.055	− .008	2.413	− .006	3.962	− .003
					1.847	− .004	2.556	− .005	4.209	− .002
					1.978	− .008	2.970	−0.003	4.794	+0.002
4077.885	Sr	8	.926	.000	1.093	− .003				
					1.241	− .001				
					1.510	− .004				
					1.784	− .001				
4226.904	Ca	20	.320	− .005	1.030	− .003				
			0.788	−0.004	1.285	−0.003				
Weighted mean	....	....	....	+0.0006	....	−0.0038	....	−0.0054	....	−0.0023
No. of lines	....	....	....	8	....	12	....	4	....	8
No. of measures	....	....	....	51	....	67	....	25	....	44

TABLE IX

Residuals for Lines to the Red of Much Stronger Lines Showing Anomalous Dispersion

Influencing Line	Element	Intensity	Distance 0–1 A	Residual	Distance 1–2 A	Residual	Distance 2–3 A	Residual	Distance 3–5 A	Residual
3933.825	Ca	1000	....	....	1.638	−0.010	2.140	−0.002	3.654	−0.002
							2.296	− .009	4.335	− .005
									4.614	− .004
									4.727	− .001
3944.160	Al	15	0.724	+0.002	1.100	+ .002				
			.873	+ .001	1.198	+ .008				
			.968	+ .002	1.313	+ .001				
					1.833	− .001				
3961.674	Al	20	.815	− .001	1.321	+ .001				
					1.578	− .005				
3968.625	Ca	700	.788	+ .001	1.159	− .002	2.850	−0.004	3.688	− .009
					1.274	− .008	....	....	4.095	.000
					1.794	− .004	....	....	4.431	+0.006
					1.915	− .004				
4077.885	Sr	8	.630	− .001	1.685	− .003				
			.746	+ .004						
4226.904	Ca	20	0.702	−0.004	1.199	− .003				
					1.975	−0.002				
Weighted mean	....	....	....	+0.0003	....	−0.0018	....	−0.0049	....	−0.004
No. of lines	....	....	....	8	....	14	....	3	....	7
No. of measures	....	....	....	62	....	84	....	21	....	37

very strong lines showing great anomalous dispersion. For standards of reference the mean displacements for all lines of a given intensity were used. For each line within 1 A of the strong lines, there are, on an average, 8 lines of the same intensity beyond this limit, upon which the standard displacement depends. If mutual influence produces a measurable effect, this effect should be increasingly manifest as the influencing lines are approached and should reach its maximum value in their near neighborhood. Within 1 A of these strong lines there are 8 lines to the violet showing a mean residual of +0.0006 A based upon 51 measures; there are 8 lines to the red showing a mean residual of +0.0003 A based upon 62 measures. The result then is practically zero, and it seems improbable that an effect of the order assumed by Professor Julius could fail to give an indication of its presence. The results given in Tables VIII and IX show no systematic variation as one approaches the influencing lines, nor any systematic difference between the displacement of lines on the violet and red sides of the strong lines. Observations will be made during the approaching spot maximum upon a larger number of spots, with particular attention to lines under the possible influence of stronger lines, and the mass of data greatly increased; but as far as the present observations are concerned, they fail to show a dependence of mutual influence upon the intensity of the controlling lines, upon their power to produce anomalous dispersion effects, or upon the nearness of the influenced lines.

ANOMALOUS DISPERSION IN THE LABORATORY AND IN THE SUN

The following extract from one of the earlier papers of Professor Julius gives the idea that he himself was at that time under the impression that a straightforward comparison could be made between laboratory results for anomalous dispersion and solar phenomena:

> A careful examination of the anomalous dispersion of a great number of substances will, of course, have to be made before it can be made out in how far our view will account for the facts already known or yet to be revealed in the chromosphere spectrum. Amongst other things, it must then appear

whether those elements whose lines are most conspicuous in the chromosphere light do actually cause uncommonly great anomalous dispersion—a wide field for experimental research, the exploration of which has only just commenced.[1]

A comparison between Geisler's results and the latest flash spectrum data of Mitchell shows, however, that lines showing very small anomalous dispersion in the laboratory are often conspicuous in the chromospheric spectrum, while lines exhibiting it to a high degree in the laboratory may fail in conspicuousness. The lines of the violet magnesium triplet at λ 3833, intensities 10, 15, and 25, are assigned heights of 6000–7000 km, while neighboring lines of mean intensity 5 show a mean height of 600 km. Though the magnesium lines show weak anomalous dispersion and are only four times as strong, they show heights nearly tenfold as great as the comparison lines. The Fe line λ 3860, intensity 20, anomalous dispersion weak, has a height of 6000 km. Comparison lines of mean intensity 6 show a mean height of 800 km. The D lines of sodium show the phenomenon in the laboratory to an enormous degree. Mitchell finds their height to be 1000 km, while neighboring lines of mean intensity 5 upon the same plate are given a mean height of 360 km; so that the D lines, though 5 times as strong, produce arcs only about 3 times as long as the comparison lines. Of the calcium lines, λ 4226.9 shows anomalous dispersion to the highest degree; it is given a height of 5000 km against 14,000 km for the H and K lines, which show less anomalous dispersion. The two factors that appear to be more consistently effective than anomalous dispersion in producing conspicuous chromospheric reversals of the Fraunhofer lines seem to be great normal intensity and enhancement under reduced pressure.

When Mr. Adams made a direct and apparently rational comparison between laboratory results and displacements at the limb and could find no relation between them, it was answered that such a simple comparison could not serve the purpose, because a peculiar feature of the explanation from the point of view of anomalous dispersion is that both very strong and very weak anomalous dispersion make the displacements small, whereas intermediate values give larger displacements. It is of interest to apply this new criterion to the displacements found by Mr. Adams. The results based upon the same lines that Mr. Adams used are as follows. The figures in parentheses show the number of lines involved in the comparison:

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[1] *Astrophysical Journal*, **12**, 194–195, 1900.

	Anomalous Dispersion			
	Strong	Moderate	Weak	Very Weak
Displacements....	+0.006 A (21)	0.002 A (2)	0.0044 A (13)	0.0061 A (30)

The contradiction between theory and observation appears quite evident.

As the displacements at the edge of the penumbrae of eccentrically located sun-spots were considered by Professor Julius to be due to anomalous dispersion, it seemed of interest to apply the foregoing criterion to these displacements. The results were unfavorable to the dispersion theory as I apprehended it. It is now shown by Professor Julius that the criterion did not fit this case, but that the test of mutual influence is to apply. In my list of 506 lines there are 131 lines in the near neighborhood of stronger lines, that is, under the influence of these lines, but they show no systematic differences from lines not so situated.

It is worthy of note that Professor Julius takes no account of laboratory results when considering center and limb displacements; but to prove that very strong and very weak anomalous dispersion make the displacements small and that intermediate values give larger displacements, he classifies the lines simply according to line-intensity, irrespective of their known anomalous dispersion. This appears to assume that anomalous dispersion is proportional to line intensity. Likewise, there is no attempt to correlate the effects of mutual influence with what is known of the power of the influencing lines to produce anomalous dispersion phenomena. It seems again to be assumed that all lines of the same intensity are equally effective. The behavior of the great winged lines of magnesium and of many of the strongest lines of iron whose cores are bounded by broad shadings seems to contradict the assumption. These wings or shadings are explained as "dispersion bands" and it seems probable that their breadth and intensity would bear a direct relation to the power of the lines to produce anomalous dispersion effects; but the lines referred to show weak or very weak anomalous dispersion. The D lines of sodium showing anomalous dispersion out of all proportion to other lines are not bordered by shadings that are proportionately conspicuous or that are comparable with those bordering the H and K lines of calcium. The extent and intensity of the wings appear to depend upon the strength of a line and upon the form of its emission-curve rather than upon the amount of its anomalous dispersion. The disappearance of the wings at the limb, a very striking phenomenon, has as yet received no explanation from the point of view of anomalous dispersion.

In Professor Julius' last paper[1] he seems to have freed the anomalous dispersion theory from the ordinary canons for correlating laboratory and solar observations on the ground that the relations of anomalous dispersion are peculiar to itself, so that it becomes increasingly difficult to correlate the two classes of phenomena. Taken in connection with the great flexibility of the theory this renders it difficult of proof or disproof by observations.

THE RELATIVE LEVEL OF THE ELEMENTS

Professor Julius says: "St. John attempts to corroborate his intensity-and-level hypothesis by considering the atomic weight of the elements in connection with the displacements of their lines in the spot spectrum. We may doubt whether the data suffice for the purpose."

He refers to a statement of mine, where I say: "But on the chart it will be noticed that the lines of the heavy elements, such as barium, lanthanum, neodymium, cadmium, cerium, lead, and ytterbium, originate at lower levels than the lines of like intensity of iron." I regret that I made the mistake of writing neodymium instead of niobium, and I am glad to have the error called to my attention. On the chart the lines of neodymium are not below those of iron of the same intensity.

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[1] *Astrophysical Journal*, **40**, 19–20, 1914.

Professor Julius says further: "But if we refer to Table I of the first paper we find that the evidence is not very strong." As Professor Julius has raised the question, it may be well to present the facts relating to the heavy elements in some detail.

In comparing the displacements of the lines of the heavy elements with those of iron, Professor Julius falls again into the error of using displacements of the iron lines deduced from the six separated regions, though 19 of the 21 lines of the heavy metals are in the violet, for which region the generalized iron scale is slightly high. The correct procedure is, of course, to compare these displacements with those for iron in the same spectral region and measured upon the same or interconnected plates, as in Table X.

TABLE X

DISPLACEMENTS OF THE HEAVY ELEMENTS COMPARED WITH IRON

Element	Intensity	λ	Observed	Mean for Fe	Residuals	Residuals by Elements
Ba.....	2	4130	0.027	0.020	+0.007	+0.005
	5	5853	.029	.026	+ .003	
Ce......	1	4111	.028	.022	+ .006	+ .002
	3	4129	.022	.014	+ .008	
Fe, Ce..	1	4137	.014	.022	− .008	
Cd.....	3	4678	.038	.033	+ .005	+ .005
La......	2	3628	.019	.018	+ .001	+ .002
	1	3694	.027	.022	+ .005	
	1	3995	.014	.022	− .008	
	1	4086	.026	.022	+ .004	
	2	4123	.023	.020	+ .003	
	2	4196	.024	.020	+ .004	
	4	4204	.020	.014	+ .006	
	2	4287	.019	.020	− .001	
	1	4333	.026	.022	+ .004	
Nd?....	2	3994	.020	.017	+ .003	.000
	1	4109	.016	.022	− .006	
	1	4116	.025	.022	+ .003	
Nb.....	1	4232	.030	.020	+ .010	+ .010
Pb.....	1	4387	.026	.020	+ .006	+ .006
Yt......	3	3694	0.020	0.014	+0.006	+0.006

Of the 21 lines 3 show large negative residuals: λ 3995, a nebulous and difficult line of intensity 1 belonging to lanthanum; λ 4137, a line attributed to cerium, where the negative evidence seems to Professor Julius to deserve being italicized; and a line of neodymium. In regard to the cerium line, a reference to Rowland's Table would have shown that it may as well be called an iron as a cerium line. It was placed upon the observing list through an inadvertence and has not been treated as a cerium line in any discussion. I regret that the full Rowland identification was omitted from the table and that the cerium line λ 4111 appears by a typographical error under the cobalt symbol. The La line λ 3995 is between lines of intensities 3 and 5. Professor Julius has evidently judged that its large negative residual is due to its being on the violet side of the line of intensity 3, but the line λ 4137 is less than a fourth of an angstrom to the red of a stronger line and therefore according to the anomalous dispersion theory should give a positive residual. The residual is large and negative. As to the neodymium lines, it is well known that their identification is only suggested and rests upon no accepted evidence.

The emphasis that Professor Julius places upon these negative residuals leads one to remark that there are two distinct plans for an investigation such as I undertook. One can select a few lines well adapted to measurement and, by concentrating upon them, results of very high accuracy may be obtained; but in that case everything depends upon the selection of the lines. On the other hand, the investigation may be extended, as in this instance, to a wide range of elements and lines. For elements represented by many lines a selection was possible, but for the heavy elements all lines were measured. In such an extended investigation dependence must be placed upon means obtained by statistical treatment, and these in the case of the heavy elements show such a preponderance of positive residuals that the general result appears to be established by the observations. Professor Julius raises the question of zircon. The measurement of the weak zircon lines in the solar spectrum is particularly difficult. The facts are given in Table XI.

The situation is not quite so bad as Professor Julius puts it. He implies that six of seven lines give decidedly negative residuals when compared with iron. But the evidence given by the mean of the first five in the list is quite indecisive. Two of the remaining three are close to the red side of stronger lines and should, according to the dispersion theory, give positive residuals; these residuals are, however, −0.004 A and −0.005 A. These two lines, as well as λ 4137, must then, from the point of view of anomalous dispersion, be affected by errors of observation. The remaining line is represented in neither the arc nor spark spectrum of zircon.[1] I regret that by a typographical error the line λ 4277 appears in the table as a Cr line.

TABLE XI

Comparative Displacements of the Zircon and Iron Lines

λ	Intensity	Observed	Mean for Fe	Residual
4149	2	0.016	0.020	−0.004
4183	1	.024	.022	+ .002
4273	2	.021	.020	+ .001
4277	0	.026	.024	+ .002
4294	2	.018	.020	− .002
4304	0	.020	.024	− .004
4359	0	.019	.024	− .005
4374	0	0.015	0.024	−0.009

In this connection it is of interest to consider an element whose level is higher than iron. When the displacements of the titanium lines are compared with those of iron of the same intensity and spectral region the result is as in Table XII.

TABLE XII

Displacements of Titanium Compared with Those of Iron

Intensity	No. of Lines	Mean Ti-Fe
0	6	−0.0020 A
1	7	− .0024
2	18	− .0028
3	10	− .0026
4	6	−0.0025
	47	−0.0026 A

Professor Julius adds: "If, however, the accuracy attained would permit of regarding such deviations as genuine, the interpretation of the displacements on the basis of the Doppler effect would be condemned. One could not reasonably admit the various absorption centers present in a gas-current at a certain level to have different proper velocities of outflow from the spot vortex."[1]

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[1] Kayser, *Handbuch der Spectroscopie*, **6**, 864, 1912.

It was precisely because the deviations were considered to be genuine and because one could not reasonably admit that the absorption centers present in a gas-current at a certain level can have different velocities, that one was led to the assumption that they are at levels differing with their velocities. It is to be observed that the deviations between the displacements of the lines of these elements and of iron are of the same order as those between iron lines of neighboring intensities which Professor Julius considers to be genuine and explicable from the point of view of his theory.

In saying that the displacements of the Fe lines of intensity 4, for example, are in magnitude between those for lines of intensities 3 and 5, it is meant that, when the data are sufficient, the mean displacement for lines of intensity 4 is less than that for lines of intensity 3 and more than that for lines of intensity 5; not that the statement is true for every line of the respective intensities, otherwise there would be no room for errors of measurement and no need for the tedious duplication of observations by extending them to ever-larger numbers of lines. In an exactly similar way and to the same degree, one is justified in saying that the displacements of the lines of the heavy elements are, as a class, greater than for the iron lines of corresponding intensities. There are several grounds for expecting deviations from the mean: (1) errors of measurement of weak lines which are inherently great are somewhat above normal because in general the intensity of development of the plates was intended for the measurement of lines of medium intensities; (2) errors in the Rowland intensities, which are quite large for the weaker lines, in which case the individual variations are so pronounced that the results for a single line must be accepted with caution unless its intensity is verified; (3) there is always the possibility of the line in question being an undetected blend, especially in the ultra-violet region where the lines are so numerous; (4) there are apparently differences characteristic of the different groups of lines of a given element.[1] It is first necessary, however, to establish the broader outlines of a general truth, if it exists, then later the individual variations may be used to push investigation a step farther.

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[1] *Astrophysical Journal*, **40**, 7, 1914.

GENERAL DISPLACEMENTS OF FRAUNHOFER LINES

Another deduction from the dispersion theory is said by Professor Julius to be proved by the center and limb displacements observed by Adams. These displacements, according to that theory, are a more pronounced effect of the general shifting of the Fraunhofer lines to the red, which is ascribed to their asymmetry. Both phenomena, the general shift and the center and limb effect, are similarly produced, are subject to the same laws, and may be judged by the same criterion. In the case of the center and limb observations the displacements are small for lines of weak intensity, increase to a maximum for lines of intermediate intensity, then decrease for lines of great intensity. Professor Julius says: "This is exactly what our theory requires. . . . . Lines of moderate intensity should show the largest displacements, as they really do."[2]

It is interesting to compare the above definite statement with the results obtained from observations of general shift. The Fe lines of the groups *a*, *b*, and *c*4 are the best lines in the iron spectrum for standards, their wave-lengths, under constant pressure, are independent of arc conditions, and the lines are of excellent quality both in the arc and in the solar spectrum. An extended series of observations on sun-arc displacements is a part of the Mount Wilson program. Some preliminary results are now being prepared for publication and their general discussion will appear later. In the list are 105 lines of groups *a*, *b*, and *c*4. A synopsis of the means taken by intensities is given in Table XIII.

For comparison the limb-center shifts are included, and also the sun-arc results of Evershed. The result based upon these lines for which sun-arc displacements can be obtained with high precision is at variance with the deduction from the anomalous dispersion theory. For lines of moderate intensity 5 the displacement is a minimum instead of a maximum. The march of the phenomenon for the iron lines shown by Fig. 2 appears not to be accidental, for the lines of group *a* taken separately, the two sections of the *b* lines in the violet and red respectively, and the lines of the same groups in the list published by Evershed[1] show a similar march of displacement with intensity.

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[1] *Mt. Wilson Contr.*, No. 74; *Astrophysical Journal*, **38**, 352, 1913.

[2] *Observatory*, **37**, 256, 1914.

TABLE XIII

SUN-ARC DISPLACEMENTS—IRON LINES

	Intensity							
	2	3	4	5	6	7	8	10–15
No. of lines	9	22	29	16	13	5	10	3
Sun-arc...	+0.0047	+0.0038	+0.0030	+0.0020	+0.0042	+0.0058	+0.0090	+0.0110
Evershed..	+ .0083	+ .0047	+ .006	+ .0055	+ .0077	+ .0135	+ .0134	+ .015
Limb-center...	+0.0066	+0.0068	+0.0071	+0.0088	+0.0083	+0.0088	+0.0079	+0.0041

There are many lines showing negative shifts for the sun-arc comparison. The evidence that these displacements to shorter wave-length are real will be found in the paper to which reference is made above. Shifts in the solar atmosphere to shorter wave-lengths appear to be unaccounted for by the anomalous dispersion theory.

A further and very serious difficulty for the theory appears in classifying the lines into groups by sun-arc displacements, that is, in putting into each category lines showing similar behavior in the sun, as the lines fall into the same groups as when classified by pressure-shift in the laboratory. The hallmark of pressure-shift is evident, but discussion is reserved for the paper on "Pressures in the Solar Atmosphere" now in preparation. It is of interest here only from its bearing upon the deduction from the anomalous dispersion theory.

In reference to the limb-center displacements Professor Julius remarks: "Adams himself considers pressure as the effective agent

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[1] *Kodaikanal Observatory Bulletin*, No. 36, 1913.

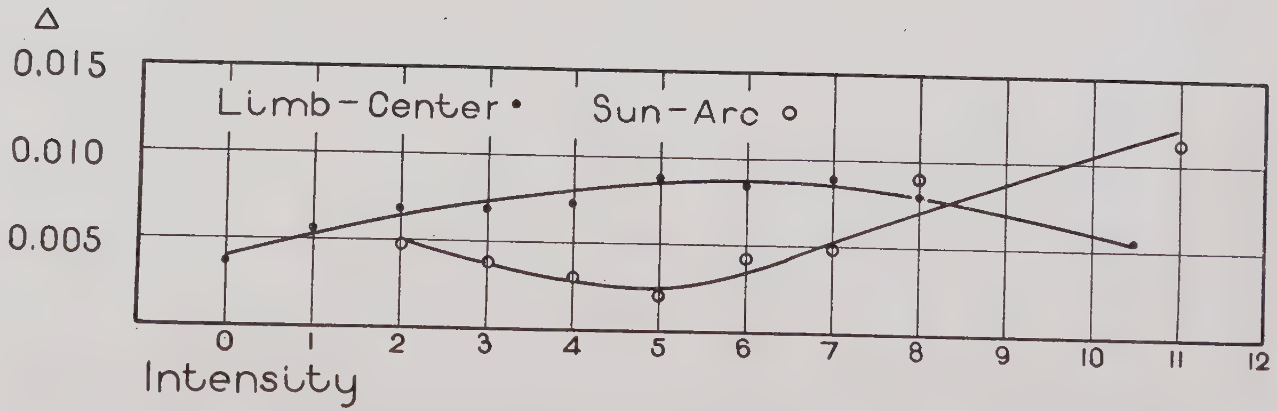

FIG. 2.—Limb-center displacements are from the observations of Adams. Sun-arc displacements are of Fe lines of groups *a*, *b*, *c*4.

in producing these displacements; it therefore did not occur to him to classify the shifts according to line intensity."[1]

It would more accurately represent Mr. Adams' attitude to say that he did not classify the shifts according to line intensity because he recognized that the characteristic behavior of the elements and classes of lines would be concealed rather than revealed by the treatment of heterogeneous data *en masse.* It seems to be implied that Mr. Adams considered pressure the only effective agent concerned in the center and limb displacements. Such an inference is hardly consonant with Mr. Adams' discussion of the question of the heavy elements and the enhancement of lines, and his treatment of scattering, motion, and level. In respect to the latter he begins a discussion with the expression: "Since the matter of level appears to be the most significant factor in determining the amount of the displacement for any given line . . . . ;[2] and it was at his suggestion that I took up the question of level and center-limb displacements in my second paper on "Radial Motion in Sun-Spots."[3]

ERRORS OF OBSERVATION

Professor Julius says: "A very marked fact not explained by St. John's theory, viz., the large deviation of individual displacements from the means for each intensity class and spectral region, is shown to be in harmony with our interpretation on the basis of anomalous dispersion."[4]

There are 131 lines in the lists of those to the violet or red of stronger lines. In such cases the large deviations explained only by the dispersion theory should be in evidence. The mean deviation for these lines without regard to sign is 0.003 A, which compares favorably with the precision obtainable in other solar spectrum measurement where the difficulties of measurement are of the same order. The mean of the 25 largest deviations of this class is 0.007 A, while for 25 lines not under the influence of stronger lines it is 0.009 A.

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[1] *Astrophysical Journal*, **40**, 27, 1914.

[2] *Mt. Wilson Contr.*, No. 43, p. 26; *Astrophysical Journal*, **31**, 55, 1910.

[3] *Mt. Wilson Contr.*, No. 74, p. 23; *Astrophysical Journal*, **38**, 341, 1913.

[4] *Astrophysical Journal*, **40**, 25, 1914.

As to the lack of proportionality between displacements and wave-lengths, Professor Julius says: "One cannot admit such fluctuations to be entirely accidental or due to observational errors. The important retrograde differences between the values for the mean λ 4732 and mean λ 5235, e.g., far exceeds the mean errors of the result of 76 and 86 observations, and must be genuine."[1]

On this point I am unable to make so definite a statement. I can only say I was impressed by the great difference in absolute values obtained from exposures taken in immediate succession even upon the same plate and in the same region. The area of maximum displacement is very narrowly localized at the edge of the penumbra, and unless the seeing is exceedingly fine the slit cannot be placed on the exact point; and when, as was sometimes the case, observations had to be made under conditions of poor seeing when the edge of the penumbra nearly disappeared, still greater variations were found. Although Professor Julius considers the high values of the displacements in the region λ 4634–λ 4829 to represent a genuine divergence, he has nevertheless treated them as accidental in determining his normal displacements by a smoothing-out process. Consequently the normals are high for the violet and green regions and low for the region of mean λ 4732. In selecting the six spectral regions there was no thought of using them for determining the relation between displacement and wave-length. It was plain that an impracticable number of observations for each region would be required to eliminate the accidental differences between the regions. When, however, the results were finally assembled, the variation of displacement with wave-length was so evident, in spite of the deviations for the separate regions, which my experience told me might be due to observational conditions, that it seemed a fair assumption to refer these displacements to the Doppler effect and to leave final appeal to observation until the approaching spot maximum.

In considering the data in my original list of displacements, a clear distinction should be kept in mind between errors of measurement and errors of observation. In any one of the six spectral regions the relative displacements of the different lines are subject only to errors of measurement, while the relative values for the means of the segregated regions are affected by the large variations in absolute displacements from plate to plate inherent in the difficulties of the observations.

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[1] *Op. cit.*, **40**, 24, 1914.

SOME OUTSTANDING DIFFICULTIES

1. The regular progression of the heights given by flash spectrum lines with solar intensities leads directly to the same relative levels as those deduced from the displacements in the penumbrae of spots. The explanation on the theory of anomalous dispersion would seem to require anomalous dispersion to be directly proportional to the intensities of lines, and the arcs given by flash lines to be directly proportional to the anomalous dispersion of the lines, neither of which appears to be in harmony with observation.

The 346 Fe lines, unenhanced and identified only with iron, in Mitchell's Table I, show the following results:

	No. of Lines									
	4	19	30	55	72	49	40	28	24	25
Solar intensities......	00	0	1	2	3	4	5	6	7–8–9	10–40
Heights in km.......	275	279	288	344	369	397	425	488	590	806

2. The regular change in the intensities in the flash lines relative to the intensities of the corresponding solar lines is a marked phenomenon and is not an obvious deduction from the anomalous dispersion theory. There are in Mitchell's Table I, 1181 unenhanced lines of intensity 6 or less assigned to single elements. The differences, flash—solar intensity, expressed in intensity intervals, are as follows:

	No. of Lines									
	18	69	164	166	189	185	166	109	71	44
Solar intensity	0000	000	00	0	1	2	3	4	5	6
Flash—solar.........	+3.4	+2.3	+1.4	+0.6	+0.1	−0.7	−1.1	−2.1	−2.7	−3.2

3. Some two years ago I published observations showing displacements when the slit of the spectrograph is perpendicular to the

radius of the solar disk passing through the center of the umbra.[1] It is implied by Professor Julius that such displacements are only occasional and occur when the region of minimum density has not a symmetrical shape.[2] Of the spots examined at the time of publication, 70 per cent showed such displacements and practically all spots examined since then by more refined methods of observation have shown these displacements. They occur and persist for weeks in regular and symmetrical spots, so that one can hardly admit that "there is no difficulty in accounting for the occasional displacements in question on the basis of unequal refraction at opposite edges of the spots." It may be possible to imagine a density distribution capable of producing the effect "since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion, if only we are at liberty to assume the required density distribution,"[3] but that such an artificial condition characterizes a regular spot during at least the greater part of its life seems improbable, and the mind reverts to the obvious explanation based on vortex motion.

4. There are some lines displaced in the opposite direction in the penumbrae of spots, but their small number does not lessen the difficulty for the dispersion theory. Not only are they among the most important chromospheric lines, but they often show displacements that are pronounced and obvious without measurement, they appear in every observation, and indicate a comparatively stable condition during the life of a spot. Up to the present the anomalous dispersion theory has been unable to suggest an explanation, and the Doppler effect due to the inflow of the upper chromospheric vapors is as yet the only one proposed.

5. No explanation is furnished by the dispersion theory of the differences characteristic of the elements such as those shown by the displacements of the lines of iron, titanium, and the heavy elements when lines of the same region and intensity are compared.

EFFECTIVE LEVELS

The idea of levels has been employed in all spectroheliographic work, particularly by Hale and Deslandres, and has been previously discussed by Abbot[1] and by the writer.[2] It is a familiar observation that the different arc lines of iron, for example, are not produced with equal relative intensity in all parts of the arc. Some appear only at the poles, others only in the core, while others are characteristic of the flame of the arc. For these three classes of lines the effective centers of radiation are not uniformly distributed throughout the mass of radiating vapor. If such a body of vapor were backed by a suitable continuous spectrum, the absorption lines would be referable to different portions of the arc. Suppose, for example, that the core of the arc could be put into rotation with a velocity greatly differing from that of the outer layers. Such a difference in motion could conceivably be detected by the inclination of the spectrum lines if the slit of the spectrograph were normal to the axis of the arc, since different portions of the arc are effective in the production of the classes of lines considered. Again let us consider systems of electrons emitting a number of lines of very different intensities in a mass of vapor of solar proportions with a similar distribution of temperature and density. For the strongest lines the absorption coefficient is the greatest and there will be a spherical core from which no light of these wave-lengths reaches the surface, that is, the emergent light of these wave-lengths comes from a spherical shell. The total radiation is an integrated effect and its center of gravity locates the effective radius of the shell or the effective layer. From the levels of the shorter radii of the shell the light emitted at the outer surface is small owing to the absorption; from the levels of the longer radii it is again small owing to the lower temperature; it is evident that the really effective level lies between these extremes. The lines will appear dark, as Abbot suggests, in contrast to the light that is not selectively absorbed, since such radiation comes from lower and hence hotter levels. If attention be directed to the weakest lines of the system, it is seen that light of these wave-lengths will reach the surface from a greater depth owing to the smaller coefficient of absorption, that low levels will be more effective and the higher levels less effective for the weakest than for the strongest lines, so that the center of gravity of the integrated effect is lower, that is, the effective level is lower for the weakest lines than for the strongest lines. From this point of view it is evident that the existence of tangential velocities varying with the depth could be detected spectroscopically. That these lines of different intensities may belong to a series does not seem to require modification of the discussion.

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[1] *Mt. Wilson Contr.*, No. 54, p. 20; *Astrophysical Journal*, **34**, 76, 1911.

[2] *Ibid.*, **40**, 29, 1914.

[3] *Ibid.*, **25**, 111, 1907.

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[1] *The Sun*, pp. 97 and 251.

[2] *Mt. Wilson Contr.*, No. 74, p. 5; *Astrophysical Journal*, **38**, 345, 1913; *Mt. Wilson Contr.*, No. 88, p. 18; *Astrophysical Journal*, **40**, 373, 1914.

Professor Julius urges against this point of view that in the laboratory a gas such as NO_2 shows at the same time strong and weak lines. It is also true that the calcium spark gives ordinarily λ 4226.9, H, K, and many weaker lines, but de Gramont[1] has shown that K is the ultimate line, the last to disappear when the quantity of the element tends toward zero. If the quantity be sufficient to show K only, it is evident that the corresponding absorption spectrum would show K only, and that with increasing quantities of Ca vapor H, λ 4226.9, and other lines would appear in succession. It seems a clear deduction that the upper portion of the Ca envelope of the sun would be the source of the K_3 line and that a greater depth of vapor would be necessary to produce the line λ 4226.9, or the center of gravity of its effective layer would be lower. In the laboratory experiment with NO_2 the range of intensity of the absorption lines depends upon the quantity of vapor in an apparently similar way.

SUMMARY

1. In the lists published by Professor Julius there are 43 lines to the violet and 39 lines to the red of stronger lines. Among the 506 lines of my original list there are 24 other lines to the violet and 25 to the red of stronger lines within half an angstrom of stronger lines. Using the Julius normals the mean residual for the additional lines to the violet is +0.0023 A, and for those to the red −0.0034 A, results opposite to those given by the lines selected by him, and at variance with the theory.

2. In deducing the normal displacements for a given intensity and region from those for the separate regions, one is dealing with non-homogeneous series of observations. The use of these normals

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[1] *Comptes Rendus*, **159**, 5–12, 1914.

introduces residuals that are systematically negative for lines in Professor Julius' lists which are to the violet of stronger ones, because such lines are all in the violet and green regions, for which the normals are too large. The apparent positive residual for lines to the red of stronger ones depends upon 7 lines in the blue series for which the normals are 0.009 A less than the mean for all lines of the same intensity and region.

3. Using as standard displacements in any homogeneous series the means from all lines of a given element and intensity in that region, the resulting residual for the 67 lines to the violet is −0.0003 A and for the 64 lines to the red −0.0004 A. For the 131 lines the sum of the favorable residuals is 0.212 A, of the unfavorable residuals 0.218 A. Tested, then, by standards deduced from the homogeneous group of observations to which they are to be applied, the lines in the near neighborhood of stronger lines show no systematic differences from lines not so situated.

4. Influencing lines showing very strong anomalous dispersion exert no greater effect upon neighboring lines than those showing very weak anomalous dispersion.

5. New measurements of weak lines in the broad shadings of the strongest lines in the solar spectrum show no systematic differences for lines near them nor any systematic progression in the displacements as the strong lines are approached.

6. Criteria given by Professor Julius relating anomalous dispersion to the chromospheric and limb spectra are applied with results unfavorable to the theory, and consideration is given to the increasing indefiniteness of the correlation between laboratory results on anomalous dispersion and solar observation.

7. Professor Julius compared the displacements of the lines of the heavy elements with the standard displacements of the iron scale, which are high for the violet region, and by this procedure reduced the positive residuals. The displacements of the 21 lines of the heavy metals exceed those for the same intensities of iron on the average by +0.003 A when comparison is made with iron lines upon the same plates.

8. The general displacements of the Fraunhofer lines to the red are a minimum for lines of medium intensities, a result opposite to

that required by the anomalous dispersion theory; and there are, moreover, many lines displaced to shorter wave-length, a phenomenon yet unaccounted for by the theory.

9. Classified by their displacements in the solar atmosphere the iron lines fall into the same classes as when classified by pressure-shift, a result apparently not to be explained by anomalous dispersion.

10. A consideration of errors shows that for a given region they are not abnormal for weak lines near stronger ones, and that the departures from regularity, when unconnected regions are compared, may be referred to the difficult observing conditions.

11. The general conclusion from this review of solar observations is that the deductions from the anomalous dispersion theory which are susceptible of definite and quantitative tests are not supported by the observational data, and that observations are outstanding which have not yet been explained by the theory.

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THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRA OF VANADIUM AND CHROMIUM

By ARTHUR S. KING

The electric furnace spectra to be treated in this paper cover the range of wave-length from λ 3150 to λ 6850 for vanadium, and from λ 3550 to λ 7000 for chromium. The method of classification as to the initial temperature at which a spectrum line appears and its rate of increase in intensity as the temperature rises is similar to that used in previous studies of the spectra of iron[1] and titanium.[2]

APPARATUS AND METHODS

The tube resistance furnace was operated *in vacuo* according to the method described in previous papers. A distinct gain in the photography of the spectrum was obtained by the use of the vertical concave-grating spectrograph[3] recently mounted in the Pasadena laboratory. The spectra under examination being very rich in lines, especially in the region of shorter wave-length, the second order of this 15-foot grating was used to advantage in the crowded region extending to about λ 4500. The scale is approximately 1 mm = 1.85 A, and a range of about 850 A is obtained on a single plate. The first order gives a spectrum of high intensity and its scale was sufficient for the region toward the red. Seed "Gilt-Edge 27" plates were used throughout the spectrum, being bathed with the Wallace solution of pinacyanol, pinaverdol, and homocol for the region from λ 5000 into the red. A number of photographs of the whole spectrum were also made with a 1-meter concave grating to record the lines most presistent at low temperature and to observe the general changes in intensity with wave-length for the several furnace temperatures and for the arc.

[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

[2] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

[3] *Mt. Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 205, 1914.

The three temperatures on which the classification for both vanadium and chromium is based were given by a Wanner pyrometer as 2000–2150° C. for the low, 2300–2350° C. for the medium, and 2500–2600° C. for the high temperature plates. With vanadium, 2000° C. appears to be about the lower limit for the appearance of a spectrum, though the lines produced at this temperature are fairly numerous. The case is different for chromium, the melting-point of which is considerably lower. Temperatures between 1700° and 1800° C. sufficed to give a number of the most persistent lines in this spectrum, which are listed in a separate table. The somewhat richer spectrum given at 2000°, however, was better suited to show the development of the lines from this into the medium and high temperatures.

The metals used in the furnace were metallic vanadium and crystalline chromium, both supplied by Eimer and Amend. These substances were of fair purity and as the furnace tubes used were of regraphitized Acheson graphite, the spectra obtained showed but few impurity lines of sufficient strength to be disturbing.

EXPLANATION OF THE TABLES

Wave-lengths.—The wave-lengths in the first column of Tables I and II are those given by Exner and Haschek[1] for the arc spectrum, supplemented occasionally by those of Hasselberg,[2] indicated by *H*, usually in cases where close doublets were not resolved by Exner and Haschek. Several chromium lines which are very wide in the arc in air were sometimes found to be resolved into as many as three sharp lines by the vacuum furnace. A vacuum arc furnished by special fittings inside the furnace chamber also gave sharp lines in these cases, which were measured from neighboring chromium lines and the wave-lengths entered in the table.

An asterisk after the wave-length denotes that an explanatory remark for the given line is to be found at the end of the table.

Arc intensities.—These were estimated by the writer from arc spectra taken on the same plates as the furnace spectra, the vanadium arc being given by the metal in the carbon arc, while that of chromium was formed between lumps of the metal. The arc photograph used was in each case of moderate exposure, the spectrum in general being comparable in intensity with that of the high-temperature furnace. Numerous nebulous lines, especially in the chromium table, are indicated by *n* after the intensity value. A vacuum source, either arc or furnace, serves in general to sharpen such lines. The letters *R* and *r*, both for arc and for furnace lines, indicate complete and partial self-reversal, respectively.

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[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Kgl. svenska vet. akad. handl.*, **26**, 1894; **32**, 1899; see also Kayser, *Handbuch der Spectroscopie*, **5**, 337; **6**, 750.

Furnace intensities.—In estimating the intensities of furnace lines an effort was made to obtain correct relative intensities for the lines for each temperature, a line distinctly outlined on the plate being given the intensity "1," a fainter appearance being indicated as "trace" (*tr*). The appearance of a line is usually much altered as the temperature rises, but the relative change of different lines with increase of temperature is shown in the tables.

Classification.—The method of assigning lines to classes is the same as that used for the titanium spectrum and is but slightly modified from the system used for iron. Class I lines are relatively strong at low temperatures and strengthen slowly at higher temperatures. Many of them are given the same intensity for the three temperatures and for the arc. Class II lines appear at low temperature but strengthen more rapidly than those of Class I as the tube becomes hotter. The basis of division between these classes, while sometimes rather arbitrary, is usually well defined. The lines of Class III are absent or faint at low temperature, appear at medium temperature, and are usually considerably stronger at high temperature. Class IV lines appear only at the highest furnace temperature, sometimes faintly at medium; while those of Class V are usually absent in the furnace or, if present, are faint compared to the arc intensity. For vanadium, as for titanium, this class is limited almost entirely to the enhanced lines, which are indicated by *V E*. Faint lines given in the arc tables are not entered in this list unless they appear also in the furnace.

The use of *A* after the class number indicates that the line in question is relatively weak in the arc, being usually not more than half as strong as in the high-temperature furnace. The significance of lines of this character will be considered in the discussion.

TABLE I

Temperature Classification of Vanadium Lines

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3164.96....	5	.....	.....	.....	V E	3241.30....	4	4	2	.....	III
3168.25....	6	.....	.....	.....	V E	3242.14....	1	2	tr	.....	III A
3170.47....	2	2	2	1	II	3243.42....	3	4	2	.....	III
3183.53....	30r	50R	30R	10r	II	3246.96....	1	1	.....	.....	IV
3184.11....	60R	100R	60R	20r	II	3248.81....	3	3	2	.....	III
3185.51....	40R	80R	40R	15r	II	3249.69....	10	8	6	5	II
3187.81....	10	tr	.....	.....	V E	3250.12....	2	2	1	.....	III
3188.21....	3	1	.....	.....	IV	3250.90....	4	.....	.....	.....	V E
3188.60....	10	tr	.....	.....	V E	3252.01....	4	.....	.....	.....	V E
3189.16....	1	1	.....	.....	IV	3253.00....	1	1	.....	.....	IV
3190.80....	15	1	.....	.....	V E	3254.91....	10	5	3	1	II
3194.07....	6	5	4	2	II	3255.79....	9	5	4	1	III
3194.53....	2	1	.....	.....	IV	3256.60....	1	1	.....	.....	IV
3194.70....	1	tr	.....	.....	IV	3257.00....	1	tr	.....	.....	IV
3195.05....	1	tr	.....	.....	IV	3259.67....	5	4	3	tr	III
3198.13....	20	30R	15r	6	II	3261.21....	6	3	1	.....	III
3199.97....	6	3	2	.....	III	3262.20....	5	4	1	.....	III
3201.35....	2	4	2	tr	III A	3263.39....	15	15r	8	5	II
3202.51....	25	35R	18r	6	II	3266.02....	5	2	tr	.....	IV
3204.30....	3	3	3	2	I	3266.20....	4	3	2	.....	III
3205.36....	5	2	1	.....	III	3267.83....	20	1	.....	.....	V E
3205.69....	15	6	4	3	II	3271.25....	20	1	.....	.....	V E
3207.01....	1	1	.....	.....	IV	3271.53....	3	2	2	.....	III
3207.51....	20	30R	15r	5	II	3271.80....	12	12r	7	4	II
3208.46....	6	.....	.....	.....	V E	3272.30....	1	1	tr	.....	III
3210.22....	4	3	1	.....	III	3273.16....	7	4	3	1	II
3210.54....	2	2	tr	.....	IV	3276.23....	20	1	.....	.....	V E
3211.68....	1	1	tr	.....	III	3278.02....	5	10r	6	3	II A
3212.57....	15	6	4	3	II	3279.97....	6	.....	.....	.....	V E
3214.04....	2	2	1	.....	III	3281.25....	3	.....	.....	.....	V E
3214.86....	6	.....	.....	.....	V E	3282.67....	5	.....	.....	.....	V E
3215.47....	4	4	3	2	II	3283.46....	15	15r	8	6	II
3217.20*...	10	7?	5?	2?	II?	3284.47....	6	5	4	1	III
3218.45....	1	3	2	tr	III A	3288.55....	2	2	1	.....	III
3218.98....	5	3	2	tr	III	3289.51....	6	.....	.....	.....	V E
3225.76....	1	4	1	.....	III A	3291.81....	4	4	4	3	I
3226.23....	4	4	3	2	II	3295.56....	1	1	.....	.....	IV
3227.21....	3	3	1	.....	III	3298.28....	15	15r	10	8	II
3227.53....	4	2	tr	.....	IV	3298.85....	6	.....	.....	.....	V E
3228.31....	3	3	1	.....	III	3299.20....	3	3	2	.....	III
3229.72....	4	2	tr	.....	IV	3299.32....	1	1	.....	.....	IV
3230.76....	6	6	5	3	II	3300.08....	2	2	1	.....	III
3232.07....	2	.....	.....	.....	V E	3308.36....	3	4	4	1	III
3233.33....	6	4	3	tr	III	3309.03....	1	1	.....	.....	IV
3233.68....	2	.....	.....	.....	V E	3309.30....	8	4	3	1	II
3233.93....	3	.....	.....	.....	V E	3313.14....	2	2	1	.....	III
3234.86....	2	3	2	tr	III	3314.11....	3	2	tr	.....	IV
3238.00....	8	.....	.....	.....	V E	3316.01....	1	1	.....	.....	IV
3239.06....	1	1	.....	.....	IV	3319.14....	4	3	1	.....	III

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3319.91	1	1	.....	.....	IV	3408.13	3	3	2	tr	III
3320.30	3	3	2	.....	III	3408.60	1	4	3	1	II A
3321.80	5	3	1	.....	III	3409.21	4	3	1	.....	III
3324.37	1	1	.....	.....	IV	3411.10	1	1	.....	.....	IV
3324.55	3	2	1	.....	III	3413.90	1	1	.....	.....	IV
3326.52	1	1	.....	.....	IV	3414.32	5	5	4	2	II
3327.30	tr	tr	.....	.....	IV	3416.65	2	5	4	2	II A
3328.12	2	1	.....	.....	IV	3417.17	5	5	4	2	II
3328.54	2	1	.....	.....	IV	3418.63	5	5	4	2	II
3330.01	12	8	6	3	II	3424.00	3	3	1	.....	III
3332.60	tr	tr	.....	.....	IV	3425.20	6	4	3	1	II
3333.70	2	2	1	.....	III	3425.37	1	tr	.....	.....	IV
3334.28	tr	tr	.....	.....	IV	3426.06	1	5	4	2	II A
3336.35	1	1	.....	.....	IV	3426.87	1	2	1	.....	III A
3336.49	2	1	.....	.....	IV	3427.60	1	1	.....	.....	IV
3336.93	2	1	.....	.....	IV	3432.19	1	1	.....	.....	IV
3340.31	1	1	.....	.....	IV	3437.91	1	1	.....	.....	IV
3342.42	2	1	.....	.....	IV	3439.13	tr	tr	.....	.....	IV
3345.15	tr	tr	.....	.....	IV	3442.14	2	1	.....	.....	IV
3356.50	10	7	5	2	II	3442.45	2	2	1	.....	III
3362.2	1	tr	.....	.....	IV	3443.69	1	1	.....	.....	IV
3363.70	4	5	3	tr	III	3445.00	1	1	.....	.....	IV
3365.71	10	7	5	3	II	3445.93	2	2	tr	.....	IV
3367.02	4	4	3	1	II	3447.23	tr	tr	.....	.....	IV
3367.18	1	1	.....	.....	IV	3453.12	tr	tr	.....	.....	IV
3369.1	1	1	.....	.....	IV	3453.65	1	1	.....	.....	IV
3371.27	3	1	.....	.....	IV	3455.02	3	2	tr	.....	IV
3372.94	tr	1	.....	.....	IV A	3455.36	1	1	.....	.....	IV
3374.17	3	4	2	.....	III	3455.70	1	1	.....	.....	IV
3376.20	8	6	5	2	II	3455.95	tr	tr	.....	.....	IV
3377.51	10	8	6	3	II	3457.04	4	4	2	.....	III
3377.75	15	10	8	6	II	3457.23	3	.....	.....	.....	V E
3379.49	2	1	.....	.....	IV	3460.23	1	3	1	.....	III A
3384.71	5	5	4	1	III	3463.53	2	6	4	3	II A
3387.50	2	1	.....	.....	IV	3465.48	tr	tr	.....	.....	IV
3389.64	tr	tr	.....	.....	IV	3482.31	1	1	.....	.....	IV
3390.51	2	2	1	.....	III	3486.05	6	2	1	.....	III
3390.91	6	5	4	1	III	3487.14	2	2	1	.....	III
3394.90	2	1	.....	.....	IV	3489.58	4	4	2	.....	III
3395.64	3	3	1	.....	III	3490.40	1	1	.....	.....	IV
3396.65	3	2	tr	.....	IV	3491.54	tr	tr	.....	.....	IV
3397.70	6	5	4	1	III	3493.32	4	.....	.....	.....	V E
3397.97	4	5	4	1	III	3496.40	1	1	.....	.....	IV
3398.39	1	1	.....	.....	IV	3497.09	3	3	1	.....	III
3400.55	12	8	6	4	II	3498.34	3	3	1	.....	III
3401.49	2	2	1	.....	III	3500.47	1	1	.....	.....	IV
3402.73	9	7	5	3	II	3500.96	3	2	2	.....	III
3403.51	5	4	1	.....	III	3501.64	4	4	3	tr	III
3405.09	2	2	tr	.....	IV	3503.31	1	1	tr	.....	III
3405.30	6	5	4	2	II	3504.60	12	.....	.....	.....	V E
3406.96	6	6	4	1	III	3505.39	1	1	.....	.....	IV

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3505.84	6	6	4	1	III	3580.97	3	3	1	.....	III
3506.99	3	3	1	.....	III	3582.97	3	3	1	.....	III
3516.35	1	1	.....	.....	IV	3583.85*	8	12	8	3	II
3517.44	12	tr	.....	.....	V E	3586.34	2	.....	.....	.....	V
3518.27	tr	tr	.....	.....	IV	3589.89	15	tr	.....	.....	V E
3519.31	3	3	1	.....	III	3591.23	1	1	tr	.....	III
3520.16	5	.....	.....	.....	V E	3592.16	15	tr	.....	.....	V E
3522.00	2	.....	.....	.....	V E	3592.32	1	4	3	1	II A
3522.73	3	3	1	.....	III	3592.70	3	2	1	.....	III
3524.86	6	.....	.....	.....	V E	3593.49	12	.....	.....	.....	V E
3525.90	1	1	tr	.....	III	3595.75	1	1	tr	.....	III
3528.35	1	1	tr	.....	III	3598.26	1	1	tr	.....	III
3529.89	10	8	6	4	II	3600.16	5	5	3	1	II
3530.91	10	.....	.....	.....	V E	3604.24	1	tr	.....	.....	IV
3533.87*	10	6	6	5	I	3605.77	3	1	tr	.....	III
	6	8	4	2	II	3606.85*	8	6?	4?	3?	II?
3534.85	1	4	2	1	II A	3609.42	3	3	1	.....	III
3538.36	3	.....	.....	.....	V E	3616.86	3	2	1	.....	III
3540.66	1	5	3	1	II A	3617.44	1	1	tr	.....	III
3542.77	1	3	2	1	II A	3621.35	2	.....	.....	.....	V E
3543.62	8	6	5	3	II	3622.79	2	1	tr	.....	III
3545.32	12	.....	.....	.....	V E	3628.53	1	1	1	.....	III
3545.49	8	6	5	3	II	3629.46	2	2	1	.....	III
3551.67	3	3	2	tr	III	3634.06	3	3	2	.....	III
3552.80	1	1	tr	.....	III	3635.99	3	2	1	.....	III
3553.43	6	6	5	3	II	3637.10	1	1	1	.....	III
3555.31	3	5	4	1	III	3637.89	3	1	.....	.....	IV
3555.87	2	3	3	1	II	3638.50	2	2	1	.....	III
3556.40	4	4	3	1	II	3639.16	6	5	3	2	II
3556.96	15	tr	.....	.....	V E	3640.20	2	2	1	.....	III
3557.34	2	2	1	.....	III	3641.24	4	4	3	1	II
3560.74	3	1	tr	.....	III	3643.25	1	1	tr	.....	III
3561.55	1	1	tr	.....	III	3643.99	5	5	3	1	II
3562.29	2	2	1	.....	III	3644.49	1	2	1	.....	III A
3563.54	2	2	1	.....	III	3644.87	8	8	6	3	II
3563.67	1	1	.....	.....	IV	3645.68	3	3	1	tr	II
3565.20	1	3	2	tr	III A	3647.4	3	3	1	tr	II
3566.32	4	4	2	tr	III	3648.50	tr	tr	.....	.....	IV
3569.07	3	3	1	.....	III	3649.08	5	5	3	1	II
3569.19	1	3	2	1	II A	3652.54	2	2	1	tr	III
3571.16	4	4	2	1	II	3654.82	2	1	tr	.....	III
3571.36	2	2	1	.....	III	3656.81	6	4	2	1	II
3571.80	5	5	3	1	II	3657.62	2	6	5	4	I A
3572.45	1	2	1	.....	III A	3659.60	2	1	tr	.....	III
3572.77	2	2	1	.....	III	3661.52	2	1	1	.....	III
3573.65	5	5	3	1	II	3662.16	1	1	tr	.....	III
3574.90	3	4	2	1	II	3663.71	15	8	5	4	II
3575.27	3	3	1	.....	III	3665.28	8	6	3	2	II
3578.01	4	4	2	1	II	3667.89	15	8	5	4	II
3579.24	2	2	1	.....	III	3669.58	2	.....	.....	.....	V E
3579.46	1	tr	.....	.....	IV	3671.37	10	8	6	6	I

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3672.55....	8	6	3	2	II	3737.60....	1	5	4	2	II A
3673.57....	12	10	5	4	II	3738.11....	5	4	3	1	II
3675.64....	3	3	1	.....	III	3738.89....	8	8	5	2	II
3675.89....	20	20r	20	20	I	3740.39....	6	5	3	1	II
3676.84....	10	8	4	2	II	3741.63....	6	5	3	1	II
3677.23....	2	6	6	6	I A	3744.04....	1	1	1	.....	III
3680.25....	15	6	5	3	II	3746.00....	8	.....	.....	.....	V E
3683.30....	30	30R	20	20	I	3747.26....	3	4	4	2	II
3684.50....	3	3	2	1	II	3748.11....	8n	5	3	1	II
3686.42....	8	8	6	5	I	3750.27....	1	1	tr	.....	III
3686.89....	1	1	tr	.....	III	3751.07....	6	.....	.....	.....	V E
3687.63*...	12?	6?	5?	3?	II?	3751.95....	5	8	6	4	II A
3688.23....	50	50R	30r	20	II	3753.44....	4	3	1	tr	II
3690.49....	40	40R	20	20	II	3755.85....	4	3	1	tr	II
3692.40....	50	50R	30r	20	II	3756.18....	3	5	4	2	II
3694.79....	3	4	2	1	II	3757.04....	tr	tr	.....	.....	IV
3695.50....	30	10	6	4	II	3758.70....	2	2	1	tr	II
3696.03....	40	50R	25r	20	II	3758.95....	1	1	tr	.....	III
3699.63....	3	4	3	1	II	3759.45....	4	3	1	tr	II
3700.49....	3	.....	.....	.....	V E	3760.95....	3	4	3	2	II
3700.78....	tr	1	tr	.....	III A	3761.56....	3	3	2	1	II
3703.73....	100	100R	40r	25	II	3763.29....	6	8	5	2	II
3704.16....	2	3	2	1	II	3764.95....	2	1	tr	.....	III
3704.85....	60	60R	25	20	II	3765.74....	1	1	tr	.....	III
3705.20....	30	30R	15	20	I	3766.55....	2	2	1	tr	II
3705.97....	1	1	tr	.....	III	3769.21....	4	4	3	1	II
3706.19....	4	4	3	2	II	3769.98....	1	1	tr	.....	III
3708.87....	6	5	4	2	II	3770.15....	2	1	1	.....	III
3711.56....	1	1	tr	.....	III	3770.67....	3	6	5	2	II A
3713.46....	1	1	tr	.....	III	3771.11....	8	.....	.....	.....	V E
3713.70....	1	1	tr	.....	III	3771.32....	1	4	3	1	II A
3714.11....	5	8	8	8	I A	3771.81....	1	5	5	4	I A
3715.00....	1	1	1	.....	III	3772.29....	2	1	tr	.....	III
3715.61....	10	.....	.....	.....	V E	3772.87....	2	1	tr	.....	III
3717.69....	1	1	tr	.....	III	3774.25....	3	1	tr	.....	III
3718.26....	2	.....	.....	.....	V E	3775.33....	3	5	3	1	II
3719.05....	4	2	1	.....	III	3775.82....	4	3	2	tr	III
3721.07....	1	1	tr	.....	III	3776.30....	4	3	2	1	II
3721.55*...	3	6	5	4	I A	3777.02....	2	3	2	tr	III
3722.14....	4	4	2	1	II	3777.30....	2	6	6	6	I A
3722.35....	3	1	1	.....	III	3777.61....	1	1	tr	.....	III
3723.47....	3	3	2	tr	III	3778.46....	2	.....	.....	.....	V E
3723.71....	1	1	tr	.....	III	3778.82....	25	25R	20	20	I
3727.48....	10	.....	.....	.....	V E	3779.78....	4	6	4	2	II
3728.48....	2	.....	.....	.....	V E	3781.55....	3	5	3	2	II
3729.18....	4	4	3	1	II	3781.89....	1	1	1	.....	III
3730.33*...	3	6	4	2	II A	3782.71....	3	3	2	1	II
3731.17....	2	1	tr	.....	III	3783.08....	1	1	1	.....	III
3732.21....	1	5	4	2	II A	3784.83....	2	5	7	6	I A
3732.90....	10	.....	.....	.....	V E	3784.95....	1	1	tr	.....	III
3734.57....	5	5	3	1	II	3787.30....	5	4	3	1	II

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3787.68....	3	3	3	1	II	3835.70....	4	6	6	3	II
3788.93....	1	1	1	.....	III	3836.19....	5	5	6	4	I
3790.47....	20	20R	20	20	I	3836.63....	1	tr	.....	.....	IV
3790.62....	8	8	5	3	II	3839.11....	10	7	7	6	I
3791.46....	2	8	6	5	I A	3839.52....	8	5	4	2	II
3793.79....	8	12	12	15	I A	3840.25....	4	5	4	3	I
3794.51....	2	1	tr	.....	III	3840.85....	60r	60R	40R	30	II
3795.12....	50	50R	35r	35	II	3842.02*...	5	12r	8	12	I A
3796.63....	3	3	1	tr	II	3842.84....	3	5	4	2	II
3798.39....	4	4	4	4	I	3843.10....	2	3	2	1	II
3798.80....	2	4	1	.....	III A	3843.61....	4	6	5	2	II
3799.41....	1	tr	.....	.....	IV	3844.58....	20	20r	20	20	I
3800.10....	25	25R	20	20	I	3845.01....	4	4	4	2	II
3803.01....	2	3	2	tr	III	3846.11....	3	2	1	1	II
3803.60....	25	25R	20	20	I	3847.49....	20	20R	20	20	I
3803.93....	6	6	5	2	II	3849.43....	6	5	5	4	I
3804.03....	6	10	8	10	I A	3851.34....	5	4	4	3	I
3804.72....	3	3	2	.....	III	3852.25....	2	4	3	2	II A
3805.05....	2	3	2	tr	III	3854.23....	1	1	1	.....	III
3806.92....	8	6	5	4	II	3855.50....	30	30R	30r	30	I
3807.63....	20	20R	20	20	I	3856.08....	60r	60R	50R	50	I
3808.26....	3	1	tr	.....	III	3858.82....	5	8	6	4	II A
3808.67....	40	40R	25r	25	II	3859.50....	6	8	7	4	II
3809.23....	1	1	tr	.....	III	3861.77....	6n	6	5	3	II
3809.74....	15	15	15	15	I	3862.40....	12	12r	12	15	I
3811.47....	2	1	1	.....	III	3864.02....	6	6	5	3	II
3813.63....	60	60R	25r	25	II	3864.26....	3	3	3	4	I
3815.66....	10	12	12	12	I	3864.70....	1	1	.....	.....	IV
3818.00....	8	12	12	15	I A	3865.05....	35	30R	25r	25	I
3818.12....	4	4	3	1	II	3867.48....	2	4	3	1	II A
3818.41....	60	60R	30r	30	II	3867.75....	15	15r	15	15	I
3818.91....	1	1	tr	.....	III	3870.73....	2	3	2	1	II
3820.11....	15	15r	15	15	I	3871.20....	8	6	6	4	I
3821.65....	15	15r	15	15	I	3872.91....	4n	6	4	2	II
3822.21....	30	30R	20r	20	I	3873.78....	4	5	4	2	II
3822.85....	1	1	tr	.....	III	3874.50....	1	1	1	.....	III
3823.05....	15	15r	15	15	I	3875.22....	35	30R	25r	25	I
3823.40....	15	15r	15	15	I	3875.55....	3	5	6	4	I
3823.55....	1	2	1	.....	III A	3876.02....	20	20R	20r	20	I
3823.92....	4	3	1	tr	II	3876.26....	20	20R	20r	20	I
3824.14....	5	6	6	3	II	3876.84....	1	1	tr	.....	III
3825.18....	1	1	tr	.....	III	3879.38*...	2?	4?	3?	?	III?
3825.47....	4	3	2	1	II	3879.80*...	3?	2?	2?	?	III?
3826.90....	6	8	5	3	II	3884.06....	3	3	3	1	II
3828.70....	60r	60R	30r	30	II	3884.60....	4	4	3	1	II
3828.93....	4	3	2	1	II	3885.00....	2	.....	.....	.....	V E
3830.44....	2	2	1	.....	III	3885.70....	1	1	.....	.....	IV
3831.99....	3	3	2	1	II	3885.93....	2	2	2	tr	III
3832.99....	4	3	2	.....	III	3886.74....	6	6	5	4	I
3833.36....	3	3	2	.....	III	3888.20....	2	2	1	.....	III
3834.95....	tr	tr	.....	.....	IV	3888.47....	3	3	2	tr	III

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3889.37....	1	1	tr	.....	III	3936.47....	5	5	6	5	I
3890.39....	25	25R	20	25	I	3937.69....	3	3	2	1	II
3891.30*...	4	3	2	1	II	3938.35....	3	2	1	tr	II
	2	4	3	1	II A	3939.04....	2n	2	1	tr	II
3892.63....	2	2	1	tr	II	3939.47....	4	4	3	1	II
3893.03....	25	25R	20	25	I	3940.75....	2	2	1	tr	II
3893.89....	1	1	tr	.....	III	3941.43....	3	3	2	1	II
3894.20....	4	4	3	2	II	3942.18....	6	6	5	3	II
3896.31*...	6	10	8	5	II	3943.80....	12	10	10	8	I
3896.80....	2	2	1	.....	III	3944.66....	1	1	tr	.....	III
3896.96....	2	2	1	.....	III	3945.32....	2	2	1	.....	III
3897.23....	6	4	3	2	II	3946.03....	1	1	tr	.....	III
3898.08*...	6	4?	3?	1	II	3950.39....	4	4	2	tr	III
3898.41....	5	4	3	1	II	3952.13....	6	.....	.....	.....	V E
3899.26....	4	.....	.....	.....	V E	3963.89....	4	2	1	tr	II
3900.29....	6	4	3	2	II	3964.65....	2	1	tr	.....	III
3901.29....	6	4	3	2	II	3968.25....	3	.....	.....	.....	V E
3901.83....	2	2	1	tr	II	3972.12....	2	2	1	.....	III
3902.41....	50r	50R	40R	40	I	3973.53....	2	2	1	.....	III
3902.68....	3	4	4	3	I	3975.50....	1	1	tr	.....	III
3903.38....	4	.....	.....	.....	V E	3979.30....	4	3	2	tr	III
3904.36....	1	3	2	1	II A	3979.56....	4	3	2	tr	III
3904.59*...	2	2	1	.....	III	3980.69....	3	2	1	.....	III
	3	3	2	tr	III	3984.50....	6	6	3	1	II
3906.90....	6	6	6	6	I	3984.76....	6	6	4	2	II
3907.32....	2n	1	1	tr	II	3988.98....	5	5	3	1	II
3908.47....	4n	3	2	tr	III	3990.77....	20	15	12	10	I
3909.55....	tr	tr	.....	.....	IV	3991.24....	1	1	tr	.....	III
3909.81....	4	4	2	tr	III	3992.98....	12	8	6	6	I
3910.03*...	20	20R	20r	25	I	3995.05....	1	1	1	.....	III
3910.93....	5	6	6	5	I	3997.28....	3	.....	.....	.....	V E
3912.37....	10	8	8	8	I	3998.91....	15	12	12	8	I
3913.01....	4	8	6	4	II A	4000.25....	1	1	tr	.....	III
3913.71....	2	1	tr	.....	III	4001.81....	1	1	.....	.....	IV
3914.01....	1	2	2	1	II A	4003.10....	2	.....	.....	.....	V E
3914.46....	5	5	5	3	I	4003.33....	tr	tr	.....	.....	IV
3915.26....	2	1	1	tr	II	4003.69....	2	1	.....	.....	IV
3915.51....	2	2	1	tr	II	4005.88....	6	.....	.....	.....	V
3916.53....	3	.....	.....	.....	V E	4009.92....	2	1	1	.....	III
3917.29....	2	2	2	1	II	4011.45....	3	3	1	.....	III
3920.14....	2	2	1	.....	III	4015.21....	1	tr	.....	.....	IV
3920.64....	5	5	5	4	I	4022.05....	3	1	.....	.....	IV
3922.09....	6	6	6	6	I	4023.32....	4	2	.....	.....	IV
3922.61....	12	12	12	12	I	4023.54....	5	.....	.....	.....	V E
3924.83....	10	8	5	4	II	4030.05....	2	6	5	tr	III A
3925.40....	10	15r	15	20	I	4031.39....	2	2	.....	.....	IV
3926.82....	1	1	1	.....	III	4032.02....	5	2	.....	.....	IV
3930.13....	10	6	6	6	I	4032.64....	1	tr	.....	.....	IV
3931.50....	5	4	4	2	II	4033.00....	2	6	6	tr	III A
3934.20....	20	15r	15	15	I	4034.86....	1	4	2	.....	III A
3935.30....	6	6	4	2	II	4035.76....	4	.....	.....	.....	V E

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4036.90	2	.....	.....	.....	V E	4124.24	5	6	3	.....	III
4040.46	2	1	.....	.....	IV	4128.30	60	60R	35	40	I
4041.75	3	.....	.....	.....	V	4129.01	5	4	2	.....	III
4042.80	5	3	1	.....	III	4131.35	1	6	4	.....	III A
4048.76	4	8	8	3	II A	4132.15	60	60R	35r	40	I
4051.12	10	6	2	.....	III	4133.95	3	6	3	.....	III A
4051.58	12	6	2	.....	III	4134.65	60	60R	35r	40	I
4052.62	1	5	4	tr	III A	4136.25	4	3	1	.....	III
4053.43	3	2	tr	.....	IV	4136.56	3	8	8	1	III A
4057.26	10	4	2	.....	III	4139.42	4	4	tr	.....	IV
4064.13	10	5	1	.....	IV	4141.52	2	1	.....	.....	IV
4067.89	3	1	.....	.....	IV	4142.00	2	5	3	tr	III A
4068.15	4	8	8	3	II A	4142.81	2	6	4	tr	III A
4070.93	4	6	5	2	II	4143.06	2	2	1	.....	III
4071.68	8	5	2	.....	III	4149.01	2	5	5	tr	III A
4072.31	3	3	1	.....	III	4150.86	2	1	.....	.....	IV
4083.09	4	4	2	.....	III	4152.84	2	1	.....	.....	IV
4090.80*	25	12	?	12	I	4153.50	2	7	6	tr	III A
4092.11	3	3	1	.....	III	4156.02	1	5	2	.....	III A
4092.55	8	8	6	.....	III	4158.15	1	3	1	.....	III A
4092.89	50	50R	30r	40	I	4159.85	8	10	15	10	I
4093.65	5	6	4	.....	III	4160.56	1	5	2	.....	III A
4094.44	3	2	.....	.....	IV	4169.45	2	2	tr	.....	IV
4095.69	25	15	15	7	II	4171.49	3	4	tr	.....	IV
4097.09	3	2	.....	.....	IV	4174.20	5	4	2	.....	III
4099.99	60	60R	35r	40	I	4175.32	1	3	tr	.....	IV A
4102.37	20	15	15	6	II	4177.26	2	1	.....	.....	IV
4103.56	1	1	.....	.....	IV	4179.61	15	15	20	15	I
4104.53	12	7	2	.....	III	4181.05	1	1	.....	.....	IV
4104.94	15	12	5	.....	III	4182.26	2	2	.....	.....	IV
4105.37	60	60R	30r	35	I	4182.81	10	8	12	10	I
4107.63	4	4	2	.....	III	4190.03	12	12	15	12	I
4108.37	5	4	1	.....	III	4191.71	10	10	9	4	II
4109.20	2	2	.....	.....	IV	4195.77	1	1	.....	.....	IV
4109.98	50	50R	30r	40	I	4197.86	2	2	.....	.....	IV
4111.98	100R	100R	50R	50	I	4198.80	4	8	8	2	III A
4112.53	5	4	1	.....	III	4200.35	4	7	7	1	III A
4113.70	12	12	10	tr	III	4201.05	1	6	5	tr	III A
4114.70	3	2	.....	.....	IV	4210.03	20	15	20	15	I
4115.36	60	60R	30r	40	I	4216.54	1	4	2	1	II A
4115.6	2	3	1	.....	III	4218.88	4	10	10	2	III A
4116.64H	50	50R	50r	30	I	4219.67	2	6	4	tr	III A
4116.85H	4	15	15	10	I A	4222.49	2	1	.....	.....	IV
4118.37*	8	6	2	.....	III	4223.14	1	1	.....	.....	IV
4118.81	8	8	8	tr	III	4224.28	5	3	1	.....	III
4119.25	1	1	.....	.....	IV	4227.90	4	4	1	.....	III
4119.62	8	8	10	1	III	4229.86	4	3	1	.....	III
4120.71	8	8	8	tr	III	4232.66	15	9	4	.....	III
4121.15	1	tr	.....	.....	IV	4233.15	12	7	3	.....	III
4123.34	6	4	1	.....	III	4234.19	12	10	15	10	I
4123.71	60	60R	35r	40	I	4234.70	8	8	15	10	I

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4235.91....	10	6	2	.....	III	4355.14....	5	5	1	.....	IV
4236.77....	1	4	2	.....	III A	4356.12....	10	12	15	15	I
4239.13....	2	1	.....	.....	IV	4356.96....	1	1	.....	.....	IV
4240.25....	3	3	tr	.....	IV	4357.64....	2	2	.....	.....	IV
4240.52....	4	6	2	.....	III	4360.76....	3	3	.....	.....	IV
4241.48....	3	3	1	.....	III	4361.58....	2	2	.....	.....	IV
4246.85....	1	1	.....	.....	IV	4363.52....	1	1	.....	.....	IV
4252.96....	1	1	.....	.....	IV	4363.70....	5	12	12	4	II A
4257.51....	6	4	2	.....	III	4364.39....	4	2	.....	.....	IV
4259.46....	8	8	10	8	I	4365.90....	3	2	.....	.....	IV
4261.34....	2	1	.....	.....	IV	4368.23....	10	10	10	15	I
4262.29....	6	5	2	.....	III	4368.79....	4	3	.....	.....	IV
4265.28*...	8n	10	3	.....	III	4369.24....	2	2	.....	.....	IV
4266.06....	1	tr	.....	.....	IV	4373.42....	4	4	tr	.....	IV
4267.47....	1	tr	.....	.....	IV	4374.01....	4	5	1	.....	IV
4268.81....	20	12	8	1	III	4375.25....	1	1	.....	.....	IV
4269.90....	5	4	1	.....	III	4375.50....	4	2	.....	.....	IV
4270.48....	4	2	.....	.....	IV	4376.97....	1	tr	.....	.....	IV
4271.69....	12	10	6	tr	III	4378.07....	2	2	.....	.....	IV
4277.13....	12	8	6	tr	III	4379.41....	150r	150R	75r	60	II
4279.10....	2	4	1	.....	III A	4380.74....	4	5	1	.....	IV
4284.25....	15	10	8	tr	III	4381.20....	1	4	1	.....	IV A
4286.57....	5	5	2	.....	III	4384.36....	1	9	6	1	III A
4287.95....	4	2	.....	.....	IV	4384.91....	125r	125R	60r	50	II
4288.96....	1	2	.....	.....	IV A	4385.50....	1	1	.....	.....	IV
4291.47....	4	4	1	.....	III	4387.40....	3	8	3	.....	III A
4292.00....	15	10	4	.....	III	4390.19....	100	100R	50r	40	II
4296.29....	15	6	3	.....	III	4390.81....	1	1	.....	.....	IV
4297.82....	12	5	3	.....	III	4391.85....	2	2	.....	.....	IV
4298.19....	12	8	3	.....	III	4392.27....	5	12	15	5	II A
4302.28....	tr	1	.....	.....	IV A	4393.26....	4	4	1	.....	III
4303.68....	3	1	.....	.....	IV	4394.03....	4	8	5	.....	III A
4305.61....	3	3	.....	.....	IV	4394.99....	3	4	2	.....	III
4306.40....	15	12	15	15	I	4395.45....	80	80R	40	35	II
4307.36....	12	10	12	12	I	4397.54....	1	tr	.....	.....	IV
4309.67....	2	1	.....	.....	IV	4399.59....	2	3	1	.....	III
4309.99....	20	20r	20	15	I	4400.78....	60	60r	30	30	II
4314.06....	3	2	.....	.....	IV	4403.85....	4	5	2	.....	III
4330.26....	30	30r	20	25	I	4405.23....	4	10	8	1	III A
4332.56....	1	1	.....	.....	IV	4406.30*...	6	20	5	.....	III A
4333.03....	30	30r	20	20	I	4406.88....	80	80R	50r	50	I
4334.28....	4	8	2	.....	III A	4407.80....	70	70R	40r	40	I
4336.30....	2	1	.....	.....	IV	4408.35....	70	70R	40r	40	I
4341.21....	40	40r	20	25	I	4408.70....	90	90R	60r	50	I
4342.37....	4	5	2	.....	III	4412.33....	12	12	15	10	I
4343.00....	6	4	2	.....	III	4413.86....	2	6	2	.....	III A
4350.88....	1	1	.....	.....	IV	4414.73....	2	2	.....	.....	IV
4351.00....	2	12	6	tr	III A	4416.63....	20	20r	20	20	I
4352.64....	2	1	.....	.....	IV	4416.80....	2	6	1	.....	IV A
4353.08....	50	50R	30r	40	I	4420.12....	12	12	15	12	I
4353.51....	2	1	.....	.....	IV	4421.79....	20	20r	20	20	I

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4422.41	3	3	.....	.....	IV	4492.49*	1	2	.....	.....	IV A
4422.65	2	1	.....	.....	IV	4495.17	1	1	.....	.....	IV
4423.39*	8	20	8	tr	III A	4496.25	8	8	2	.....	III
4424.10	2	1	.....	.....	IV	4497.03H	5	6	2	.....	III
4424.75	4	10	2	.....	III A	4497.55	2	1	.....	.....	IV
4425.90	4	8	2	.....	III A	4498.30	tr	1	.....	.....	IV A
4426.20	20	20r	20	20	I	4501.01	1	1	.....	.....	IV
4427.48	5	4	2	.....	III	4501.44	1	1	.....	.....	IV
4428.70	15	12	15	15	I	4502.18	8	10	6	tr	III
4429.98	15	15	20	20	I	4506.30H	2	2	.....	.....	IV
4430.71	3	4	.....	.....	IV	4506.41H	1	tr	.....	.....	IV
4433.05*	1	6	1	.....	IV A	4506.77	2	2	.....	.....	IV
4434.73	5	7	2	.....	III	4509.49	3	3	.....	.....	IV
4436.30	15	15	15	20	I	4511.65	2	2	.....	.....	IV
4438.01	20	20r	20	25	I	4513.81	2	2	.....	.....	IV
4439.15	1	1	.....	.....	IV	4514.39	6	5	2	.....	III
4441.89	25	25r	20	30	I	4515.74	2	2	.....	.....	IV
4443.40	5	3	.....	.....	IV	4517.73*	3	1	.....	.....	IV
4444.40	20	20r	20	25	I	4520.35	3	10	2	.....	III A
4445.98	1	1	.....	.....	IV	4520.71	2	.....	.....	.....	V E
4449.73	5	7	3	.....	III	4523.03	tr	tr	.....	.....	IV
4451.10	4	3	.....	.....	IV	4524.41	15	10	5	.....	III
4452.23	20	12	12	1	III	4525.34	5	5	1	.....	IV
4452.89	2	3	.....	.....	IV	4528.18	5	2	.....	.....	IV
4453.31	1	1	.....	.....	IV	4529.47	4	2	.....	.....	IV
4456.69	3	1	.....	.....	IV	4529.80	8	8	3	.....	III
4457.67	15	15	20	30	I	4530.99	4	1	.....	.....	IV
4457.96	8	8	3	.....	III	4534.11	4	1	.....	.....	IV
4458.58	1	1	.....	.....	IV	4535.74	1	1	.....	.....	IV
4459.99	30	30R	25	35	I	4537.84	6	6	1	.....	IV
4460.58	50	50R	30r	40	I	4540.19	6	5	1	.....	IV
4461.20	4	3	.....	.....	IV	4545.59	25	15	8	1	III
4462.59	20	12	10	1	III	4549.80	10	5	.....	.....	IV
4464.44	2	2	.....	.....	IV	4552.04	3	4	tr	.....	IV
4464.94	2	3	.....	.....	IV	4553.25	7	4	.....	.....	IV
4465.69	2	3	.....	.....	IV	4560.95	20	12	6	tr	III
4467.05	2	4	.....	.....	IV A	4570.62	6	4	1	.....	III
4467.82	1	1	.....	.....	IV	4572.01	15	8	5	.....	III
4468.20	8	9	3	.....	III	4577.38	40	30r	25	35	I
4468.96	4	4	1	.....	III	4578.90	15	10	4	.....	III
4469.91	15	10	8	1	III	4579.29	7	5	1	.....	IV
4471.95	1	1	.....	.....	IV	4580.62	40	30r	25	35	I
4473.43	1	1	.....	.....	IV	4581.45	2	2	.....	.....	IV
4474.26	10	10	3	.....	III	4583.98	5	3	1	.....	III
4474.93	12	12	5	tr	III	4586.11	2	2	tr	.....	IV
4476.08	2	3	.....	.....	IV	4586.59	50	30R	30	40	I
4480.21	6	6	2	.....	III	4588.98	1	tr	.....	.....	IV
4489.09	20	15	8	tr	III	4591.43	12	4	tr	.....	IV
4491.00	5	6	2	.....	III	4594.36	60	40R	40	50	I
4491.35	2	3	1	.....	III	4603.17	2	1	.....	.....	IV
4491.64	1	1	.....	.....	IV	4606.33	15	15	20	15	I

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4609.79	4	3	1	.....	III	4784.68	5	15	15	5	II A
4611.08	2	4	2	.....	III A	4786.72	20	8	4	.....	III
4611.91	2	2	.....	.....	IV	4793.12	3	3	.....	.....	IV
4619.88	8	2	.....	.....	IV	4795.30	3	1	.....	.....	IV
4619.97H	25	20	20	15	I	4797.12	20	8	4	.....	III
4624.55	8	12	8	1	III	4798.14	2	3	.....	.....	IV
4626.64	7	12	7	1	III	4799.20	1	1	.....	.....	IV
4635.34	15	10	20	15	I	4799.97	5	12	15	7	II A
4636.32	2	2	.....	.....	IV	4807.72	25	8	5	.....	III
4640.25	8	10	6	tr	III	4819.20	2	1	.....	.....	IV
4640.92	7	10	5	tr	III	4827.63	30	25	30	30	I
4644.69	3	4	2	.....	III	4828.99	1	1	.....	.....	IV
4646.16	1	4	3	.....	III A	4830.86	1	1	.....	.....	IV
4646.60	15	12	12	1	III	4831.82	35	35	30	30	I
4666.31*	4	3?	3?	.....	III?	4832.61	30	30	25	25	I
4670.70	25	15	20	3	III	4833.20	3	6	1	.....	IV A
4687.08*	6	?	?	.....	?	4833.99	1	tr	.....	.....	IV
4699.49*	3	?	?	.....	?	4843.17	2	2	.....	.....	IV
4705.26*	4	?	?	.....	?	4848.78	3	4	1	.....	III
4706.35*	8	5?	?	.....	?	4849.00	1	tr	.....	.....	IV
4706.77*	12	5?	?	.....	?	4851.69	40	40r	30	30	I
4707.63*	4	8?	2?	.....	III A	4859.31	2	1	.....	.....	IV
4709.91*	4	4?	?	.....	?	4862.79	5	5	1	.....	IV
4710.75*	12	4?	?	.....	?	4864.91	40	30r	25	20	I
4714.30*	10	?	?	.....	?	4875.65	40	30r	25	25	I
4716.10*	5	5?	?	.....	?	4877.40	tr	1	.....	.....	IV A
4717.89	10	5	1	.....	IV	4880.74	8	12	5	.....	III
4721.75	6	3	1	.....	III	4881.73	50	40R	30	30	I
4723.09	8	4	1	.....	III	4882.38	2	4	1	.....	III A
4729.71	6	5	2	.....	III	4885.81	2	2	.....	.....	IV
4730.59	3	3	1	.....	III	4887.00	2	6	2	.....	III A
4737.92	1	1	.....	.....	IV	4890.29	1	1	.....	.....	IV
4738.52	1	1	.....	.....	IV	4891.41	1	1	.....	.....	IV
4739.29	1	2	.....	.....	IV A	4891.80	4	4	1	.....	III
4742.81	5	4	tr	.....	IV	4894.41	4	4	1	.....	III
4746.81	5	5	1	.....	IV	4896.60	1	4	.....	.....	IV A
4747.33	1	1	.....	.....	IV	4900.21	tr	1	.....	.....	IV A
4748.72	7	7	2	.....	III	4900.81	6	4	tr	.....	IV
4751.22	8	9	2	.....	III	4904.58*	12	12	4	.....	III
4751.80	6	6	2	.....	III	4905.10	3	3	tr	.....	IV
4754.19	7	6	1	.....	IV	4906.09	tr	1	.....	.....	IV A
4757.55H	4	4	1	.....	III	4908.86	1	2	.....	.....	IV A
4757.68H	8	8	2	.....	III	4916.42	2	1	.....	.....	IV
4758.94	2	4	1	.....	III A	4925.84	10	10	8	1	III
4765.86	1	1	.....	.....	IV	4932.21	4	8	4	.....	III A
4766.81	10	8	3	.....	III	4933.80	1	1	.....	.....	IV
4773.30	1	tr	.....	.....	IV	4942.98	1	2	.....	.....	IV A
4776.54H	10	8	3	.....	III	4966.27	2	1	.....	.....	IV
4776.70H	5	4	1	.....	III	5002.50	4	4	.....	.....	IV
4778.61	tr	tr	.....	.....	IV	5014.79	5	3	1	.....	III
4781.55	tr	1	.....	.....	IV A	5047.45*	1	2?	1?	.....	IIIA?

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
5051.81*...	2	1	.....	.....	IV	5517.40....	1	8	4	2	II A
5064.30*...	3	.....	.....	.....	V	5534.02....	1	1	.....	.....	IV
5105.32*...	2	3?	1?	.....	III?	5542.91....	1	6	4	2	II A
5128.70*...	7	6?	4?	.....	III	5446.13....	2	7	4	tr	III A
5138.60*...	5	3?	2?	.....	III	5447.26....	8	12	9	4	II
5148.87*...	4	2?	1?	.....	III?	5557.67....	1	6	5	2	II A
5159.52*...	3	?	?	.....	III?	5558.98....	3	5	3	.....	III
5166.93....	1	1	.....	.....	IV	5561.90....	2	4	2	.....	III A
5170.11....	1	1	tr	.....	III	5574.20....	1	4	2	1	II A
5176.95....	4	2	1	.....	III	5584.77....	10	10	10	8	I
5178.73....	1	1	.....	.....	IV	5586.21....	2	2	1	.....	III
5179.30....	1	tr	.....	.....	IV	5588.69....	1	3	1	.....	III A
5180.96....	1	1	.....	.....	IV	5592.63....	12	10	8	8	I
5183.05....	1	1	.....	.....	IV	5593.24....	1	10	5	4	II A
5192.22....	1	1	.....	.....	IV	5598.10....	1	2	1	.....	III A
5193.20....	7	5	4	.....	III	5601.60....	2	3	2	.....	III
5193.84....	1	1	tr	.....	III	5604.41....	1	1	.....	.....	IV
5195.00....	10	7	2	.....	III	5605.18....	8	10	10	5	II
5195.65....	5	3	1	.....	III	5624.87....	20	12	12	15	I
5206.80....	1	1	tr	.....	III	5625.18....	10	10	10	5	II
5213.82....	1	1	tr	.....	III	5626.27....	8	12	10	6	II
5216.76....	3	2	tr	.....	IV	5627.85....	30	12	15	20	I
5225.90....	3	2	1	.....	III	5632.71....	1	8	6	4	II A
5233.87....	2	2	tr	.....	IV	5646.29....	10	12	10	5	II
5234.26....	8	4	2	.....	III	5651.70....	1	1	.....	.....	IV
5240.35....	1	1	tr	.....	III	5657.10....	1	1	.....	.....	IV
5241.03....	9	5	2	.....	III	5657.70....	12	10	10	6	II
5261.12....	1	1	.....	.....	IV	5665.49....	1	1	.....	.....	IV
5271.20....	1	1	.....	.....	IV	5668.55....	12	12	10	6	II
5272.87....	1	1	.....	.....	IV	5671.05....	30	20	20	20	I
5282.70....	1	1	.....	.....	IV	5683.37....	2	2	1	.....	III
5330.58....	1	3	1	.....	III A	5687.93....	1	1	.....	.....	IV
5353.56....	5	3	1	.....	III	5691.3.....	1	1	.....	.....	IV
5383.61....	2	1	.....	.....	IV	5698.71....	60	30	30	40	I
5385.33....	3	1	tr	.....	III	5703.81....	40	20	20	30	I
5388.50....	1	1	tr	.....	III	5704.6.....	2	1	.....	.....	IV
5398.12....	1	tr	.....	.....	IV	5706.25....	2	1	.....	.....	IV
5402.17....	8	5	2	.....	III	5707.20....	30	20	20	25	I
5415.47....	10	6	2	.....	III	5707.95....	1	1	.....	.....	IV
5418.29....	2	1	.....	.....	IV	5709.11....	2	2	1	.....	III
5424.28....	4	3	1	.....	III	5716.37....	3	1	.....	.....	IV
5434.40....	3	2	tr	.....	IV	5725.80....	6	5	2	.....	III
5437.89....	1	1	.....	.....	IV	5727.28....	60	30	30	40	I
5458.31....	1	1	.....	.....	IV	5727.89....	20	20	20	8	II
5487.42....	2	3	1	.....	III	5730.10....	tr	1	.....	.....	IV A
5488.10....	10	5	3	.....	III	5731.50....	30	35	30	12	II
5490.19....	2	2	1	.....	III	5733.29....	1	1	.....	.....	IV
5505.09....	2	3	1	.....	III	5733.63....	1	tr	.....	.....	IV
5507.95....	8	6	2	.....	III	5734.21....	5	4	2	.....	III
5511.40....	1	1	.....	.....	IV	5737.25....	25	20	20	12	II
5515.27....	1	15	10	6	II A	5743.66....	18	20	20	8	II

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
5747.92....	2	2	1	.....	III	6135.49....	15	15	15	10	II
5749.05....	4	4	3	.....	III	6150.32....	15	20	25	30	I
5750.89....	2	2	1	.....	III	6170.55....	8	20	20	15	I A
5752.92....	3	4	2	.....	III	6182.06....	1	4	2	.....	III A
5761.67....	2	7	4	tr	III A	6189.55....	3	15	15	4	II A
5772.65....	6	6	5	.....	III	6190.70....	1	4	2	.....	III A
5776.90....	4	15	10	3	II A	6199.40....	30	30	30	35	I
5782.81....	2	6	3	.....	III A	6214.04....	15	20	20	15	I
5783.12....	1	1	.....	.....	IV	6214.93....	1	1	.....	.....	IV
5783.81....	2	2	1	.....	III	6216.52....	30	30	30	35	I
5784.64....	5	3	1	.....	III	6218.52....	3	2	.....	.....	IV
5786.43....	7	4	2	.....	III	6221.39....	1	7	3	.....	III A
5788.80....	3	4	3	.....	III	6224.70....	15	20	20	15	I
5790.80....	1	2	1	.....	III A	6230.92....	30	25	25	30	I
5800.2.....	2	2	tr	.....	IV	6233.31....	12	20	20	15	I A
5807.38....	3	2	1	.....	III	6236.49....	1	1	.....	.....	IV
5815.00....	1	1	.....	.....	IV	6238.38....	tr	tr	.....	.....	IV
5817.30....	3	4	2	.....	III	6240.30....	6	15	15	8	II A
5817.85....	5	2	1	.....	III	6243.02....	15	20	20	25	I
5826.83....	1	1	1	.....	III	6243.37....	30	20	20	30	I
5830.95....	7	3	1	.....	III	6243.70....	3	8	4	.....	III A
5846.65....	8	4	1	.....	III	6245.35....	2	12	8	3	II A
5850.57....	2	4	2	.....	III A	6249.37....	tr	1	.....	.....	IV A
5853.96....	1	2	tr	.....	IV A	6252.02....	30	30	30	30	I
5855.70....	tr	1	.....	.....	IV A	6257.03....	8	20	20	10	II A
5863.41....	tr	1	.....	.....	IV A	6258.73....	8	25	30	10	II A
5924.82....	2	4	3	.....	III A	6261.39....	5	25	20	6	II A
5979.11....	2	4	2	.....	III A	6266.49....	7	20	20	8	II A
5981.02....	2	7	4	tr	III A	6268.98....	8	25	30	10	II A
5984.85....	1	4	2	.....	III A	6274.80....	15	20	20	20	I
6002.52....	2	7	4	1	II A	6282.52....	2	3	1	.....	III
6002.89....	4	10	8	3	II A	6285.32....	20	20	20	25	I
6008.90....	tr	4	2	.....	III A	6293.02....	20	20	20	25	I
6016.34....	1	1	.....	.....	IV	6296.69....	15	20	20	20	I
6018.16....	tr	4	2	.....	III A	6298.89....	tr	1	.....	.....	IV A
6022.00....	tr	tr	.....	.....	IV	6304.60....	2	1	.....	.....	IV
6025.64....	1	2	1	.....	III A	6309.89....	1	1	.....	.....	IV
6039.95....	25	15	15	15	I	6311.70....	3	2	1	.....	III
6048.89....	tr	2	1	.....	III A	6318.59....	tr	1	.....	.....	IV A
6058.33....	5	20	10	3	II A	6321.49....	2	1	.....	.....	IV
6063.57....	tr	1	tr	.....	III A	6324.87....	2	3	1	.....	III
6067.47....	1	1	tr	.....	III	6327.00....	6	8	5	1	III
6081.70....	25	20	20	20	I	6339.23....	5	7	4	tr	III
6087.70....	1	3	1	.....	III A	6344.12....	1	1	.....	.....	IV
6090.45....	50	25	25	25	I	6349.61....	5	7	4	tr	III
6104.91....	tr	4	2	.....	III A	6355.72....	1	2	1	.....	II A
6107.21....	2	8	4	tr	III A	6357.47....	4	6	4	.....	III
6111.90....	25	25	25	12	II	6358.99....	3	4	2	.....	III
6119.70....	40	20	20	25	I	6361.42....	3	4	2	.....	III
6128.49....	2	6	4	tr	III A	6374.67....	1	2	1	.....	III A
6135.21....	2	4	3	.....	III A	6379.53....	3	4	1	.....	III

TABLE I—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
6393.47....	2	3	1	.....	III	6543.71....	3	15	8	1	III A
6417.19....	tr	1	.....	.....	IV A	6550.26....	1	1	.....	.....	IV
6418.87....	tr	1	.....	.....	IV A	6558.23....	2	7	3	tr	III A
6423.50....	tr	1	.....	.....	IV A	6566.10....	2	8	4	tr	III A
6425.3.....	1	1	.....	.....	IV	6579.19....	1	4	3	.....	III A
6430.68....	3	5	3	.....	III	6606.22....	5	15	15	3	III A
6431.82....	2	4	3	.....	III A	6608.06....	2	5	3	.....	III A
6433.37....	1	3	2	.....	III A	6623.80....	tr	2	tr	.....	IV A
6435.35....	1	3	2	.....	III A	6625.10....	5	15	15	3	III A
6438.25....	tr	1	.....	.....	IV A	6633.53....	1	4	1	.....	III A
6445.35....	tr	1	.....	.....	IV A	6644.02....	tr	2	tr	.....	IV A
6448.02....	tr	1	.....	.....	IV A	6662.6.....	tr	2	tr	.....	IV A
6451.20....	tr	1	.....	.....	IV A	6753.20....	5	15	15	3	III A
6452.55....	8	20	15	4	II A	6766.70....	4	12	12	2	III A
6467.19....	1	4	2	.....	III A	6785.19....	3	10	10	1	III A
6488.22....	2	2	tr	.....	IV	6812.63....	2	7	6	tr	III A
6490.94....	1	1	.....	.....	IV	6830.16*...	1	2	2	.....	III A
6504.38....	8	20	15	4	II A	6832.67*...	1	3	3	.....	III A
6508.96....	1	1	tr	.....	III	6842.11*...	1	2	2	.....	III A
6531.65....	20	20	20	6	II						

REMARKS ON TABLE I

λ	
3217.20	Furnace line may be partly Ti.
3533.87	Components of double line about 0.07 A apart.
3583.85	Wide in furnace and arc. May be double.
3606.85	Blend with Fe.
3687.63	Blend with Fe.
3721.55	Probably double.
3730.33	Wide in furnace and arc. Probably double.
3842.02	Exceptionally strong at low temperature.
3879.38 3879.80	Disturbed by carbon.
3891.30	Double line, not fully resolved.
3896.31	Probably double.
3898.08	Blend with Fe.
3904.59	Double line, not fully resolved.
3910.03	Probably double. Reversal is stronger on red side.
4090.80	Blended with impurity line on medium temperature plates.
4118.37	Double line, not resolved.
4265.28	Wide in furnace and arc.
4406.30	Probably double.
4423.39	Probably double.

4433.05	Wide in furnace.
4492.49	Wide in furnace.
4517.73	Wide in furnace.
4666.31	Disturbed by carbon.
4687.08 to 4716.10	These lines are in a dense carbon fluting and intensities are uncertain. They are not strong in the furnace and probably belong in Classes III and IV.
4904.58	Probably double.
5047.45 to 5159.52	Disturbed by carbon.
6830.16 6832.67 6842.11	λ's according to Shaw, *Astrophysical Journal*, **30**, 127, 1909.

TABLE II

TEMPERATURE CLASSIFICATION OF CHROMIUM LINES

λ (EXNER AND HASCHEK)	ARC	FURNACE			CLASS	λ (EXNER AND HASCHEK)	ARC	FURNACE			CLASS
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3550.81....	4	.....	.....	.....	V	3743.10....	5	4	1	.....	III
3558.66....	8n	5	1	.....	IV	3743.71....	20	8	4	2	II
3566.28....	10n	5	1	.....	IV	3744.03....	20	6	3	2	II
3572.90....	2	2	.....	.....	IV	3744.67....	5	4	2	.....	III
3573.80....	5	3	1	.....	III	3747.42....	2	1	tr	.....	III
3574.19....	4	2	1	.....	III	3748.75....	5	3	1	.....	III
3574.97....	5	2	1	.....	III	3749.15....	20	10	5	1	III
3575.06....	3	1	.....	.....	IV	3751.34....	1	tr	.....	.....	IV
3578.81*...	200R	400R	200R	100r	II	3755.99....	2	1	tr	.....	III
3584.47....	12n	.....	.....	.....	V	3757.30....	5	3	1	.....	III
3587.12....	2n	.....	.....	.....	V	3757.80....	12	5	3	tr	III
3593.64*...	160R	320R	160R	80r	II	3758.18....	6	4	2	.....	III
3599.54....	1	1	.....	.....	IV	3767.60....	4	3	tr	.....	IV
3601.81....	6	3	2	.....	III	3768.23H..	3n	3	1	.....	III
3602.70....	1	1	tr	.....	III	3768.37....	12	8	3	1	II
3603.89*...	3	?	1	.....	III	3768.86....	5	4	1	.....	III
3605.49*...	140R	280R	140R	70r	II	3769.14....	1	1	tr	.....	III
3609.65....	3	2	1	.....	III	3786.38....	2	1	.....	.....	IV
3610.20....	2	1	tr	.....	III	3789.87....	4	8	5	2	II A
3613.80....	1	1	.....	.....	IV	3790.35....	2	1	.....	.....	IV
3615.77....	4	10r	5	3	II A	3790.62....	5	2	tr	.....	IV
3619.57....	2	2	.....	.....	IV	3791.52....	7	4	1	.....	III
3632.98....	6	5	2	.....	III	3792.29....	7	4	1	.....	III
3636.70....	10	5	3	1	II	3792.55....	1	1	.....	.....	IV
3639.97....	15	5	4	1	II	3793.44....	7	4	1	.....	III
3640.55....	4	3	1	.....	III	3794.00....	7	4	1	.....	III
3641.61....	3	3	1	.....	III	3794.75....	5	3	1	.....	III
3641.99....	8	4	2	.....	III	3797.29....	7	4	1	.....	III
3646.30....	1	1	.....	.....	IV	3797.85....	12	6	3	.....	III
3648.66....	2	3	1	.....	III	3804.95....	15	8	4	1	III
3649.11....	10	5	3	tr	III	3806.97....	4	1	.....	.....	IV
3654.07....	8	4	2	.....	III	3808.06....	6	3	1	.....	III
3656.40....	10	5	3	tr	III	3810.4.....	2n	1	.....	.....	IV
3663.01....	2	2	1	.....	III	3812.36....	5	2	.....	.....	IV
3663.36....	5	4	2	.....	III	3814.71....	4	1	.....	.....	IV
3666.14....	1	1	.....	.....	IV	3815.60....	10	5	3	.....	III
3666.78....	3	1	.....	.....	IV	3817.96....	4	3	2	.....	III
3676.45....	2	1	.....	.....	IV	3818.62....	8	5	2	.....	III
3679.18....	1	1	.....	.....	IV	3819.72....	15	6	3	.....	III
3679.97....	2	2	1	.....	III	3823.68....	8	8	7	4	II
3681.82....	1	1	.....	.....	IV	3825.51....	10	3	1	.....	III
3685.70....	5n	5	3	.....	III	3826.58....	10	5	2	.....	III
3686.95....	6n	5	2	.....	III	3830.20....	15n	.....	.....	.....	V
3687.42....	5n	3	1	.....	III	3831.20....	5	8	7	3	II A
3687.66....	4	4	2	.....	III	3832.50....	2	5	4	tr	III A
3713.10....	1	1	.....	.....	IV	3834.94....	8	3	2	.....	III
3716.68....	3n	1	.....	.....	IV	3836.22....	4	2	1	.....	III
3730.96....	10	10R	10r	8	I	3841.43....	20	7	5	1	III
3732.19....	12	12R	12r	8	I	3842.22....	4n	2	.....	.....	IV

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
3849.10	10	8	4	1	III	3963.85	30	12	9	5	II
3849.45	20n	8	3	.....	III	3969.22	5	5	4	tr	III
3849.60	8	10	7	6	I	3969.90	25	10	9	4	II
3850.20	20	12	4	1	III	3971.41	5	4	3	tr	III
3852.33	8	10	7	6	I	3972.83	2	2	1	.....	III
3853.30	3	2	1	.....	III	3976.44	1	1	.....	.....	IV
3854.36	12	8	3	.....	III	3976.85	25	20	12	4	II
3854.95	4n	1	.....	.....	IV	3978.80	4	5	3	.....	III
3855.41	5	3	2	.....	III	3979.95	3	4	2	.....	III
3855.73	8	5	3	.....	III	3981.41	5	6	3	.....	III
3856.41	5	3	2	.....	III	3984.08	20	12	8	4	II
3857.75	15	10	5	1	III	3984.50	10	8	5	1	III
3859.0	10n	6	3	.....	III	3990.14	6	3	tr	.....	IV
3862.69	3	1	.....	.....	IV	3991.30	20	12	8	4	II
3868.40	2	.....	.....	.....	V	3991.82	10	8	4	tr	III
3870.4	3n	2	1	.....	III	3992.29	1	1	.....	.....	IV
3874.70	10	5	3	.....	III	3992.99	15	10	4	.....	III
3875.3	4n	tr	.....	.....	IV	3994.13	4	4	2	.....	III
3879.39	8	5	2	.....	III	4000.12	2	.....	.....	.....	V
3881.41	5n	2	1	.....	III	4001.61	8	3	tr	.....	IV
3883.48	15	18r	15	10	I	4004.08	2	.....	.....	.....	V
3885.36	15	15r	12	8	I	4012.61	8	2	.....	.....	IV
3886.92	15	15r	12	8	I	4014.81	3	1	.....	.....	IV
3894.20	15	12r	10	7	I	4018.35	3	3	1	.....	III
3897.79	2	tr	.....	.....	IV	4022.40	8	2	.....	.....	IV
3902.24	2	.....	.....	.....	V	4023.90	2	.....	.....	.....	V
3903.05*	12	8?	6?	4?	II?	4025.18	7	6	3	tr	III
3903.29	8	8	7	5	I	4025.63	1	1	.....	.....	IV
3907.91	2	tr	.....	.....	IV	4026.31	10	10	5	1	III
3908.91	25	25R	15	10	II	4027.23	8	8	4	.....	III
3911.97*	10n	3	1	.....	III	4028.21	1	1	.....	.....	IV
3912.13*		4	2	.....	III	4033.44	3	3	2	.....	III
3915.98	6	5	2	.....	III	4037.42	3	3	2	.....	III
3916.41	12	10r	8	6	I	4039.20	10	4	1	.....	III
3917.19	2	1	tr	.....	III	4041.97	2	.....	.....	.....	V
3917.80	4	3	1	.....	III	4042.40	4	3	1	.....	III
3919.32	35r	35R	20r	12	II	4043.86	3	.....	.....	.....	V
3921.21	20	15r	12	9	I	4046.93	3	2	tr	.....	IV
3926.82	3	1	.....	.....	IV	4048.91	10	4	tr	.....	IV
3928.82	25	20r	15	10	I	4049.95	2	1	.....	.....	IV
3941.67	20	15r	12	10	I	4050.20	2	1	.....	.....	IV
3945.64	2	1	.....	.....	IV	4051.49	2	.....	.....	.....	V
3946.10	3	2	tr	.....	IV	4056.19	3	.....	.....	.....	V
3949.00	1	tr	.....	.....	IV	4056.95	2	.....	.....	.....	V
3949.71	2	1	.....	.....	IV	4058.94	10	4	tr	.....	IV
3951.26	3	2	tr	.....	IV	4060.82	2	.....	.....	.....	V
3951.93	2	1	tr	.....	III	4065.88	6	.....	.....	.....	V
3952.55	4	3	1	.....	III	4067.09	10	7	3	.....	III
3953.30	3	2	tr	.....	IV	4075.00	3	1	.....	.....	IV
3958.22	1	tr	.....	.....	IV	4076.20	4	1	.....	.....	IV
3960.91	1	1	tr	.....	III	4077.25	5	5	1	.....	IV

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4077.85	4	1	.....	.....	IV	4183.25	1	1	.....	.....	IV
4081.90	3	2	.....	.....	IV	4185.10	3	4	1	.....	III
4085.20	2	1	.....	.....	IV	4185.53	2	2	.....	.....	IV
4090.50	3	1	.....	.....	IV	4186.51	3	1	.....	.....	IV
4092.35	3	1	.....	.....	IV	4190.31	3	1	.....	.....	IV
4097.80*	20n	5	2	.....	III	4191.45	10	8	3	1	II
4098.11*		7	3	tr	III	4191.94	4	4	1	.....	III
4098.33*		7	3	tr	III	4192.27	5	1	.....	.....	IV
4099.19	4	2	tr	.....	IV	4193.80	8	3	tr	.....	IV
4101.33	4	2	tr	.....	IV	4194.50	2	3	1	.....	III
4104.01	2	1	.....	.....	IV	4195.10	7	2	1	.....	III
4105.00	6	3	1	.....	III	4197.40	7	2	tr	.....	IV
4106.22	2	.....	.....	.....	V	4198.68	8	2	tr	.....	IV
4108.56	3	2	1	.....	III	4200.26	5	3	1	.....	III
4109.73	8	4	2	.....	III	4203.75	10	8	3	.....	III
4111.02*	20n	8	4	tr	III	4204.43	3	4	1	.....	III
4111.51*		6	3	.....	III	4204.64	7	1	.....	.....	IV
4111.82*		3	1	.....	III	4207.10	4	.....	.....	.....	V
4120.78	8	5	2	.....	III	4208.50	6	2	.....	.....	IV
4121.45	4	.....	.....	.....	V	4209.51	10	4	tr	.....	IV
4121.98	7	4	1	.....	III	4209.92	6	5	2	.....	III
4122.35	5	3	1	.....	III	4211.50*	6	5	1	.....	IV
4123.54	10	6	1	.....	IV	4212.81	4n	1	.....	.....	IV
4126.69	18	10	5	2	II	4213.36	4	3	tr	.....	IV
4127.10	3	3	1	.....	III	4216.52	8	6	1	.....	III
4127.46	4	1	.....	.....	IV	4217.75	15	10	3	.....	III
4127.81	5	5	2	.....	III	4221.73	8	2	.....	.....	IV
4128.56	3	.....	.....	.....	V	4222.90	6	6	2	.....	III
4129.36*	20n	10	5	tr	III	4224.68	4	.....	.....	.....	V
4131.55	10	2	.....	.....	IV	4230.68	4	4	1	.....	III
4134.55	3	.....	.....	.....	V	4232.41	3	2	tr	.....	IV
4146.38	4	tr	.....	.....	IV	4233.05	2	2	.....	.....	IV
4146.65	2	1	.....	.....	IV	4234.70	3	3	1	.....	III
4146.86	4	2	.....	.....	IV	4237.90	2	2	tr	.....	IV
4152.93	4	1	.....	.....	IV	4239.12	8	7	2	.....	III
4153.23	4	3	1	.....	III	4240.89	10	8	2	.....	III
4154.00	20	10	5	1	III	4243.01	1	1	.....	.....	IV
4161.55	12	1	.....	.....	IV	4248.45	2	2	tr	.....	IV
4163.79	20	4	1	.....	III	4248.87	2	1	.....	.....	IV
4165.70	10	1	.....	.....	IV	4252.39	2	1	tr	.....	III
4170.00	6	2	.....	.....	IV	4254.51	500R	1000 R	500R	250r	II
4170.40	4	1	.....	.....	IV	4255.70*	6	?	1	.....	III
4171.86	3	3	1	.....	III	4257.54	2	1	.....	.....	IV
4172.95	4	1	.....	.....	IV	4259.32	2	1	.....	.....	IV
4174.36	1	1	.....	.....	IV	4261.51	8	8	3	.....	III
4174.53	1	1	.....	.....	IV	4261.80	4	2	tr	.....	IV
4175.01	15	15	5	1	III	4262.30	4	2	tr	.....	IV
4175.43	3	1	.....	.....	IV	4262.56	2	1	tr	.....	III
4176.16	5	4	1	.....	IV	4263.30	12	5	1	.....	IV
4178.12	3n	.....	.....	.....	V	4267.01	1	1	.....	.....	IV
4179.42	12	6	1	.....	IV	4268.95	2	tr	.....	.....	IV

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4270.15....	4	2	tr	.....	IV	4385.15....	20	20r	20	20	I
4273.10....	6	2	1	.....	III	4387.54H..	2	2	1	.....	III
4275.00....	400R	800R	400R	200r	II	4387.64H..	5	4	2	.....	III
4280.57....	12	4	1	.....	III	4391.93....	8	8	8	6	I
4284.90....	2	3	1	.....	III	4392.50....	1	1	.....	.....	IV
4289.90....	350R	700R	350R	175r	II	4393.74....	2	1	.....	.....	IV
4292.14....	6	1	.....	.....	IV	4395.61....	2	1	.....	.....	IV
4293.71....	4	3	1	.....	III	4397.44....	3	3	1	.....	III
4295.91....	15	15	4	.....	III	4400.01....	3	3	1	.....	III
4296.46....	1	tr	.....	.....	IV	4403.55H..	3	3	1	.....	III
4296.80....	1	tr	.....	.....	IV	4403.68H..	5	1	tr	.....	IV
4297.21....	5	5	2	.....	III	4406.46....	2	1	.....	.....	IV
4297.89....	12	2	tr	.....	IV	4410.49....	4	4	1	.....	III
4299.85....	4	3	2	.....	III	4411.15H..	2	1	tr	.....	III
4301.35....	6	2	tr	.....	IV	4411.26H..	5	5	2	.....	III
4305.61....	5	4	2	.....	III	4412.44....	6	10	10	6	I A
4307.67....	1	tr	.....	.....	IV	4414.02....	5	3	1	.....	III
4319.80....	8	6	2	.....	III	4422.8.....	2	1	.....	.....	IV
4320.78....	4	3	1	.....	III	4423.55....	3	4	1	.....	III
4321.44....	3	2	tr	.....	IV	4424.20H..	2	2	1	.....	III
4321.81....	3	2	tr	.....	IV	4424.49....	10	7	3	tr	III
4323.70....	5	1	.....	.....	IV	4425.31....	3	1	.....	.....	IV
4325.25....	15	10	4	.....	III	4428.66....	5	5	2	.....	III
4332.75....	2	1	.....	.....	IV	4430.09....	2	1	.....	.....	IV
4337.74....	30	30R	25	20	I	4430.72....	4	1	tr	.....	III
4338.56....	3	1	.....	.....	IV	4432.35....	7	7	3	.....	III
4338.95....	3n	1	.....	.....	IV	4434.15....	2	2	1	.....	III
4339.62....	40	40R	30r	20	I	4442.45....	4	2	1	.....	III
4339.90....	20	20r	20	12	I	4443.89....	4	1	.....	.....	IV
4340.31....	8	8	3	.....	III	4450.5.....	3n	1	.....	.....	IV
4343.33....	4	3	1	.....	III	4458.72....	12	10	4	tr	III
4344.68....	40	40R	35r	25	I	4459.57....	4n	4	1	.....	III
4345.27....	2	2	.....	.....	IV	4459.92*...	6	?	?	?	?
4346.99....	10	7	3	.....	III	4460.90....	4n	4	1	.....	III
4351.22....	20	20r	20	15	I	4461.50....	1	tr	.....	.....	IV
4351.98....	60	60R	40r	25	I	4462.91....	3	3	1	.....	III
4354.13....	2	2	tr	.....	IV	4464.85....	2	2	tr	.....	IV
4356.95....	4	3	1	.....	III	4465.09....	4	4	1	.....	III
4357.69....	2	1	tr	.....	III	4465.31H..	2n	1	.....	.....	IV
4359.82....	20	20r	20	20	I	4465.53....	5	5	2	.....	III
4363.31....	6	6	3	.....	III	4466.35....	3	2	tr	.....	IV
4368.45....	2	2	tr	.....	IV	4467.74....	4	3	1	.....	III
4371.48....	20	20r	20	20	I	4468.55....	1	1	.....	.....	IV
4373.42....	8	8	8	6	I	4469.99....	1	1	.....	.....	IV
4373.81....	2	.....	.....	.....	V	4473.96....	4n	4	2	.....	III
4374.32....	12	8	4	1	II	4475.50....	8n	8	3	.....	III
4375.49....	8	5	2	.....	III	4477.20....	2n	4	2	.....	III A
4376.95....	3	.....	.....	.....	V	4480.45*...	2	2	tr	.....	IV
4377.69....	4	3	1	.....	III		1	1	.....	.....	IV
4381.30....	6	5	2	.....	III	4483.02....	5	3	1	.....	III
4383.05....	2	2	1	.....	III	4488.21....	5	5	2	.....	III

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4489.62	5	2	tr	.....	IV	4595.79	6	1	.....	.....	IV
4490.8*	2n	1	.....	.....	IV	4600.30	5	5	3	.....	III
4491.90	3n	5	3	.....	III	4600.90	20	20	20	20	I
4492.01	3	2	1	.....	III	4601.21	4	4	2	.....	III
4492.50	6	3	2	.....	III	4613.53	15	15	15	15	I
4495.50	2	1	.....	.....	IV	4616.28	25	25	25	25	I
4497.03	25	25R	25r	25	I	4619.70	8	5	2	.....	III
4498.87	6	6	4	.....	III	4622.10	10	10	4	.....	III
4500.42	7	6	4	.....	III	4622.62	5	3	1	.....	III
4501.25	6	5	3	.....	III	4622.95	3	2	tr	.....	IV
4501.90	3	3	2	.....	III	4626.35	20	20	20	20	I
4502.43	1	1	.....	.....	IV	4632.39*	2	?	.....	.....	IV ?
4503.25	2	1	.....	.....	IV	4633.49	2	1	.....	.....	IV
4506.99	4	1	.....	.....	IV	4637.36	4	4	2	.....	III
4512.10	10	8	3	.....	III	4637.95	4	4	2	.....	III
4514.70	8	6	2	.....	III	4639.78*	2	?	.....	.....	IV ?
4515.60	4	3	1	.....	III	4646.35	40	40	40	40	I
4521.32	4	1	.....	.....	IV	4646.68*	3	2	tr	.....	IV
4525.00	2	.....	.....	.....	V	4646.99	3	2	tr	.....	IV
4526.28	3	2	.....	.....	IV	4648.29	5	5	2	.....	III
4526.66	15	12	10	3	II	4649.01	5	3	tr	.....	IV
4527.53H*	6	?	?	.....	III ?	4649.61	5	2	tr	.....	IV
4527.65H*	4	?	?	.....	III ?	4651.49	20	20	20	20	I
4530.02	5	5	2	.....	III	4652.38	30	30	30	30	I
4530.91	20	15	10	3	II	4654.94	3	2	tr	.....	IV
4535.33	6	6	3	.....	III	4656.38*	2	?	.....	.....	IV ?
4535.89*	15	8 ?	6 ?	2	II	4663.53	7	3	tr	.....	IV
4539.94	5	5	3	.....	III	4664.05	8	4	1	.....	III
4540.64	12	8	5	2	II	4665.00	8	5	2	.....	III
4540.88	10	6	3	.....	III	4666.10	4	1	.....	.....	IV
4541.23	5	4	3	.....	III	4666.40	4	3	1	.....	III
4541.65	4	2	1	.....	III	4666.72	7	3	1	.....	III
4542.76	5	3	1	.....	III	4667.34	2	1	.....	.....	IV
4543.89	2	2	1	.....	III	4669.52	6	3	1	.....	III
4544.78	12	10	6	2	II	4680.70	4	2	1	.....	III
4545.47	5	5	3	.....	III	4681.10*	3	?	?	.....	IV ?
4546.10	20	20	20	20	I	4689.55	8	6	2	.....	III
4555.01	2	2	.....	.....	IV	4694.15	5	3	1	.....	III
4555.30	1	1	.....	.....	IV	4695.35*	3	?	1 ?	.....	III ?
4556.35	6	3	1	.....	III	4697.25	5	4	2	.....	III
4563.89	2	2	tr	.....	IV	4698.71	20	12	5	1	III
4564.36	3	.....	.....	.....	V	4700.80	4	3	tr	.....	IV
4565.70	12	12	12	12	I	4706.29	2	1	.....	.....	IV
4569.80	8	6	2	.....	III	4708.20	15	6	3	tr	III
4571.28	1	1	.....	.....	IV	4718.63	20	8	4	1	III
4571.89	10	8	4	.....	III	4723.31	4	3	1	.....	III
4575.30	2	1	.....	.....	IV	4724.60	4	3	1	.....	III
4580.26	20	20	20	20	I	4727.32	6	4	1	.....	III
4584.30	2	1	.....	.....	IV	4729.92	2	1	.....	.....	IV
4586.31	2	1	.....	.....	IV	4730.90	8	6	3	.....	III
4591.61	20	20	20	20	I	4737.52	10	6	3	.....	III

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
4745.49....	2	3	1	.....	III	5122.27....	4	5	4	3	I
4752.29....	6	2	tr	.....	IV	5123.63....	6	8	6	4	I
4754.92....	2	2	tr	.....	IV	5139.81....	12	2	.....	.....	IV
4755.33....	2	2	tr	.....	IV	5144.85*...	7	?	?	1	II ?
4756.30....	15	10	6	1	III	5161.94*...	3	?	?	.....	III ?
4764.48....	5	3	1	.....	III	5166.41....	15	3	1	.....	III
4764.87....	1	1	tr	.....	III	5177.57....	7	2	tr	.....	IV
4766.88....	2	1	tr	.....	III	5184.73....	10	3	1	.....	III
4767.47....	1	tr	.....	.....	IV	5192.18....	10	3	1	.....	III
4768.02....	3	2	tr	.....	IV	5193.67....	4	1	.....	.....	IV
4775.32....	1	1	.....	.....	IV	5196.61....	15	4	1	.....	III
4789.53....	20	12	8	3	II	5200.37....	6	2	tr	.....	IV
4790.55....	2	4	2	.....	III A	5204.71....	150r	120R	70R	50	II
4792.72....	15	8	3	.....	III	5206.24....	200r	150R	80R	60	II
4796.33....	2	1	.....	.....	IV	5208.60....	300r	200R	100R	75	II
4801.24....	15	9	4	.....	III	5212.40....	3	1	.....	.....	IV
4806.48....	1	3	1	.....	III A	5214.27....	6	2	tr	.....	IV
4810.95....	1	2	tr	.....	III A	5221.10....	3	1	.....	.....	IV
4814.50....	1	2	tr	.....	III A	5221.93....	8	3	1	.....	III
4829.53....	18	15	9	2	II	5222.85....	2	2	tr	.....	IV
4837.06....	2	2	1	.....	III	5224.24....	3	1	.....	.....	IV
4846.52....	1	1	.....	.....	IV	5224.70....	3	1	.....	.....	IV
4861.40....	4	4	2	.....	III	5225.19....	25	10	3	tr	III
4862.01....	15	12	6	1	III	5226.00....	4	3	1	.....	III
4870.99....	20	10	4	1	III	5227.09....	4	2	.....	.....	IV
4874.82....	1	1	.....	.....	IV	5228.25....	3	1	.....	.....	IV
4880.21....	1	1	.....	.....	IV	5230.35....	2	2	1	.....	III
4885.13....	1	3	1	.....	III A	5239.12....	5	5	2	tr	III
4885.92H..	4	5	3	.....	III	5240.62....	3	1	.....	.....	IV
4886.11H..	1	2	tr	.....	IV A	5241.62H..	1	1	.....	.....	IV
4887.20....	20	6	4	1	II	5243.53....	7	3	1	.....	III
4887.88....	1	1	tr	.....	III	5247.72....	40	20r	20	20	I
4888.71....	4	4	2	.....	III	5255.08H..	10	4	tr	.....	IV
4903.47....	8	6	3	.....	III	5255.27H..	15	6	1	.....	IV
4921.15....	3	2	tr	.....	IV	5261.91....	6	2	.....	.....	IV
4922.49....	20	10	6	1	III	5264.35....	50	30R	30	20	I
4936.50....	10	5	2	.....	III	5265.33....	8	4	1	.....	III
4942.68....	8	30	20	10	II A	5265.90....	25	20r	20	12	I
4954.99....	10	5	2	.....	III	5272.18....	8	4	1	.....	III
4965.10....	6	20	15	8	II A	5273.59....	6	2	1	.....	III
5013.49....	6	5	2	.....	III	5275.33....	20n	12	4	2	II
5022.04....	2	10	7	4	II A	5275.85H..	15n	12	6	3	II
5048.95....	2	8	6	3	II A	5276.20H..	20n	12	6	3	II
5052.08....	8	10	8	6	I	5278.39....	2	.....	.....	.....	V
5066.06....	5	2	1	.....	III	5280.48....	4	2	.....	.....	IV
5067.87....	10	3	1	.....	III	5287.34....	4	2	.....	.....	IV
5068.45....	2	8	6	3	II A	5296.86....	50	25r	25	20	I
5073.10....	12	10	8	6	I	5297.48....	20n	12	8	3	II
5092.03....	3	8	6	3	II A	5298.14H..	15n	10	6	2	II
5110.93*...	7	3	2 ?	1 ?	II ?	5298.46....	60	30r	30	25	I
5113.30*...	5	3	?	1 ?	II ?	5300.89....	25	15	15	12	I

TABLE II—*Continued*

λ (Exner and Haschek)	Arc	Furnace			Class	λ (Exner and Haschek)	Arc	Furnace			Class
		High Temp.	Medium Temp.	Low Temp.				High Temp.	Medium Temp.	Low Temp.	
5304.33....	4	1	.....	.....	IV	5720.03....	2	2	1	.....	III
5313.02....	5	2	tr	.....	IV	5729.41....	1	1	.....	.....	IV
5318.92....	4	2	.....	.....	IV	5738.75....	1	1	tr	.....	III
5328.53....	50n	15	10	6	II	5746.64....	2	1	tr	.....	III
5329.33....	20n	10	6	3	II	5753.89....	2	.....	.....	.....	V
5329.95....	5n	4	2	1	II	5781.42....	8	5	3	tr	III
5340.65*...	4	?	.....	.....	IV ?	5782.10....	8	5	3	tr	III
5345.00....	2	2	1	.....	III	5783.38....	20	10	5	1	III
5345.99....	70	40R	40	30	I	5784.17....	20	10	5	1	III
5348.50....	50	30R	30	20	I	5785.29....	20	8	4	1	III
5387.14....	5	2	tr	.....	IV	5785.95*...	15	6	4	1	III
5387.73*...	3	?	tr	.....	III	5786.19*...	?	10	10	5	I A
5390.60....	3	1	.....	.....	IV	5787.25....	3	3	2	.....	III
5391.56....	2	?	.....	.....	IV	5788.22....	40	12	8	2	III
5400.75....	8	3	1	.....	III	5788.59....	4	2	1	.....	III
5405.19....	3	?	.....	.....	IV	5791.30....	50	20	15	3	III
5410.01....	100	60R	60	40	I	5792.00....	1	1	tr	.....	III
5442.60....	3	?	.....	.....	IV	5838.88....	2	2	1	.....	III
5464.12....	4	?	.....	.....	IV	5844.84....	2	2	1	.....	III
5480.71....	4	2	.....	.....	IV	6330.30....	25	50	50	50	I A
5628.82....	8	4	2	.....	III	6363.03....	15	30	30	30	I A
5638.37....	1	1	tr	.....	III	6501.43....	2	15	10	10	I A
5642.59....	2	2	1	.....	III	6529.36....	2	.....	.....	.....	V
5648.40....	1	1	tr	.....	III	6538.12....	3	18	15	15	I A
5649.52....	2	2	tr	.....	IV	6573.10....	2	12	12	12	I A
5658.82....	1	1	tr	.....	III	6597.80....	2	.....	.....	.....	V
5664.24....	8	4	2	.....	III	6630.25....	4	20	20	20	I A
5681.42....	2n	.....	.....	.....	V	6661.30....	12	3	.....	.....	IV
5682.63....	4n	2	1	.....	III	6669.50....	3	1	.....	.....	IV
5694.94....	10	4	2	.....	III	6881.7.....	2	1	tr	.....	III
5698.53....	20	6	3	tr	III	6882.4.....	5	2	1	.....	III
5700.75....	1	1	tr	.....	III	6883.2.....	10	4	2	.....	III
5702.52....	10	5	2	.....	III	6924.4.....	10	5	2	.....	III
5712.87....	2	.....	.....	.....	V	6925.4.....	6	3	1	.....	III
5712.98....	6	7	3	tr	III	6978.75....	15	8	2	.....	III

REMARKS ON TABLE II

λ	
3578.81 3593.64 3605.49	Widely reversed at high temperature.
3603.89	Concealed at high temperature by λ 3605.49.
3903.05	Difficult blend with Class II Fe line.
3911.97 3912.13	Measured by writer in vacuum arc. Blend into wide and diffuse line in arc in air.

λ	
4097.80	As for λ 3912.
4098.11	
4098.33	
4111.02	As for λ 3912.
4111.51	
4111.82	
4129.36	Measured by writer in vacuum arc.
4211.50	May be close double.
4255.70	Concealed at high temperature by λ 4254.51.
4459.92	Blend with strong V line in furnace.
4480.45	Double, scarcely resolved.
4490.8	Very diffuse in arc.
4527.53	Blend with strong Ti line in furnace.
4527.65	
4535.89	Blend with Ti.
4632.39	Disturbed by carbon.
4639.78	Disturbed by carbon.
4646.68	λ Rowland.
4656.38	Disturbed by carbon.
4681.10	Disturbed by carbon.
4695.35	Disturbed by carbon.
5110.93	Disturbed by carbon.
5113.30	
5144.85	Disturbed by carbon.
5161.94	
5340.65	Disturbed by carbon.
5387.73	
5785.95	λ Rowland.
5786.19	λ Rowland. Faint in arc, blended with λ 5785.95.

GENERAL CHARACTERISTICS OF THE VANADIUM AND CHROMIUM SPECTRA

1. *Low temperature lines of chromium.*—As has been noted, some chromium lines appeared at a temperature lower than that used for the low-temperature column of Table II. Table III is a list of these lines with their intensities estimated from a photograph taken with the 1-meter concave grating, a bathed film with filter being used so that the spectrum from λ 3500 into the red was registered. The temperature read 1728° C. and the prolonged exposure made it probable that the spectrum for this temperature was fairly complete. Almost all of the lines as far as λ 5000 were

obtained also on a plate taken at the same temperature with the 15-foot concave grating.

The intensities in Table III are quite different from those in the low-temperature column of Table II, given at 2000–2100°. This is probably in some measure due to differences in contrast and color-curve between the film and the plate, but chiefly to the fact that the lines of Class I retain a high relative intensity at the minimum temperature for their appearance, while those of Class II, which make up the remainder of Table III, are weaker than in Table II.

TABLE III

CHROMIUM LINES GIVEN BY A TEMPERATURE OF 1700–1800°

λ	I	λ	I	λ	I
3578.81	25	4344.68	40	5048.95	tr
3593.64	20	4351.22	20	5052.08	3
3605.49	20	4351.98	30	5068.45	1
3615.77	1	4359.82	40	5073.10	5
3730.96	3	4371.48	50	5092.03	1
3732.19	3	4373.42	10	5122.27	1
3883.48	8	4385.15	40	5123.63	2
3885.36	7	4391.93	8	5204.71	70
3886.92	7	4412.44	4	5206.24	80
3894.20	6	4497.03	60	5208.60	100
3903{.05 .29}	10	4546.10	60	5247.72	50
		4565.70	25	5264.35	50
3908.91	10	4580.26	50	5265.90	30
3916.41	6	4591.61	35	5296.86	60
3919.32	12	4600.90	50	5298.46	60
3921.21	8	4613.53	40	5300.89	20
3928.82	8	4616.28	50	5328.53	1
3941.67	12	4626.35	50	5329.30	tr
4254.51	200	4646.35	60	5345.99	80
4275.00	175	4651.49	40	5348.50	70
4289.90	150	4652.38	50	5410.01	80
4337.74	30	4942.68	10	6330.30	12
4339.62	20	4965.10	6	6363.03	6
4339.90	20	5022.04	tr	6630.25	3

2. *Successive development of the classes.*—The low-temperature spectrum, described for chromium in the preceding section, occurs for each of the elements which have been studied with the furnace at a stage probably not more than 200° above the melting-point. The medium temperature (about 2300°) produces a rich spectrum. The Class II lines are well developed and the Class III lines are present. Those lacking are the weaker arc lines, some lines which

are strong but diffuse in the arc, and the enhanced lines. At the highest temperatures used the number is increased by the addition of the Class IV lines, and in the case of vanadium, as with titanium, by the faint appearance of some of the stronger enhanced lines. A striking change produced by the high temperature is the general widening of the lines present at lower temperature, together with an increase in the number of reversals.

3. *Changes with the wave-length.*—Photographs made with the 1-meter concave grating showed the spectra of vanadium and chromium extending farther into the ultra-violet as the temperature rises, a condition also observed for iron and titanium. The limits of the vanadium spectrum on these films are roughly λ 3200, λ 2800, and λ 2500 for the three temperatures. The arc spectrum extends to shorter wave-length. Aside from the extension toward shorter waves nothing definite appears in the way of a general change. Low-temperature lines occur in groups throughout the spectrum, beginning with the limit in the ultra-violet. At the red end, while a number of lines occur in each spectrum for which the furnace is more favorable than the arc, these are not relatively strong at low temperature, but are for the most part in Class III.

The tendency in all light-sources for lines of shorter wave-length to reverse more easily is perhaps nowhere so striking as in the furnace spectra. If reversal were dependent merely on the presence of rarer vapor at a lower temperature, lines in the red region should reverse as easily as in the violet, since a number of red lines are given by the low-temperature furnace. Reversal is, however, clearly a function of wave-length. The high-temperature furnace gives numerous and wide reversals of lines in the region of shorter wave-length, presumably by reason of the cooler vapor near the ends of the tube; but in the green reversal becomes more difficult and in the yellow and red even the strong low-temperature lines of these elements remain hard and sharp in the furnace spectrum.

4. *Absence of the band spectra.*—The arc spectra of vanadium and chromium each show a series of bands shaded toward the red. The heads of these have been measured[1] as approximately λλ 5470, 5737,

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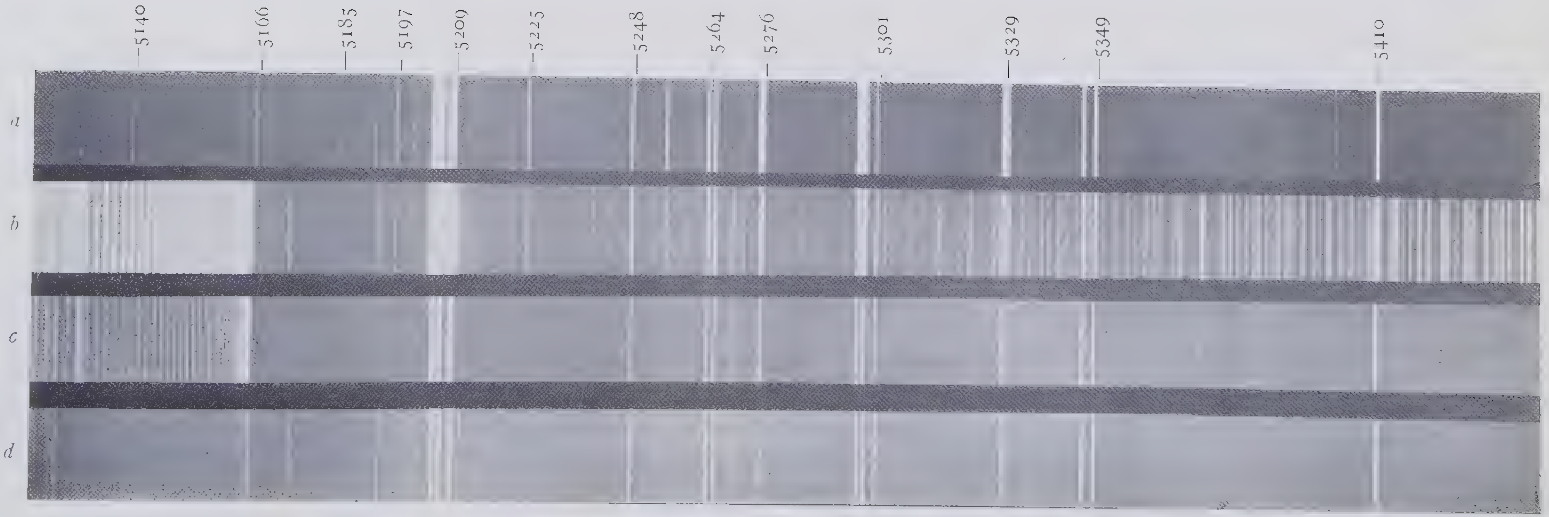
[1] Kayser, *Handbuch der Spectroscopie*, **5**, 376; **6**, 786.

6087, 6478 for vanadium, and λλ 5565, 5795, 6052, 6395 for chromium. These appear distinctly on my arc photographs but in none of the furnace spectra. The present evidence is that these bands are due to oxides formed in the arc and are not low-temperature spectra of the metals themselves.

5. *Comparison with the arc spectrum.*—In Tables I and II lines of intensity 1 in the arc are entered only when they occur also in the high-temperature furnace. The number of arc lines in Exner and Haschek's tables which are thus omitted from the furnace list is much greater for chromium than for vanadium. Of the stronger arc lines 35 chromium lines occur in Class V, while this class for vanadium is limited almost entirely to the enhanced lines (*V E*). In short, the relative behavior of the two elements is similar to that of titanium and iron, in that when the vanadium spectrum has once appeared (its high melting-point rendering the initial temperature higher than that of chromium), the furnace is highly efficient in producing a rich spectrum, giving at high temperature a spectrum closely comparable in number of lines with that of the arc.

The most interesting difference between the arc and furnace spectra is probably the large number of lines relatively much stronger in the furnace than in the arc and designated by *A* after the class number. It is difficult to reconcile the behavior of these lines with a continuous change in temperature alone between the arc and the furnace. It would appear that certain conditions needed for the emission of these lines are stronger in the furnace than in the arc. No discontinuity is apparent in the development of the spectrum from one furnace temperature to another. A line strong at low temperature is strong also at medium and high temperatures, but it may be a weak line in the arc, and the same is true for lines which develop at the successive temperature stages. A Class III line, for example, may show a normal increase from medium to high temperature and then drop to a low intensity in the arc. Further investigation may show that the emission of such lines is greatly localized in the arc and that the furnace is better suited to produce a vapor in the same condition as the given region of the arc, but at present the reason for the difference is obscure.

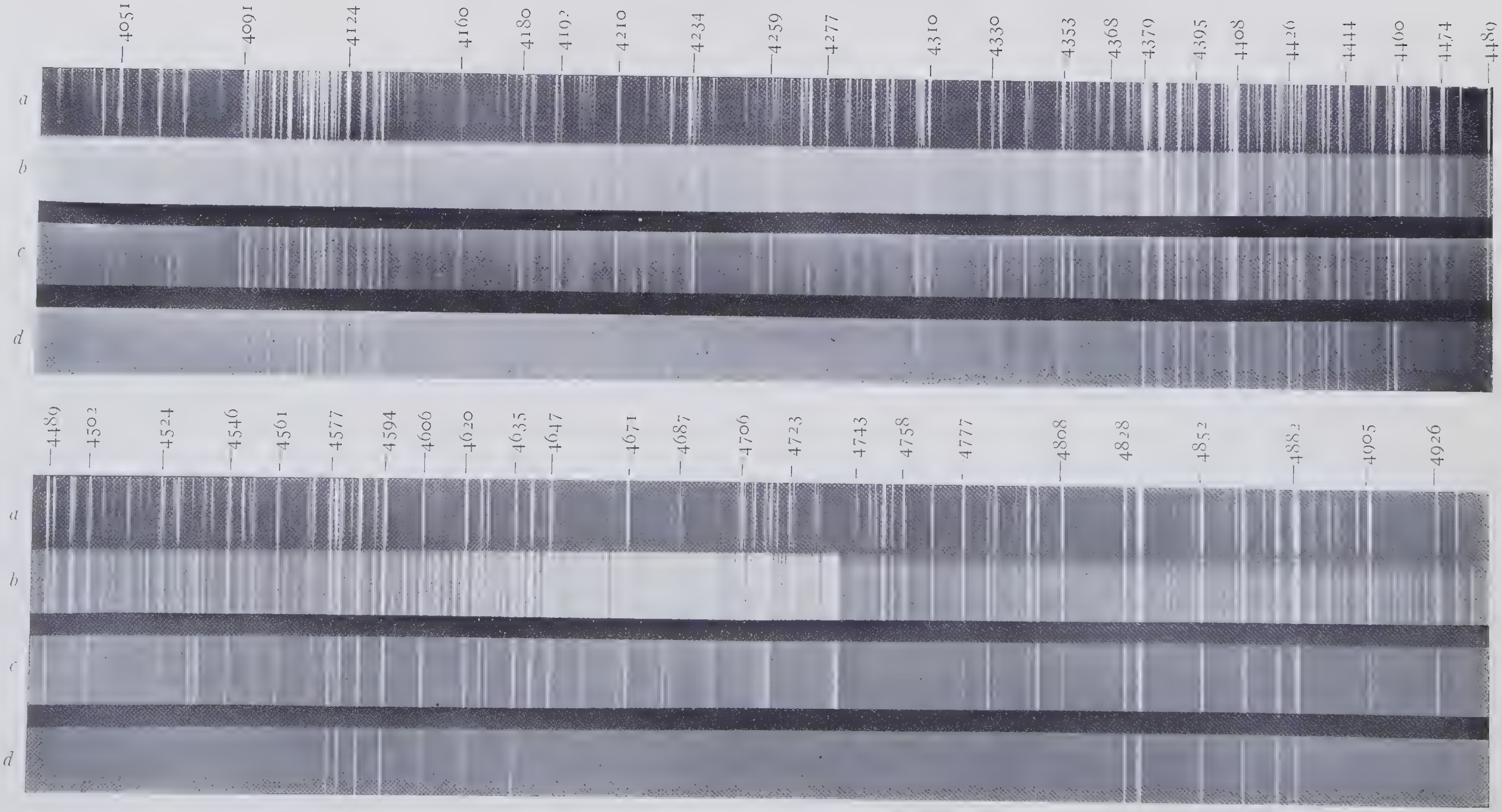
PLATE V



FURNACE AND ARC SPECTRA OF CHROMIUM

a. Arc spectrum of chromium.
b. Spectrum of the electric furnace at 2600° C.
c. Spectrum of the electric furnace at 2350° C.
d. Spectrum of the electric furnace at 2100° C.

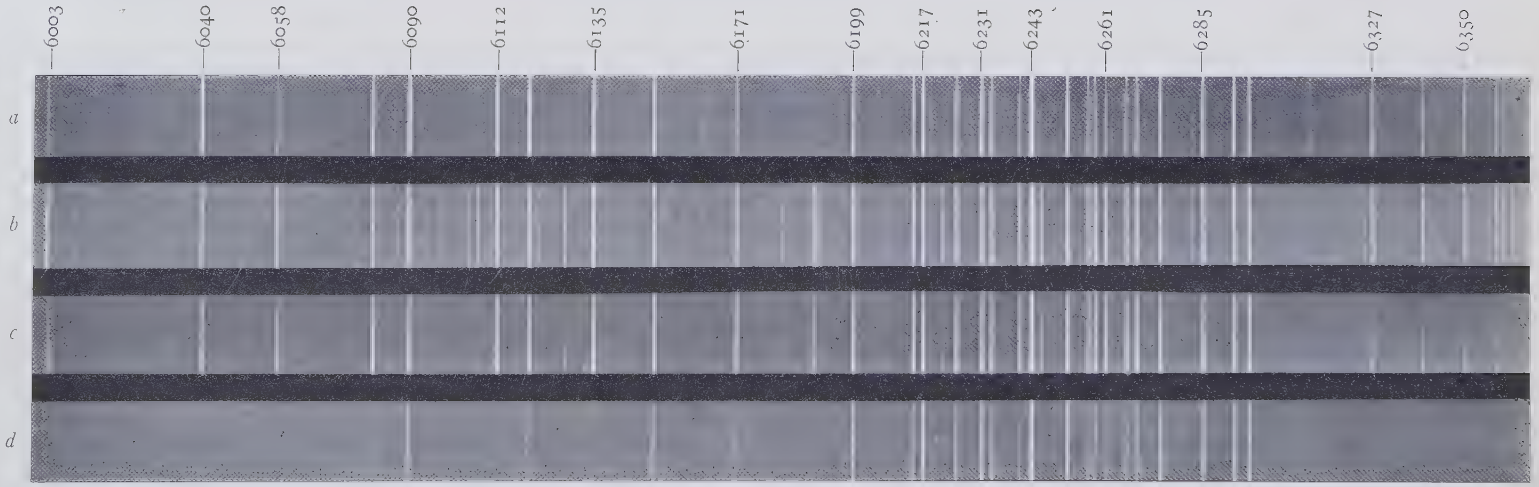
PLATE III



FURNACE AND ARC SPECTRA OF VANADIUM

a. Vanadium in carbon arc.
b. Spectrum of the electric furnace at 2600° C.
c. Spectrum of the electric furnace at 2350° C.
d. Spectrum of the electric furnace at 2100° C.

PLATE IV



Furnace and Arc Spectra of Vanadium

a. Vanadium in carbon arc.
b. Spectrum of the electric furnace at 2600° C.
c. Spectrum of the electric furnace at 2350° C.
d. Spectrum of the electric furnace at 2150° C.

EXPLANATION OF THE PLATES

The two sections of Plate III show the vanadium spectrum from λ 4000 to λ 4950 for the arc and for three furnace temperatures. An interesting region in the red is reproduced in Plate IV. The large changes in relative intensity of many lines are brought out, together with the maintenance of a high intensity at low temperature by the lines of Class I. The development of a typical carbon fluting is well shown in Plate III. The low temperature is barely able to produce the strong head at λ 4737.

Plate V gives a similar series of arc and furnace spectra for chromium from λ 5100 to λ 5425. Most of the prominent lines here belong to Class I and the chief change in them is a widening at high temperature. Strong lines in other classes occur, however, at λλ 5197, 5225, 5255, and the groups at λλ 5276 and 5329. The growth of the strong carbon band λ 5165 is also shown in this plate.

SUMMARY

1. The electric furnace spectra of vanadium and chromium have been studied for the ranges λ 3150–λ 6850 and λ 3550–λ 7000, respectively, the classification of lines being based on a comparison of the spectra given by three furnace temperatures with reference to the temperature at which a line appears and its rate of strengthening with increase of temperature.

2. The leading features in the development of the vanadium and chromium spectra are similar to those observed for iron and titanium, the vanadium spectrum being very similar to the latter.

3. The chromium spectrum near the temperature at which the vapor begins to radiate shows a predominance of the lines of Class I, which change little at higher temperatures.

4. Certain chromium lines, very diffuse in the arc in air, may be resolved into sharp components in a vacuum source, either furnace or arc.

5. A large number of lines belonging to various furnace classes are relatively weak in the arc.

6. The extension of the spectrum into the ultra-violet increases as the temperature rises. In other respects the distribution of

lines at different temperatures shows no definite relation to the wave-length.

7. The ability of lines to show self-reversal in the furnace distinctly increases with decreasing wave-length.

8. The absence from the furnace of the banded spectra which appear in the vanadium and chromium arcs indicates that they are probably due to oxides of the metals.

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THE FLASH SPECTRUM WITHOUT AN ECLIPSE REGION λ 4800–λ 6600

By WALTER S. ADAMS and CORA G. BURWELL

In a brief communication[1] published in 1909 Hale and Adams described the methods employed in photographing the chromospheric spectrum with the 60-foot tower telescope and the powerful spectrograph used in conjunction with it. Some results of the observations in two limited regions of the spectrum were given to indicate the character of the photographs obtained and the degree of accuracy of the measured wave-lengths. Observations made since that time by Mr. Hale with the 150-foot tower telescope have shown that the use of the larger solar image probably will lead to a distinct advance in the quality of the chromospheric photographs. In spite of this consideration, however, it has seemed desirable to publish the results already obtained, not only because they give some indication of the possibilities in the way of investigations of the chromospheric spectrum with the use of a solar image of moderate size, but also because the results almost certainly will differ to some extent from those obtained with the larger image on account of difference in the level of the observations. Since the slit is held rigorously tangent to the solar image during the exposures, it should be possible under the finest conditions of definition to reach a somewhat lower level in the chromosphere with the larger image.

The method employed in obtaining the photographs was that described in the communication by Hale and Adams already referred to. The light from the sun's limb falls upon a diagonal prism so placed as to reflect the light horizontally to a second prism immediately above the slit. The first prism is mounted upon a slide with a screw adjustment allowing of motion toward or away from the second prism. After the sun's image has been brought tangent to the slit the observer selects a bright line of the chromospheric spectrum and brings it into the field of view of an eye-piece, mounted in an opening near the end of the plate-holder. During the exposure this line is maintained at maximum brightness by guiding with the screw controlling the position of the first diagonal prism, thus moving the sun's image slightly on the slit. The objective employed in the 60-foot tower telescope is corrected for visual light and hence is not well adapted for photography in the blue and violet portions of the spectrum. For this reason, and on account of the difficulty of seeing with sufficient distinctness in this region the bright lines necessary for guiding, only a very few photographs have been obtained to the violet of λ 4800.

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[1] *Mt. Wilson Contr.*, No. 41; *Astrophysical Journal*, **30**, 222, 1909.

The second order of the grating was used for all of the photographs, corresponding to a linear scale on the negatives of about 1 mm=0.9 angstrom.

The exposure times varied from four minutes in the yellow and green region to eight minutes in the red.

Table I contains the results of the measures. The portion of the spectrum between λ 4800 and λ 5500 has been studied much more extensively than that to the red of λ 5500, and the wave-lengths given depend upon a larger number of determinations. For lines of greater wave-length than λ 5900 only two measures are available.

The photographs upon which these results are based were taken largely to test the methods employed, and in the case of some of the earlier photographs the diagonal prisms used were too small to give the full length of the brightest and longest chromospheric lines. As a result we have not attempted to measure the lengths of the lines and use them as a basis for discussion of level in the solar atmosphere. It is evident, however, that with a slit of sufficient length to admit the longest lines, measures of the size of the arcs made in this way would have a marked advantage over those from eclipse photographs taken without a slit in that the exposure times would be uniform for lines of all lengths and not progressively longer as in eclipse spectra for the lines of greatest length. The photographs taken without an eclipse have, moreover, the advantage of accurate guiding.

The results given in Table I are for the most part self-explanatory. The measured wave-lengths of the bright lines are in the first column, and following these the lines in Rowland's table with the corresponding elements and intensities. We have preferred not to add a large number of somewhat questionable identifications to those made by Rowland, but have included several taken mainly from Hasselberg's arc tables which sun-spot observations have verified. The chromospheric intensity given in the fifth column is on an arbitrary scale extending from 0, a line just visible on the continuous background, to 50, which is the intensity of D_3 on our photographs.

The abbreviations used in the last column are as follows:

dr = double reversal
w = wide
bf = bright fringes
E = enhanced line

The term "bright fringes" is used in the case of dark lines which show a faint bright line on either side, in general too weak for measurement. They are true double reversals but very faint. These lines, on which no measurements have been made, are indicated in the first column of the table by the absence of figures beyond the decimal point, these being given in the second column (from Rowland's table).

THE LEVEL OF THE OBSERVATIONS

There are three features in these results that indicate a very low level in the solar atmosphere for the point under observation.

1. Essentially all of the stronger lines are double reversals, similar in type to those of the hydrogen lines, but of course very much narrower. This is an indication of low level.

2. A large proportion of the bright lines are very faint absorption lines in the solar spectrum. St. John has shown clearly that in general the fainter the line the lower the level of the gas producing the absorption.

3. The carbon fluting with its head at λ 5165 is very strong. This fluting is known from visual observations to originate at a very low level in the solar atmosphere.

In view of these facts, quite apart from the obvious consideration that a slit tangent to a solar image 16.8 cm in diameter under good conditions of definition should reach a point very close to the

TABLE I

WAVE-LENGTHS AND INTENSITIES FOR CHROMOSPHERE LINES

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
4802.888	.879	....	0000	1	
4804.220	.232	La	000	1	
4804.530	.706	Co?	0	0	dr
.892				0	
4805.301	.285	Ti	3	2	dr E
4805.605	.606	Ti	0	1	
4807.308	......	....	....	0	
4807.726	.725	Cr, V	000	1	
4807.	.900	Fe	1	.....	bf
4809.204	......	La	....	2	
4810.730	.724	Zn	3	1	
4811.532	.542	Nd	000	2	
4813.670	.661	Co	1	2	
4814.147	.166	....	000N	1	
4814.676	......	....	....	0	
4815.262	.239	....	0000	0	
4815.831	.820	....	0000	0	
4816.011	.013	....	0000	1	
4817.856	.988	Ni? Fe?	2	0	dr
4818.033				0	
4819.173	.205	....	0000	0	
4819.818	.830	Y	0000	0	
4820.514	.593	Ti	1	1	
4823.543	.697	Mn	5	1	dr
.856				0	
4824.230	.325	Fe, Cr	3	1	dr E
.484				0	
4825.664	.666	Ti	000	3	
4825.906	.907	....	000	0	
4827.917	......	....	....	0	
4828.729	......	....	....	1	
4831.710	......	....	....	0	
4832.466	.460	Nd	0000	1	
4834.548	.695	Fe	1	0	dr
.834				0	
4835.890	6.059	Fe	2	1	dr
6.217				1	
4836.310	.313	Ti	0000	0	
4836.935	7.044	Cr	00	0	dr
7.180				0	
4838.543	.699	....	2	2	dr
.957	.837	Fe, Ni	1	1	
4839.594	.734	Fe	3	0	dr
.869				0	
4840.197	.193	....	0000	0	
4840.431	.449	Co	2	1	
4840.925	1.074	Ti	3	0	dr
1.250				0	
4843.	.336	Fe	3	.....	bf
4844.401	.408	Mn	0000	1	w
4846.766	......	....	....	0	
4848.121	.110	....	0000	0	
4848.282	.438	Cr	2	0	dr E
.729	.605	Ti	0000	0	
4849.242	.357	....	0	1	dr
.453				0	
4849.664	.845	Cr	00	0	dr
50.073				0	
4851.680	.689	Ca, V	1	1	w
4852.	.743	....	2	.....	bf
4854.584	.535	....	0000	0	
4855.030	.059	Fe, Y	1	4	
4855.	.600	Ni	3	.....	bf
4856.195	.203	Ti	1	0	
4857.	.579	Ni	1	.....	bf
4859.204	.221	Nd	000	1	
4859.803	.928	Fe	4	0	dr
60.050				0	
4861.092	.527	H, La	30	20	dr
.898				15	
4862.621	.783	....	0	0	dr
.923				0	
4864.356	.362	....	0000	0	
4864.484	.505	Cr	1	0	E
4864.906	.919	V	0	0	
4865.793	.798	....	1	0	
4866.331	.465	Ni	2	0	dr
.593				0	
4866.906	.930	....	0000	0	
4867.585	.724	....	00	0	dr
.847				0	
4868.048	.056	Co	1	2	
4868.252	.451	Ti	0	1	dr
.442				(2)	
.709				1	
4870.316	.323	Ti	1	1	
4870.982	.996	Ni, Cr	3	0	
4871.	.512	Fe	5	.....	bf
4872.	.332	Fe	4	.....	bf
4873.639	.630	Ni	2	0	
4874.189	.196	Ti	0	1	E
4874.368	.379	....	0000	0	
4874.689	.693	....	0000	1	w
4875.668	.671	V	1	1	
4876.	.586	Cr	1	.....	bf E
4878.169	.313	Ca	3	1	dr
.548	.407	Fe	4	1	
4881.750	.739	V	1N	1	
4882.	.336	Fe	3	.....	bf
4882.650	.670	Ce	000	1	

TABLE I—*Continued*

OBSERVED λ	ROWLAND			CHR. INT.	REMARKS
	λ	Element	Intensity		
4883.798 .876 .942	.867	Y	2	2 (4) 2	dr
4885.106 .421	.264	Ti	2	0 1	dr
4885.	.620	Fe	3	.....	bf
4886.094	.132	Cr	00	0	
4887.182	.187	Ni, Cr	2	1	
4887.729	.715	....	0000	0	
4889.	.187	Fe?	3	.....	bf
	.294	Fe	2	.....	
4890.	.948	Fe	6	.....	bf
4891.	.683	Fe	8	.....	bf
4892.012	.047	Ti	000	1	
4893.	.030	Fe	1	.....	bf
4893.182	.228	Ti, Sc	000	0	
4893.987	.997	....	00	1	
4894.142	.141	Ce	0000	0	
4896.	.625	Fe	1	.....	bf
4900.102	.095	Ti, La	2	1	
4900.300	.301	Y	2	3	
4900.814	.808	....	000	0	w
4901.056 .270	.152	Ti	0	0 0	dr
4902.248	.257	....	0000	0	
4903.	.502	Fe	5	.....	bf
4903.909	.896	....	0000	0	
4904.403 .761	.597	....	3	0 0	dr
4905.143 .487	.310	Fe?	0	0 0	dr
4908.032 .325	.209	....	0N	0 0	dr
4908.628	.673	Co	000	0	
4909.439 .703	.566	Fe	2	0 0	dr
4910.	.505	Fe	2	.....	bf
4910.965	.952	....	000N	0	
4911.387	.374	Ti	1	1	E
4912.226	.199	Ni	1	0	
4913.424	.450	....	0000	0	
4913.810	.803	Ti	2	1	
4914.175	.150	Ni	2	0	
4914.560	.583	Nd	0000	2	
4917.023	.028	....	0000	0	
4917.	.410	Fe	2	.....	bf
4918.536	.543	Ni, Ti	2	1	
4918.884	.886	Ni	0	1	
4919.019 .324	.174	Fe	6	0 0	dr
4920.044	.047	....	00	2	
4920.220	.241	....	0000	0	
4920.552 .851	.685	Fe	10	0 1	dr
4921.158	.147	La	0	3	
4921.982	.963	La, Ti	1	4	He?
4922.449	.446	Cr	2	0	
4923.544	.540	....	00d	0	
4924.012 .189	.107	Fe	5	5 4	dr E
4924.	.956	Fe	3	.....	bf
4925.573 .876	.746	Ni, V	1	0 1	dr
4926.034	......	....	....	0	
4928.	.050	Fe	2	.....	bf
4928.513	.511	Ti, Co	0	2	
4933.300 .711	.514	Fe	2	0 0	dr
4934.152 .381	.214 .277	Ba-Fe?	3 4	1 2	dr
4935.056	.048	....	0000	0	
4936.012	.015	Ni	2	1	
4936.546	.512	Cr	1	0	
4937.	.902	Ti	000	.....	bf
4938.471	.467	Ti?	000	1	
4938.708	......	....	....	0	
4939.243 .575	.416	Fe	2	0 0	dr
4941.476	.496	Co, Sc	0000	0	
4941.763	.752	Ti	0000	0	
4942.548 .793	.660	Cr	2	0 0	dr
4943.631	.623	Ce	0000	1	
4944.049	.087	Ce	0000	0	
4944.784	.751	Ce	000	1	
4946.	.568	Fe	3	.....	bf
4947.	.778	....	00	.....	bf
4950.150 .412	.291	Fe	2	0 0	dr
4953.251 .536	.392	Ni	2	0 0	dr
4954.878 5.116	.986	Cr	2	0 0	dr
4955.528	......	....	....	0	
4957.	.785	Fe	8	.....	bf
4958.302 .542	.431	Ti	00	1 0	dr
4959.282	.320	Nd	0000	1	
4961.100 .350	.235	....	00	0 0	dr
4961.583	.564	....	0000	0	
4962.486	.467	Sr?	000	0	
4962.	.751	Fe	2	.....	bf

TABLE I—*Continued*

Observed λ	Rowland λ	Rowland Element	Rowland Intensity	Chr. Int.	Remarks
4965.235	.351	Ni	0	0	dr
.474				0	
4968.	.080	Fe	3	.....	bf
4969.966	0.098	Fe	3	0	dr
70.229				0	
4970.564	......	La	....	2	
4971.374	.531	Ni, Ce	1	0	dr
.661				1	
4973.146	.281	Ti-Fe	4	0	dr
.411				0	
4974.638	.642	....	0000	1	w
4975.491	.530	Ti	00	2	
4976.169	.314	Ni	0	0	dr
.633	.508	Ni	1	0	
4978.368	.372	Ti	00	1	
4978.	.785	Fe	3	.....	bf
4979.366	.391	....	000	0	
4980.132	.352	Ni-	4	0	dr
.588				0	
4981.462	.453	....	0000	0	
4981.773	.912	Ti	4	0	dr
2.049				0	
4984.	.297	Ni	2	.....	bf
4985.	.730	Fe	3	.....	bf
4986.290	.403	Fe	1	0	dr
.551				0	
4986.982	......	La	....	1	
4988.291	.313	....	000	1	
4989.010	.130	Fe	2	0	dr
.264	.325	Ti	00	1	
4990.106	.147	Nd	000	1	
4990.544	.625	Fe	0	0	dr
.739				0	
4991.115	.247	Ti	3	1	dr
.348				0	
4991.	.452	Fe, La	2	.....	bf
4993.533	.531	....	0	2	
4994.	.316	Fe	3	.....	bf
4995.220	.208	....	0000Nd?	0	
4997.148	.283	Ti	0	0	dr
.407				0	
4999.554	.689	Ti, La	3	1	dr
.815				0	
5001.166	.165	Ti	0	1	
5001.639	.654	....	000	0	
5002.	.044	Fe	5	.....	bf
5002.491	.510	V	0000	0	
5005.591	.581	....	000	1	
5005.	.896	Fe	4	.....	bf
5006.	.306	Fe	5	.....	bf
5007.285	.398	Ti	3	1	
.646	.461	Fe	2	1	
5009.633	.829	Ti, Co	00	1	dr
10.001				1	
5011.927	......	Ce	....	1	
5012.108	.252	Fe	4	1	dr
.434	.335	....	1	1	
5012.	.625	Ni	1	.....	bf
5013.890	.871	Ti	0	1	
5013.967	.953	Ce	0000	0	
5014.250	.369	Ti	2	1	dr
.585	.457	Fe	3	0	
5016.216	.340	Ti	2	0	dr
.469				0	
5017.	.762	Ni	3	.....	bf
5018.519	.629	Fe	4	5	dr E
.727				4	
5019.740	......	....	....	0	
5020.050	.208	Ti	2	1	dr
.347				1	
5022.222	.414	Fe	3	0	dr
.574				0	
5023.	.052	Ti	2	.....	bf
5024.242	......	....	....	0	w
5024.900	5.027	Ti	3	0	dr
5.195				0	
5025.725	.749	Ti	1	0	
5025.950	.938	....	000	0	w
5027.	.305	Fe	3	.....	bf
5029.	.805	Fe	1	.....	bf
5030.103	.096	....	0000	1	w
5031.084	.058	Sc	0000	1	
5031.172	.199	....	3	3	
5032.244	.252	C	0000	1	w
5033.700	.714	....	000	1	w
5034.354	.354	C	000	0	
5034.568	.530	....	0000	0	
5034.982	......	....	....	0	
5035.	.542	Ni	5	.....	bf
5036.306	.332	....	0000	0	
5037.968	......	Ce	....	0	
5038.292	.579	Ti	2	1	dr
.890				0	
5039.	.428	Fe	3	.....	bf
5040.020	.138	Ti	3	1	dr
.275				1	
5040.602	.644	C	0000	1	w
5040.926	1.069	Fe	3d?	1	dr
1.380	.255	Fe	4	0	
5041.066	.795	Ca	2	1	dr
2.125	.936	Fe	4	2	
5042.	.367	Ni	1	.....	bf
5043.	.761	Ti	00	.....	bf

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5044.232 .533	.394	Ni, Co-Fe	3	1 0	dr
5045.451	.454	....	00	0	
5047.951	......	He?	....	0	
5048.464 .770	.612	Fe	3	0 0	dr
5050.	.008	Fe	6	.....	bf
5050.905	.919	C	000	0	
5051.558 .960	.683 .825	Ni Fe	1 4	1 1	dr
5052.352	.338	....	0N	0	w
5052.795	.803	....	00	0	
5053.038	.056	Ti	0	1	
5053.771	.756	....	00N	0	
5054.253	.261	Ti	000	0	
5056.290	.307	C	000	0	
5057.866	.875	....	0000	0	
5058.509 .809	.674	Fe	00	0 0	dr
5059.407	.409	....	000	0	
5060.096 .413	.258	Fe	3	1 1	dr
5061.899	.882	C	00	0	
5062.297	.285	Ti	0	2	
5063.353	.355	C	00	1	
5063.468	.479	C	000	1	
5063.909	.927	....	0000	0	
5064.244	.244	Ti	00	0	
5064.671 .974	.836	Ti	3	0 1	dr
5065.	.207 .380	Fe Fe	3 2	....	bf
5066.972	6.908 7.039	....	00 000	1	w
5068.573	......	....	....	0	
5068.800 9.099	.944	Fe	5	0 0	dr
5069.542	.592	Ti	000d	0	
5070.164	.165	....	00	0	
5071.954	.969	....	000N	1	
5072.029 .551	.257	Fe, C	3	1 1	dr
5072.	.849	Fe	2	.....	bf
5073.238	......	C	....	0	
5073.630	.637	Ti	00	1	w
5074.714 5.096	.932	Fe	5	0 0	dr
5075.509	.480	Ce	00	2	
5076.194 .731	.450	Fe, Ce	3	0 2	dr *
5077.690	......	Co	....	0	
5078.544	.541	C	000	0	
5078.883	.891	....	0000	1	
5079.003 .528	.158 .409	Fe Fe	3 4	0 1	dr
5079.	.921	Fe	4	.....	bf
5080.275	.288	C	0000	2	
5080.599 .843	.714	Ni	4	0 1	dr
5081.736	.764	Sc	000	2	
5081.943	.942	C	000	0	
5083.180	.205	C	000Nd?	1	
5083.383 .635	.518	Fe	4	0 0	dr
5083.896	.877	Sc	000	1	
5084.	.279	Ni	3	.....	bf
5084.849	.876	....	000	1	
5085.691	.668	Sc	0	0	
5086.394	.422	....	00	1	
5086.602	.570	C	000	1	
5087.096	.104	Sc	0000	0	
5087.592	.601	Y	1	4	
5088.202	.175	....	000	1	
5089.390	.387	....	000Nd?	1	w
5089.997	0.004	Sc, Nd	0000	0	
5090.810 1.100	.954	Fe	5	0 0	dr
5092.503	.483	C	00	1	
5092.984	.977	Nd	000	2	
5093.868	.858	C	0000N	0	
5095.332	.348	....	00d?	1	
5096.665	.660	....	0000	0	
5096.930	.908	C, Sc	000	1	
5098.309	.302	C	00N	1	
5098.480	.492	C	000N	0	
5098.	.885	Fe	3	.....	bf
5099.947	.957	....	000	1	
5100.821	.827	....	00	0	
5101.245	.251	C, Sc	000	0	
5101.652	.655	....	000	0	
5102.580	.599	....	000	0	w
5103.319	.297	C	0000	0	
5103.894	.909	C	000	1	
5104.089	.083	C	0000	0	
5105.371	.356	....	0000	0	
5105.571 .825	.718	Fe	4	0 0	dr
5106.525	.556	C	0000	0	
5106.776	.773	C	000	1	
5107.495 .905	.619 .823	Fe Fe	4 4	0 0	dr
5108.066	.056	C	000	1	

* Solar line double.

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5109.277	.291	C	000d?	1	
5109.	.827	Fe	2	.....	bf
5110.182	.188	C	0000	0	
5110.	.574	Fe	5d	.....	bf
5110.921	.938	Cr-C	00	0	
5111.429	.426	....	0000	0	
5111.806	.802	C	000	1	
5112.440	.458	....	000	4	*
5113.244	.240	Cr-C	0	0	
5113.596	.617	Ti	0	0	
5114.402	.431	C	000Nd	1	
5114.732	......	La	....	2	
5115.251 .754	.566	Ni	2	0 0	dr
5116.014	.045	C	0000	0	
5116.837	.849	C, Sc	0000	1	
5117.050	.071	C	000	0	
5117.331	.334	Ce	0000	0	
5118.278	.241	C	0000	1	
5119.296	.292	C, Y	00	1	
5119.541	.555	C	000	0	
5119.815	.820	C	0000	0	
5120.556	.516 .592	C Ti	000 0	 2	
5120.833	.802	C	000	0	
5121.586	.609	....	0000	0	
5122.459	.481	C	0000	0	
5122.596	.613	C	0000	0	
5122.962	.968	Co-C	000	0	
5123.163	.178	La	000	2	
5123.364	.390	Y	0	2	
5123.751 4.002	.899	Fe	3	0 0	dr
5124.209	.219	C	000	0	
5124.901	.939	C	0000	0	
5125.876	......	Co	....	0	
5126.212 .520	.371	Fe-Co	2	0 0	dr
5127.382 .698	.533	Fe, Ti	3	0 0	dr
5129.165 .701	.336 .546	Ti? Ni	3 2	0 1	dr E
5130.763	.757	C, Nd	000	5	
5131.779	.771	C	0000	2	
5132.512	.523	C, Nd	000	1	
5132.857	.843	C	00	2	
5133.655 4.008	.870	Fe	4	1 1	dr
5134.506	.505	C	0000	0	
5134.862	.849	C	0000	0	
5135.347	.355	Y	0000	0	
5135.835	.752 .880	C,– C,–	000 000	1	w
5136.411	.443	....	000	0	
5136.614	.625	C	0000	1	
5136.851	.835	C	0000	0	
5136.985	.969	....	000	1	
5137.	.250	Ni, Cr	3	.....	bf E
5137.756	.753	C,–	000	1	
5138.260	.279	C	0000	0	
5138.496	.518	V	0000	0	
5138.645	.690	C	0000	0	
5139.	.427	Fe	4	.....	bf
5139.539 .775	.644	Fe	4	0 1	dr
5140.092	.094	C	0000	0	
5140.566	.553	Ce	0000	0	
5141.389	.386	C,–	000	2	
5141.775 2.052	.918	Fe	3	1 1	dr
5142.288	.279	....	0000	0	
5142.	.693	Fe	4d?	.....	bf
5142.966	.958	Ni	2	1	
5143.	.111	Fe	3	.....	bf
5143.499	.511	C	000	0	
5143.748	.764	C	000	0	
5144.214	.203	....	0000	0	
5144.742	.758	C,–	000	0	
5144.852	.847	Cr, C	00	0	
5145.094	.098	C	0000	1	
5145.386	.403	C	0000	0	
5145.652	.636	Ti-C	0	0	
5146.304	.291	C,–	00	4	
5146.656	.659	Ni–	3	0	
5146.941	.945	Co-C	000d?	1	
5147.841	.871	C,–	000	1	
5147.983	.992	C,–	000	1	
5148.	.222 .410	Fe Fe	2 3	....	bf
5149.209	......	C-Co	....	1	
5150.340	.363	....	00	0	
5150.844	.842	C	0000	2	w
5151.	.020	Fe	4	.....	bf
5151.657	.628	C	0000N	0	
5151.953 2.229	2.087	Fe	3	0 0	dr
5153.335	.337	C	0000	1	
5153.442	.414	Fe-C	1	0	
5154.171 .346	.244	Ti-Co	2	0 0	dr E
5154.552	.579	C	000	2	w
5155.050	.028	C	0000	0	

* Blend.

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5155.701	.694	C,–	000	1	
5155.945	.935	Ni	2	0	
5156.518	.530	Co	000	0	
5156.728	.728	C	0000	1	
5156.927	......	La	....	1	
5157.779	.783	C	000	1	
5157.944	.915	C	000	1	
5158.	.152	Ni	00	.....	bf
5158.702	.701	C	000	2	
5159.072	.026	Co-C	0000N	1	
5159.627	.634	C	000	1	
5159.937	.946	....	0000	1	
5160.730	.419	C,–	00N	1	
5160.569	.554	C	0000	0	
5161.199	.194	C	000	1	
5161.362	.353	C	000	1	
5161.907	.910	C	000	1	
5162.161	.153	C?	0000	0	
5162.	.449	Fe, C	5	.....	bf
5162.716	.690	C	0000	1	
5163.064	.074	Ti, C	000	0	
5163.573	.585	C,–	000	0	
5163.769	.756	C, La	000	0	
5164.004	.007	C	0000	0	
5164.163	.172	C	0000	0	
5164.395	.404	C,–	000	1	
5164.564	.562	....	0000	0	
5164.942	.950	C	0000	0	
5165.201	.209	C	0000	2	
5165.290	.297	C	0000	1	
5165.436	.416	C	0000	0	
5166.320 .578	.454	Cr-Fe	3	0 0	dr
5167.364	.497	Mg	15	2	dr
.780	.678	Fe	5	1	
5169.	.069	Fe	3	.....	bf
5169.149 .328	.220	Fe	4	5 8	dr E
5170.970	.937	Fe	0	0	
5171.659 .925	.778	Fe	6	0 0	dr
5172.706 3.004	.856	Mg	20	5 4	dr
5173.782 4.056	.917	Ti	2	0 2	dr
5175.528 .659	.575	....	000	0 0	dr
5176.330	.305	Co	000	0	
5176.475 .939	.735	Ni	1	0 0	dr
5178.421	......	....	....	0	
5178.682	.644	V	000	0	
5179.948	.958	Nd	000	2	
5180.066 .388	.233	Fe	1	0 0	dr
5180.746	.747	....	000	0	
5181.351	.334	....	000	0	
5182.754	.761	....	0000	0	
5183.604 .935	.791	Mg	30	6 5	dr
5184.356	.445	Fe	2	0	dr
.718	.738	Fe, Ni	1	1	
5185.962 6.218	6.073	Ti	2	0 0	dr E
5187.624	.620	Ce	000	2	
5188.753 .903	.863	Ti	2	2 1	dr E
5189.726	.744	....	000	0	
5191.537 .761	.629	Fe, Ce	4	1 2	dr
5192.160	.155	Cr	00	1	
5192.784	.785	Nd	000	2	
5193.025 .255	.139	Ti	2	0 0	dr
5193.657	.669	Cr	000	0	
5194.018	.027	....	0000	0	
5196.120 .340	.227	Fe	1	1 0	dr
5196.630	.613	Cr	0	0	
5197.735	.743	....	2	9	
5198.769 9.002	.888	Fe	3	0 0	dr
5200.320	.355	Cr	00	0	
5200.577	.590	V, Y	0	3	
5201.259	.260	Ti	000	1	
5202.314	.439	Fe?	2	1	dr
.626	.516	Fe	4	1	
5203.126	.118	....	0000	0	
5204.564	.680	Cr	5	1	dr
.875	.768	Fe	3	1	
5205.887	.897	Y	0	5	bf
5206.	.215	Cr-Ti	5	0	
5206.739	.712	V	000	0	
5207.650 .985	.791	....	000N	0 0	dr
5208.028 .218	.111	Ti	00	1 0	dr
5208.483	.596	Cr	5	1	dr
.933	.776	Fe	2	0	
5210.196	.204	Co	0000	1	
5210.433 .687	.555	Ti	3	0 0	dr
5211.006	.015	....	000	0	
5211.699	.700	Fe	00	0	

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5212.497	.503	Ti, Nd	0000	2	
5212.854	.859	Co	000Nd?	2	
5213.156	.155	....	0000	0	
5214.292	.286	Cr	00	1	
5215.	.353	Fe	3	.....	bf
5216.	.437	Fe	3	.....	bf
5217.	.552	Fe	3	.....	bf
5218.328	.369	Fe	1	0	
5219.201	.186	....	0000	1	
5220.289	.250 .358	.... Ni	000 0	2	
5221.058	.078	....	000	0	
5221.	.928	Cr	0	.....	bf
5222.	.556	Cr	00	.....	bf
5222.846	.849	Ti, Cr	00	1	
5223.789	.791	Ti	000	1	
5224.246	.239	Cr	000N	0	
5224.693	.712	Ti, Cr	00	1	
5225.154	.101 .198	Cr Cr, Ti,	0 00	1	
5225.546 .833	.695	Fe	2	0 0	dr
5226.610 .785	.707	Ti	2	4 3	dr E
5227.	.362	Fe	5d?	.....	bf
5228.258	.268	Cr	000	0	
5228.405 .702	.546	....	1	0 0	dr
5230.	.030	Fe	4	.....	bf
5230.195 .554	.382	Co, Cr	00	0 0	dr
5231.144	.151	....	0000	0	
5232.676	.681	....	000	0	
5233.	.122	Fe	7	.....	bf
5234.216	.255	....	0000	0	
5234.374	.380	V, Nd	000	1	
5234.788	.791	....	2	8	dr?
5235.459	.557	Fe	1	0	dr
.838	.672	Ni	00	0	
5237.240	.254	Ce	0000	1	
5237.488	.493	Cr	1	2	E
5238.644 .892	.742	Ti	000N	0 0	dr
5239.975	.992	Sc	1	3	
5240.637	.639	....	000	0	
5241.028	.040	V	000	1	
5241.944	......	Ce	....	0	
5242.517 .804	.658	Fe	2	0 0	dr
5243.514	.526	Cr	00	0	
5243.	.946	Fe	1	.....	bf
5245.095	.118	....	0000	0	
5246.320	.310	....	0000	0	
5246.972	.946	....	000	1	
5247.461	.466	Ti	000	1	
5247.	.737	Cr	2	.....	bf
5249.747	.751	Nd	0000	4	
5250.178	.193	Co	0000	1	
5250.241 .508	.385	Fe	2	0 0	dr
5250.667 .968	.817	Fe	3	0 1	dr
5252.832	......	Ce	....	0	
5253.	.633	Fe	2	.....	bf
5254.822	.830	Co	0000	1	
5255.028	.121	Fe	3	0	dr
.280	.295	Cr	0	1	
5255.674	.687	Nd	0000	4	
5255.996	.973	Ti	0000	0	
5257.104	.100	Sr	00	2	
5257.760	.814	Co	0	1	
5259.900	.912	Ce	0000	2	
5260.113	.142	Ti	000	0	
5260.438 .673	.561	Ca	0	0 0	dr
5261.750 .979	.876	Ca-Cr	3	0 0	dr
5262.293	.321	Ti	1	1	
5262.	.419	Ca	3	.....	bf
5263.	.486	Fe	4	.....	bf
5264.055	.038	Fe	0	0	
5264.195	.329	Cr	4	0	dr
.527	.415	Ca	3	0	
5264.974	.976	....	0	4	
5265.321	.321	Cr	00	0	
5265.570	.729	Ca	3	0	dr
6.011	.893	Ni-Cr	2	0	
5266.459	.482	Co	000	1	
5266.622 .844	.738	Fe	6	0 0	dr
5267.284 .568	.447	....	00	0 0	dr
5268.515	.515	Ni	0	0	
5268.665	.670	....	000	0	
5268.713 .834	.784	Co	00	1 1	dr
5269.115	.125	....	000	0	
5269.602 .838	.723	Fe	8d?	2 1	dr
5270.293	.438	Ca	3	0	dr
.664	.558	Fe	4	0	
5271.206	.228	....	00	0	
5272.198	.171	Cr	00	0	

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5273.200	.339	Fe	3	0	dr
.638	.558	Fe-Cr	2	1	
5274.407	.408	Ce	00	3	
5275.141	.148	....	0	0	
5275.340	.340	Cr	00	1	
5275.	.454	....	1	.....	bf
5276.149	.169	Fe?	3	10	
5276.384	.344	Co	000	0	
5277.040	.047	Nd	000	1	
5280.054	.048	....	0	0	
5280.256	.239	....	00	0	
5280.803	.799	Co	00	2	
5281.647	.681	....	000N	0	
5281.858	.971	Fe	5	0	dr
2.121				0	
5283.649	.802	Fe	6	0	dr
.929				0	
5284.281	.281	....	1	6	
5287.349	.351	Cr	000	0	
5287.713	.741	Co	0000N	0	
5287.958	.958	Co	000	0	
5288.554	.705	Fe	2	0	dr
.805				0	
5289.976	.988	Y	000	0	
5291.006	0.984	La	000	0	
5292.365	.388	....	000N	0	
5292.803	.762	Fe	0	0	
5293.071	.044	....	000	0	
5293.334	.341	Nd	00	3	
5293.550	.543	....	000	0	
5296.	.872	Cr	3	.....	bf
5297.399	.407	Cr, Ti	000	0	
5298.334	.455	Cr	4	0	dr
.574				0	
5300.150	.152	Ti	00	0	
5300.784	.929	Cr	2	0	dr
1.062				1	
5302.145	......	Sc, La	....	2	w
5302.	.480	Fe	5	.....	bf
5302.818	.829	....	000	0	
5303.364	.401	Ce	0	0	
5303.718	.738	La	000	2	
5306.035	.040	....	0	1	
5306.640	.665	....	000	1	
5307.405	.541	Fe	3	0	dr
.648				0	
5308.594	.599	Ce	0	1	
5310.392	.417	Co	000	0	
5310.882	.866	....	0	0	
5311.634	.650	Nd	000	1	
5311.953	.962	....	000	0	
5312.838	.835	Co	00	1	
5313.042	.031	Cr	0	0	
5313.768	.758	....	1	1	
5316.735	.790	Fe	4	8	dr E
.842				8	
5316.965	.958	Co–	2	6	
5318.537	.534	Sc	00N	1	
5318.975	.955	Cr, Fe	0	0	
5319.276	.392	....	000	0	dr
.660	.502	....	000	0	
5320.005	9.999	Nd	00	3	
5320.	.220	Fe	0	.....	bf
5321.	.293	Fe	2	.....	bf
5322.964	.994	....	000	2	
5325.468	.460	Co	00	1	
5325.745	.738	....	2	3	
5326.144	.139	Co	000N	0	
5328.123	.236	Fe	8d?	1	dr
.367				1	
5328.	.722	Fe	4	.....	bf
5330.744	.748	Ce	000	2	
5332.727	.849	Co, Fe	1	0	dr
3.222	.089	Fe	4	0	
5333.849	.832	Co	000	1	
5334.391	.403	Sc	000	0	
5335.033	.050	Co	1	1	
5336.361	.356	Co	000	0	
5336.698	.660	Nd	0000	0	
5336.922	.974	Ti,–	4	3	w E
5337.517	......	Co	....	0	
5337.897	.910	Ce	0	2	
5339.708	.719	Co	000	0	
5340.	.121	Fe	6	.....	bf
5340.651	.639	Cr	0	0	
5341.	.213	Fe	7	.....	bf
5341.490	.514	Co	00	1	
5341.	.670	Ti	000	.....	bf
5342.892	.890	Co	1	3	
5343.552	.570	Co	0	1	
5343.	.622	Fe	2	.....	bf
5344.770	.767	Co	000N	0	
5345.	.991	Cr	5	.....	bf
5347.687	.712	Co	00	0	
5348.	.511	Cr	4	.....	bf
5349.286	.283	Co	000	0	
5349.506	.653	Ca	4	0	dr
.792				0	
5350.052	.059	Mn	00	0	
5350.283	.281	....	00	1	
5350.537	.547	....	00	2	
5351.253	.261	Ti	00	0	
5352.237	.234	Co	1	2	
5352.625	.591	....	000	0	

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5353.703	.702	Co	0	6	
5356.291	.270	Sc	000	0	
5357.160	.178	Nd	000	1	
5358.167	.306	....	00	0	dr
.460				0	
5359.098	.127	Co	000N	0	
5359.386	.389	Co	00	1	
5359.695	.714	....	000	0	
5361.666	.697	Nd	000	4	
5363.052	.058	Co–	3	8	
5366.947	.950	Co	000	0	
5368.729	.741	....	000	0	
5369.100	.125	Co	0000	0	
5370.540	.522	Cr	00N	0	
5371.534	.656	Cr?	4	2	dr
.794	.734	Fe	3	1	
5372.120	.121	Nd	000	3	
5373.800	.905	Fe, Cr	2	0	dr
4.033				0	
5375.558	.516	Sc	000	0	
5377.	.028	Fe	0	.....	bf
5377.798	.800	Mn	2N	1	E
5379.652	.775	Fe	3	0	dr
.883				0	
5380.	.516	....	0N	.....	bf
5381.152	.221	Ti, La	2	2	E
5381.468	.514	Co	0000	1	
5381.955	.980	Co	000	1	
5383.201	.209	....	000	0	
5386.092	.089	Nd	000	1	
5386.977	.989	Ce	0000	1	
5387.778	.769	Cr	00	0	
5390.194	.203	Ti	000	0	
5390.600	.573	Cr	000	0	w
5391.535	.660	Fe	2	1	dr
.921	.820	Fe	1	1	
5393.573	.584	Ce	000	3	
5394.	.876	Mn	2	.....	bf
5395.317	.422	....	0	0	dr
.555				0	
5396.453	.448	....	00	0	
5397.229	.344	Fe	7d?	1	dr
.459				1	
5399.694	.675	Mn	1Nd?	0	
5400.802	.831	Cr	0N	0	
5402.158	.151	V	0000	1	
5402.686	......	....	....	0	
5402.989	.982	Y	0	2	
5405.190	.192	Cr	00	0	
5405.	.554	....	1	.....	bf
5405.874	.989	Fe	6	0	dr
6.102				0	
5406.391	.388	....	0000N	0	
5407.413	.587	Mn	0	1	dr
.824	.688		0	1	
5409.253	.339	Fe, Ce	2	0	dr?
.444				2	
5409.833	.823	Ti	000	1	
5410.	.000	Cr	4	.....	bf
5411.448	.428	Ni	1	1	w
5411.774	.764	....	0000	0	
5412.868	.997	Mn	00	0	dr
3.152				0	
5413.889	.889	Mn	00N	0	w
5414.295	.279	Ce	00	3	
5415.	.416	Fe-V	5	.....	bf
5416.618	.587	Nd	0000	1	
5417.135	.247	Fe	0	0	dr
.362				0	
5418.211	......	....	....	0	
5418.972	.979	Ti	1	2	w E
5420.369	.510	Mn	0N	1	dr
.756	.613		0N	1	
5421.176	.134	....	00	1	
5421.780	.783	Nd	0000N	1	w
5424.764	.860	Ni	1	0	dr
.982				0	
5425.473	.464	....	1	4	
5428.704	......	....	....	0	
5429.362	.349	Ti	00	2	
5429.	.717	....	1	.....	bf
5429.802	.911	Fe	6d?	1	dr
30.012				1	
5430.	.062	....	0	.....	bf
5431.720	.747	Nd	000	1	w
5432.608	.753	Mn	1Nd?	1	dr
.924				1	
5433.195	.160	Fe	2	1	
5434.637	.740	Fe	5	0	dr
.855				0	
5444.796	.796	Co	00	3	
5446.425	.436	Sc	0000	0	
5446.643	.797	Ti	2	0	dr
.996				1	
7.245	7.130	Fe	6d?	0	
5447.726	.737	....	000	0	
5448.411	.582	....	00	0	dr
.609				0	
5449.427	......	Ce, Nd	....	0	
5451.317	.330	Ce, Nd	000	1	
5452.	.309	....	00	.....	bf
5454.313	.326	....	000	0	
5454.776	.783	Co	00	2	
5455.	.834	Fe	4	.....	bf

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5457.	.670	Mn	00	.....	bf
5460.558 .897	.721	Ti	00	0 0	dr
5462.	.705	Ni	1	.....	bf
5464.390 .602	.490	Fe	0	0 0	dr
5466.472 .779	.609	Fe	3	0 0	dr
5468.578	.601	Ce	000Nd?	2	
5470.634 1.018	.802 .883	Mn	0 0	1 1	dr
5472.499	.503	Ce	000	2	
5473.581	.594	Ce	000	1	
5473.955 4.278	4.113	Fe	3	0 0	dr
5474.278 .628	.436	Ti?	00	0 0	dr
5476.981 7.248	7.123	Ni	5	0 0	dr
5477.929	.901	Ti	00	0	
5478.558	.578	....	00	0	
5480.916 1.235	1.071	Fe	1	0 0	dr
5481.	.652	Fe, Ti	1	.....	bf
5481.925 2.186	2.078	Ti, Sc	00	0 0	dr
5483.	.307	Fe	1	.....	bf
5484.826	.846	Sc	000	0	
5485.229	.266	Nd	0000	0	
5485.901	.915	Nd	0000	2	
5487.	.354	Fe	1	.....	bf
5487.799 8.107	.959	Fe	3	0 1	dr
5488.396	.374	V-Ti	00Nd?	0	
5489.879	.893	Co	000	2	w
5490.	.367	Ti	0	.....	bf
5490.892	.905	....	0	2	
5491.890	.897	....	0000	0	w
5493.092	.095	....	00	1	
5493.584 .814	.709	Fe	1	0 0	dr
5494.223	.193	Nd	0000	0	
5497.607 .842	.735	Fe	5	1 0	dr
5498.107	.105	....	000	0	
5498.931	.955	....	0000	0	
5501.546 .807	.683	Fe	5	0 0	dr
5503.136 .433	.286	Fe	1	0 1	dr
5504.111	.117	Ti, La	0	2	
5505.950 6.221	6.095	Mn	1	0 0	dr
5506.697	.719	....	000	0	
5507.	.000	Fe	5	.....	bf
5507.974	.985	V	000	0	
5508.852	.840	....	0	0	
5510.096	.120	Y	0	3	
5510.832	.829	....	00	0	w
5510.946	.942	....	00	0	
5511.653	.644	....	000	0	
5512.280	.266	Ce	00	3	
5513.005 .360	.198	Ca	4	0 0	dr
5513.780	.769	....	0000	0	
5514.427 .890	.563 .753	Ti Ti	2 2	0 0	dr
5516.803 7.137	.950 .034	Mn Mn	0 0	0 0	dr
5519.605	.633	....	000	0	
5520.712	.727	Sc	00N	0	
5521.770	.799	Y	00	1	
5522.483 .828	.665	Fe	2	0 0	dr
5523.810	.783	....	000N	0	
5524.686	.684	....	000	0	
5525.332	.347	Co	00	1	
5525.	.765	Fe	2	.....	bf
5526.427	.405	....	000N	0	
5526.976 7.108	7.033	Sc	3	2 2	dr
5527.776	.796	Y	000d	0	
5528.092	.084	....	0000N	0	
5528.	.641	Mg	8	.....	bf
5530.892 1.109	.997	Ti	00N	0 0	dr
5532.336	.353	....	000	0	
5532.794 3.239	.968 .092	....	1 0	0 0	dr
5534.061	.011	Nd	000d	0	
5534.527	.504	....	000	0	
5535.058	.061	....	2	7	
5535.424 .830	.644	Fe	2	0 0	dr
5536.304	.300	....	000	0	
5536.518	.492	....	000	0	
5537.824 8.098	.928 .025	Mn Mn	00 00	0 0	dr
5540.210	.192	....	0000	0	
5543.980 4.324	4.157	Fe	2	0 0	dr
5544.834	.831	Y	000	2	

TABLE I—*Continued*

Observed λ	Rowland λ	Rowland Element	Rowland Intensity	Chr. Int.	Remarks
5546.164	.147	....	000	1	w
5546.	.732	Fe	2	.....	bf
5547.108	.215	Fe, V	1	0	dr
.323				0	
5549.990	......	....	....	0	
5553.308	.346	....	000N	0	
5557.137	.196	Ce	000	0	dr
.407	.287		000	0	
5558.137	.136	....	0	0	
5559.080	.068	V	000Nd?	0	
5560.278	.434	Fe	2	0	dr
.563				0	
5560.896	.911	....	0000	0	
5562.773	.933	Fe	2	0	dr
3.094				0	
5563.674	.824	Fe	3	0	dr
.984				0	
5565.709	.700	Ti	00	1	
5565.	.931	Fe	3	.....	bf
5567.337	.367	....	0000N	0	
5567.488	.621	Fe	2	0	dr
.732				0	
5569.249	.249	....	0000	0	
5569.662	.848	Fe	6	0	dr
.968				0	
5576.	.320	Fe	4	.....	bf
5578.746	.946	Ni	1	0	dr
9.106				0	
5580.663	.672	....	000	0	
5882.063	.198	Ca	4	1	dr
.324				1	
5583.174	.186	....	000	0	
5584.700	.729	V	000	0	
5584.	.988	Fe	0	.....	bf
5585.396	.397	....	000	0	
5586.886	.991	Fe	7	1	dr
7.094				0	
5587.727	.800	Fe	0	0	dr
889				0	
5588.836	.985	Ca	6	1	dr
9.122				1	
5589.431	.582	Ni	0	0	dr
.768				0	
5590.222	.343	Ca	3	1	dr
.465				1	
5591.024	.039	Ti	000	0	
5591.574	.586	Sc	000	0	
5592.378	.375	Ni	0	1	
5592.	.487	Fe, Ni	1	.....	bf
5593.858	.961	Ni	0	0	dr
4.078				0	
5594.	.691	Ca	4	.....	bf
5595.914	.906	....	0000	0	
5598.358	.524	Fe	1	0	dr
.845	.711	Ca	4	1	
5599.533	......	....	....	0	
5601.348	.505	Ca	3	0	dr
.628				0	
5602.923	.995	Fe	1	1	dr
	3.083	Ca	3		
3.305	.186	Fe	4	1	
5603.867	.993	....	00	0	dr
4.131				0	
5606.108	.122	....	000	0	
5607.	.887	....	00	.....	bf
5608.386	.393	....	000	0	
5610.461	.467	Ce	000	1	
5612.556	.573	....	000	0	
5613.904	.929	Ce	000N	0	
5614.992	.997	Ni, Ce	0	0	
5618.144	.130	....	000	0	
5618.712	.858	Fe	1	0	dr
9.016				0	
5619.669	.824	....	0	0	dr
.931				0	
5620.506	.715	Fe	0	0	dr
.850				0	
5621.422	.438	....	000N	0	
5622.970	.996	....	000	0	
5624.116	.245	Fe	1	0	dr
.373				0	
5625.102	.096	V	000	0	
5625.745	.755	....	000	0	
5627.006	.034	....	000	0	
5627.718	.723	....	000	1	
5628.190	.239	....	000N	0	
5633.351	......	Ce	....	0	
5633.994	4.171	Fe	3	0	dr
4.330				0	
5636.	.045	Fe	1	.....	bf
5638.365	.488	Fe	3	0	dr
.646				0	
5641.	.206	....	1	.....	bf
5641.551	.667	Fe	2	0	dr
.791				0	
5642.	.112	Ni	0	.....	bf
5644.348	.365	Ti	0	2	
5645.	.830	Si	1	.....	bf
5646.810	.904	....	00	0	dr
.990				0	
5648.792	.796	Ti	00	1	
5649.652	.611	Cr	00N	0	w
5649.	.898	Fe-Ni	0d	.....	bf

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
5651.532	.691	Fe	0	0	dr
.817				0	
5652.440	.542	Fe	1	0	dr
.640				0	
5653.934	4.091	Fe	1	0	dr
4.268				0	
5655.562	.715	Fe	2	0	dr
.884				0	
5658.106	.097	Y, Sc	2	1	w
5658.578	.561	Sc	0	3	
5658.643	.753	Fe	1	0	dr
	.884	Cr	0		
9.170	.052	Fe	4	0	
5661.402	.418	....	0000	0	
5662.344	.374	Ti	0	1	
5663.134	.155	Ti, Fe, Y	1	3	
5664.	.218	Ni-Cr	1N	.....	bf
5664.770	.797	....	000	0	
5665.632	.775	Si	1N	1	dr
.884				0	
5667.364	.368	Sc	0	2	
5667.634	.739	Fe	2	0	dr
.855				0	
5668.586	.593	V	000	0	
5669.136	.130	Ce, Nd	0000	1	
5670.154	.163	Ni, Ce	0	0	
5671.066	.071	V	0	1	
5671.715	.712	....	00N	0	
5672.039	.047	Sc	0	1	
5677.732	.919	Ce	00	0	dr
8.095				0	
5679.	.249	Fe	3	.....	bf
5682.695	.869	Na	5	1	dr
3.026				1	
5684.415	.415	Sc	1	2	
5687.056	.063	Sc	000	1	
5687.	.697	....	0	.....	bf
5688.267	.436	Na	6	1	dr
.619				1	
5688.736	.759	Nd	0000	3	
5689.694	.694	Ti	0	1	
5690.251	.286	....	0000	0	
5690.484	.646	Si	3	0	dr
.770				0	
5693.669	.865	Fe	3	0	dr
4.056				0	
5694.959	.962	Cr	0	1	
5695.180	.207	Ni	2	0	
5696.905	.869	....	0000N	0	
5698.051	.047	....	0000Nd?	0	
5698.	.242	Fe	0	.....	bf
5698.	.555	Fe, Cr	1	.....	bf
5698.754	.746	V	1	2	
5700.356	.402	Sc	00	0	
5701.	.323	Si	1N	.....	bf
5702.455	.543	Cr	0	0	dr
.676				0	
5702.857	.876	Ti	000	1	
5703.580	.591	....	0000	0	
5703.785	.797	V	1	1	
5705.969	6.215	Fe, Nd	3	0	dr
6.424				1	
5707.202	.204	V	0	0	
5708.470	.622	Si, Nd	3N	1	dr
.769				0	
5709.472	.601	Fe	5	0	dr
.947	.775	Ni	5	0	
5711.123	.313	Mg	6	0	dr
.462				0	
5712.	.098	Fe	3	.....	bf
5712.	.357	Fe	2	.....	bf
5714.102	.120	Ti	000	1	
5714.	.380	Fe	0	.....	bf
5715.473	......	Ce	....	1	
5716.653	.671	Ti	00	0	
5718.	.055	Fe	4	.....	bf
5723.986	4.107	....	0	0	dr
4.224				0	
5727.242	.271	Ti-V	2N	2	
5729.122	.096	Y	00d?	0	
5730.114	.116	....	0000	0	
5731.399	.437	V	00	1	w
5731.845	.984	Fe	4	0	dr
2.146				0	
5732.941	.948	....	000	0	
5737.300	.288	V	0	0	
5740.182	.195	Ti	0	0	
5748.018	.176	Fe	2	0	dr
.374				0	
5750.757	.723	....	000	0	w
5753.701	.860	–Cr	1N	0	dr
.997				0	
5754.750	.881	Ni	5	0	dr
5.024				0	
5756.390	.410	....	0000	0	
5758.738	......	....	....	0	
5758.967	.978	....	000	0	
5760.908	1.052	Ni	2	0	dr
1.194				0	
5761.678	.800	....	0	0	dr
.906				0	
5762.474	.479	Ti	000Nd?	3	
5763.084	.218	Fe	6	0	dr
.359				1	

TABLE I—*Continued*

OBSERVED λ	ROWLAND			CHR. INT.	REMARKS
	λ	Element	Intensity		
5766.547	.550	Ti	0	2	
5769.094	.120	Ce	000N	1	
5769.285	.295	La	000	2	w
5770.741	.714	Ce	000Nd?	1	w
5772.628	.630	V	000	0	
5774.243	.250	Ti	0	2	
5775.160 .422	.304	Fe	4	0 0	dr
5775.970	.969	....	0000	0	w
5780.922	1.024	Fe	0	0	dr
1.202	.130	Cr, Ti	00	0	
5782.198	.313	....	3	0	dr
.507	.390	Cu?	3	0	
5784.760 .992	.879	Fe	1	0 0	dr
5785.372 .625	.498	Fe	3	0 1	dr
5786.209	.193	Ti, Cr	0N	3	
5786.416	.373	....	000	1	
5788.	.141	Cr	4	.....	bf
5788.572	.504 .611	.... Cr	000 00	0	w
5789.551	.565	....	0000	0	
5791.064	.174	Cr	4	0	dr
.366	.243	Fe	3	1	
5793.127 .421	.292	....	3	0 0	dr
5794.020 .243	.137	Fe	2	0 1	dr
5797.816	.815	Ti, La	000	2	
5798.	.077	....	3	.....	bf
5804.473	.479	Ti	0	2	
5804.990	.996	....	0000	1	w
5805.992	.986	La	0	0	
5809.	.439	Fe	4	.....	bf
5811.825	.823	Nd	000Nd?	1	w
5812.	.139	Fe	0	.....	bf
5815.	.441	Fe	0	.....	bf
5817.159 .456	.299	V	0	0 0	dr
5817.726	.708	....	000	0	
5827.968 8.253	8.097	....	0	0 0	dr
5830.891	.895	V	000	0	
5835.482	.475	....	000	0	
5842.619	.600	Nd	000	1	
5845.346 .612	.509	....	0	0 0	dr
5846.494	.487	V	00	1	
5853.796 4.012	.902	Ba	5	1 1	dr
5855.	.300	Fe	1	.....	bf
5857.518 .820	.674	Ca	8	0 0	dr
5862.	.582	Fe	6	.....	bf
5863.906	.933	La	0000	1	
5866.548 .800	.675	Ti	3	1 0	dr
5867.	.785	Ca	2	.....	bf
5875.838	......	He	....	50	
5876.174	......	He	....	5	
5879.514	.506	....	0000	0	
5884.	.028	Fe	4	.....	bf
5889.979 90.376	0.186	Na	30	3 3	dr
5895.984 6.347	6.155	Na	20	3 3	dr
5905.725 6.073	.895	Fe	4	0 0	dr
5916.340 .623	.475	Fe	3	1 0	dr
5930.	.406	Fe	6	.....	bf
5937.828 8.222	8.035	Ti	000	0 0	dr
5949.	.566	Fe	1	.....	bf
5952.	.943	Fe	4	.....	bf
5956.770 7.084	.923	Fe	4	0 0	dr
5965.894 6.231	6.055	Ti	2	0 1	dr
5967.684	.720	....	0000	1	w
5978.696 .858	.768	Ti	1	0 0	dr
5983.760 4.040	.908	Fe	5	0 0	dr
5985.	.040	Fe	6	.....	bf
5987.140 .451	.290	Fe	5	0 0	dr
5991.605	.600	....	2	3	
6003.079 .416	.239	Fe	6	0 0	dr
6006.587	.605	....	00	0	
6008.632 .941	.785	Fe	6	0 0	dr
6013.556 .878	.715	Mn	6	1 1	dr
6016.700 7.015	.861	Mn	6	1 1	dr
6018.009	......	....	....	0	
6021.853 2.171	2.016	Mn	6	1 1	dr
6024.	.281	Fe	7	.....	bf
6027.133 .421	.274	Fe	4	1 0	dr

TABLE I—*Continued*

Observed λ	Rowland			Chr. Int.	Remarks
	λ	Element	Intensity		
6034.416	.440	Nd	0000	0	
6042.155 .447	.315	Fe	3	0 0	dr E
6043.580	......	Ce	....	1	
6049.327	.337	Co	000N	1	w
6056.178 .364	.227	Fe	5	0 0	dr
6060.860	.853	....	0000	0	
6065.557 .852	.709	Fe	7	1 1	dr
6078.420	.410	....	0000N	0	
6081.655	.665	V	0	0	
6082.628	.640	Co	00Nd?	0	
6082.	.930	Fe	1	.....	bf
6084.320	.325	....	0	2	
6085.	.470	Ti, Fe	2	.....	bf
6086.874	.885	Co	0000N	1	w
6090.406	.429	Ti, V	2	0	
6091.231 .577	.395	Ti	0	0 0	dr
6092.114	.133	....	1	0	
6098.864	.870	....	00	1	
6102.780 3.105	.937	Ca	9	1 1	dr
6103.	.400	Fe	4	.....	bf
	.514	Fe	1	.....	bf E
6108.164 .500	.334	Ni	6	1 0	dr
6111.	.290	Ni	2	.....	bf
6111.864	.872	V	0d?	1	
6113.551	.538	....	0	1	
6119.736	.740	V	1	0	
6119.956	.970	Ni	0	1	E
6122.	.434	Ca	10	.....	bf
6125.054 .403	.236	Ni	1	0 0	dr E
6135.562	.580	V	00N	1	
6136.665 .982	.829	Fe	8	1 1	dr
6137.	.915	Fe	7	.....	bf
6141.816 2.065	.938	Fe, Ba	7	2 2	dr
6146.422	.445	Ti	000	0	
6147.932	.950	Fe	2	2	E
6149.457	.458	Fe, Nd	2	2	w E
6150.338	.360	V	0Nd?	0	
6151.037	.052	Y	0000	1	
6151.	.834	Fe	4	.....	bf
6154.315 .540	.438	Na	2	0 0	dr
6157.	.945	Fe	5	.....	bf
6160.	.956	Na	3	.....	bf
6162.	.390	Ca	15	.....	bf
6165.	.577	Fe	3	.....	bf
6166.521 .790	.651	Ca	5	0 0	dr
6169.	.249	Ca	6	.....	bf
6169.	.778	Ca	7	.....	bf
6173.426 .687	.553	Fe	5	0 0	dr
6180.	.420	Fe	5	.....	bf
6188.080 .331	.210	Fe	4	0 1	dr
6191.	.393	Ni	6	.....	bf
6191.612 .930	.779	Fe	9	0 0	dr
6199.392	.398	V	0	1	
6200.370 .680	.527	Fe	6	1 1	dr
6204.	.825	Ni	1	.....	bf
6210.869	.895	Sc	00N	0	
6213.497 .811	.644	Fe	6	1 1	dr
6215.241 .489	.360	Fe	5	0 0	dr
6219.378 .660	.494	Fe	6	1 1	dr
6229.340 .539	.437	Fe	1	0 0	dr
6230.	.943	V-Fe	8	.....	bf
6232.639 3.040	.856	Fe	3	1 1	dr
6237.	.534	....	3	.....	bf
6238.592	.598	Fe	2	3	E
6239.611	.585	Sc	00	0	
6240.737 .988	.863	Fe	3	1 1	dr
6243.300	.320	....	1	0	
6245.802	.832	Sc–	1	0	
6246.398 .666	.535	Fe	8	1 1	dr
6247.777	.774	Fe	2	2	E
6252.658 .901	.773	Fe	7	1 0	dr
6256.	.572	Ni-Fe	6	.....	bf
6265.198 .494	.348	Fe	5	0 0	dr
6270.259 .608	.442	Fe	3	0 0	dr
6272.239	......	Ce	....	1	
6274.049	......	....	....	0	
6280.679 .977	.833	Fe	3d	0 1	dr
6287.023	.009	V	0000N	0	

TABLE I—*Continued*

OBSERVED λ	ROWLAND			CHR. INT.	REMARKS
	λ	Element	Intensity		
6291.022 .319	.184	Fe	4	0 0	dr
6298.	.007	Fe	5	.....	bf
6301.560 .871	.718	Fe	7	0 0	dr
6302.560 .861	.709	Fe	5	0 0	dr
6305.873	.878	Sc	0000	1	w
6311.610 .834	.722	Fe	1	0 0	dr
6314.	.876	Ni	4	.....	bf
6315.805 6.241	6.028	Fe	1	0 0	dr
6318.	.239	Fe	6	.....	bf
6322.713 3.091	.907	Fe	4	0 0	dr
6324.759	......	....	....	0	
6327.615 8.034	.820	Ni	2	0 0	dr
6330.126 .489	.316	Cr	1	0 0	dr
6331.	.067	Fe	2	.....	bf
6335.409 .689	.554	Fe	6	0 0	dr
6336.842 7.341	7.048	Fe	7	0 1	dr
6347.335	.310	....	2N	2	
6355.045 .488	.246	Fe	4	0 0	dr
6358.722 9.045	.898	Fe	6	0 0	dr
6362.579	.560	Zn	1	0	
6362.930 3.264	3.090	Cr, Fe	2	0 0	dr
6364.455 .712	.575	Fe	1	0 0	dr
6369.663	.683	Fe	0	4	
6371.592	.573	Fe	1Nd?	2	
6378.	.468	Ni	2	.....	bf
6380.762 1.140	.958	Fe	4	0 0	dr
6381.836	.844	....	0000	0	
6383.916	.932	Fe	0N	1	w E
6390.706	.715	La	000	1	
6393.	.820	Fe	7	.....	bf
6395.390	.378	Co	00Nd?	0	
6408.606	.587	Co	0000	0	
6411.637 2.070	.865	Fe	7	0 0	dr
6417.120	.133	Fe?	1	3	E
6420.	.169	Fe	4	.....	bf
6421.402 .753	.570	Fe	7	0 0	dr
6430.870 1.251	1.066	Fe	5	0 0	dr
6432.870	.895	Fe?	1	6	
6439.113 .481	.293	Ca	8	1 1	dr
6449.844 50.253	0.033	Ca	6	0 0	dr
6455.218	.230	Co	0N	1	
6455.602 6.043	.820	Ca	2	0 0	dr
6456.597	.603	Fe	3	5	E
6462.	.784	Ca	5	.....	bf
6469.	.408	....	2	.....	bf
6471.677 2.109	.885	Ca	5	1 1	dr
6475.	.846	....	2	.....	bf?
6491.810	.800	Ti	1	0	E
6493.805 4.232	4.004	Ca	6	0 0	dr
6496.978 7.291	7.128	Fe	4	2 2	dr
6498.995 9.371	9.168	Fe	1	0 0	dr
6499.676 6500.096	.880	Ca	4	0 0	dr
6516.310	.311	Fe	2	7	E
6527.198 .656	.444	....	1	1 0	dr
6546.311 .675	.479	Ti-Fe	6	0 0	dr
6554.322 .625	.470	Ti	0	1 0	dr
6555.087	.080	....	0000N	0	
6555.718	.700	....	1N	0	
6559.659 .949	.815	....	0	0 0	dr
6562.299 3.618 .902	.045	H	40	20 10 10	dr
6569.	.460	Fe	5	.....	bf
6586.547	.550	Ni	1	0	
6599.320	.353	Ti	00	0	

sun's visible edge, it seems certain that the results of these observations relate to a very low level in the reversing layer. We are, therefore, quite unable to agree with Mitchell in his conclusion that the spectra taken without an eclipse are at a higher level than the eclipse photographs. In fact the arguments which he adduces, namely, the absence from the Mount Wilson negatives of some strong Fraunhofer lines, which are prominent as bright lines on the eclipse negatives, and, similarly, the strength as bright lines of certain faint Fraunhofer lines point to precisely the opposite conclusion and can be explained only on the basis of a lower level for the photographs taken without an eclipse.

COMPARISON WITH ECLIPSE RESULTS

In the course of the discussion of his admirable eclipse results Mitchell institutes a comparison with the values published by Hale and Adams in their preliminary account of the Mount Wilson observations. From this comparison Mitchell concludes:

1. The eclipse spectrum negatives show twice as many lines as the 60-foot tower telescope photographs.

2. The eclipse spectrum wave-lengths are quite as accurate as those obtained from the tower telescope photographs.

It is difficult to understand the basis for the first of these conclusions unless Mitchell has employed for comparison purposes the violet part of the spectrum in which the visual objective of the tower telescope performs to poor advantage. Certainly in the visual region no such conclusion is tenable on the evidence either of the short region between λ 5111 and λ 5198 previously published, or of the present more complete results. A comparison for the entire region between λ 4800 and λ 5880 shows the following number of bright lines, the double reversals being counted as single lines:

	Mitchell Eclipse	Mount Wilson
Bright lines (measured)...........	901	894
Dark lines with bright fringes (not measured).....................	....	133
Total......................	901	1027

The effect of the difference of level in the reversing layer for the two sets of photographs is seen clearly in these results. Nearly all of the lines with bright fringes on the Mount Wilson photographs appear on Mitchell's negatives as strong bright lines, and the same is true of a large proportion of the Mount Wilson double reversals. A considerable number of strong lines on the eclipse plates do not appear at all as bright lines on the tower telescope plates. This is due evidently to the low level at which the latter were taken. On the other hand, the tower telescope photographs show between one and two hundred lines not seen on the eclipse photographs, essentially all of which are very faint Fraunhofer lines and belong to a low level in the reversing layer. The two series of photographs, accordingly, may be said to represent the spectrum of two sections of the reversing layer at different elevations, to the higher of which the eclipse results belong.

The statement by Mitchell that the degree of accuracy attained for the eclipse and the tower telescope results is about the same is by no means borne out by a detailed comparison. For this purpose we have selected only the single lines common to both sets of measures, omitting all blends and double reversals. A comparison of all of the lines of this character between λ 4800 and λ 5300 gives for the average deviation from Rowland's wave-lengths the following values:

Eclipse Results	Mount Wilson Results
0.029 A	0.011 A

For the entire region between λ 4800 and λ 5880 the results are

Eclipse Results	Mount Wilson Results
0.030 A	0.012 A

It appears, therefore, that the average deviation from Rowland's wave-lengths of the eclipse spectrum lines is about two and one-half times as great as that of the lines from the tower telescope photographs. The well-known very high degree of precision in the relative wave-lengths of Rowland's table necessitates ascribing essentially all of these differences to errors in the determination of the bright-line wave-lengths.

DISCUSSION OF THE RESULTS

In general the results obtained for the chromospheric spectrum agree with eclipse results as regards the elements represented by the largest proportion of bright lines. They differ widely, however, in the matter of the relative intensity of the bright lines as compared with the dark lines of the solar spectrum. With the exception of the hydrogen, magnesium, and sodium lines, and enhanced lines in general, only very few strong dark lines of the solar spectrum are represented by strong bright lines in the chromospheric spectrum taken without an eclipse. Such lines usually remain dark with a faint bright fringe on either edge. As already stated, this is almost certainly due to the low level of the point under observation. The stronger lines of the solar spectrum are accompanied by wings, similar in character to those of the H and K lines of calcium, though very much less in extent and intensity, which are due to the dense vapor at the base of the sun's reversing layer. In observations at the sun's edge these wings appear as faint emission lines, while the central portion of the line, due mainly to gas at a higher level, still remains dark. The phenomenon is in fact just the same as in the case of the well-known double reversals of the calcium and hydrogen lines at the sun's limb.

It seems probable that double reversal is a universal characteristic of all lines in the chromospheric spectrum at a low level, and that failure to observe it in any particular case is due to the faintness of the lines or insufficient resolving power of the spectroscope. Mitchell observed the effect for a few of the stronger lines, even at the level of his eclipse spectra, and several hundred examples are shown in these tables. In most cases the intensities of the two bright components are equal, and when unequal there seems to be no especial preponderance either of the violet or the red component.

The great number of very faint dark lines of the solar spectrum, which are represented by bright lines in the chromosphere, is in excellent accord with St. John's conclusions that such lines are produced at a low level in the solar atmosphere. A large number of these lines in the region λ 5050–λ 5165 may be ascribed to the green carbon fluting. Many others have been identified with reasonable certainty with lines of cobalt, scandium, titanium, vanadium, and the heavy elements, all of which are very strongly represented in the chromospheric spectrum. It seems probable that a portion of the unidentified lines may be due to the fainter enhanced lines of elements which have as yet been but very imperfectly investigated in this portion of the spectrum.

Some special lines.—The two hydrogen lines which appear in this region are H_α and H_β. Both are strong double reversals in the chromosphere with the components of nearly equal intensity. A peculiar feature is the apparent doubling of the red component of H_α. This is without doubt due to the presence of the strong atmospheric line at λ 6563.763 (the photographs were taken at low sun) which falls upon the center of the bright component and so gives the appearance of a double line.

Of the three helium lines, those at λ 4922 and λ 5048 probably are present, but the first so nearly coincides with a strong La line that it cannot be distinguished with certainty. The D_3 line is the strongest bright line on our photographs, and the scale of the plates is sufficient to make the measurement of its two components possible. The separation found, 0.336 A, agrees closely with the laboratory results.

The D lines of sodium and the b lines of magnesium are very similar in their behavior, being broad double reversals with components of moderate intensity.

A peculiar feature of the results is the remarkable number of faint cobalt lines represented in the chromosphere. In this respect cobalt is to be classed rather with titanium and vanadium than with nickel, iron, and manganese.

The enhanced lines.—It appears from these results that the enhanced lines are of an exceptionally great intensity at a moderate level in the solar atmosphere, just as eclipse observations have shown them to be at a higher level. Upon our photographs they nearly always appear as double reversals, but the separation of the components is considerably less than in the case of arc lines which are doubly reversed. In discussing possible explanations of the prominence of enhanced lines in the flash spectrum Mitchell has referred to the suggestion made by Gale and Adams that it may be due to the reduction of pressure in the upper portions of the solar atmosphere, laboratory observations having shown that in the case of several elements reduced pressure brings out the enhanced lines more strongly. A second suggestion made by the same writers may also be considered in this connection. It was found that in the spectrum of a spark under moderate pressures comparable with those at the base of the reversing layer the enhanced lines remained bright when the other lines became dark. It would appear probable, therefore, that in the solar spectrum the enhanced lines would be represented by dark lines relatively less intense than would the arc lines. The relative gain in intensity of the enhanced lines when appearing as bright lines in the chromosphere would, accordingly, be due to two causes: first, the actual increase in relative intensity of the enhanced lines due to reduction of pressure; second, the relative weakness of the dark lines in the solar spectrum with which the comparison is made.

Elements of high atomic weight.—The extraordinary prominence of the lines of certain elements of comparatively high atomic weight in the flash spectrum is a well-known feature of eclipse results. Without following Mitchell in his identification of a larger number of lines with those due to praesodymium, samarium, and erbium, we are certainly justified in concluding this behavior in the chromosphere on the part of yttrium, lanthanum, cerium, and probably neodymium.

According to the conclusions of St. John and others, elements of high atomic weight lie at low level in the solar atmosphere. Mitchell concludes from his measurement of the lengths of the arcs of the flash spectrum lines of the rare earths that these do not occur in shallow layers, but rather the reverse. It is, of course, evident that the two sets of results cannot be compared directly if Mitchell is to include among his rare earths scandium, with an atomic weight of 44, and omit barium with an atomic weight of 137. The evidence afforded by the results given here appears to be distinctly in favor of St. John's conclusions. The relative intensities of the bright lines of the heavy elements become greater as we pass from eclipse spectra to the lower level of the observations without an eclipse. Thus if we compare a lanthanum line of solar intensity 1 with an iron line of intensity 1, we find a certain ratio of intensity for the two bright lines in eclipse spectra. In the spectra taken without an eclipse the lanthanum bright line is relatively stronger than in the eclipse spectra. This can hardly be interpreted in any other way than that the radiation of the heavy elements becomes relatively stronger at low levels. The case appears to us analogous to that of the carbon fluting, the individual bright lines of which show the more strongly the nearer the observations are made to the sun's visible edge.

It seems very probable that, as St. John has stated in his discussion of Mitchell's eclipse results, the explanation of the longer arcs found by Mitchell for the rare earths is to be found in the greater intensities of these lines. The average intensity given by Mitchell for the bright lines of the Fe group is 1.07; for those of the rare earths 3.10. There can be little doubt that a bright arc three times as strong as another would on the photographic plate have a considerably greater length. The case is entirely similar to that seen in the laboratory when a photograph is taken of the spectrum of a metallic arc. The strongest lines are the longest even when the lines compared are produced in just the same part of the arc.

Chromospheric wave-lengths and anomalous refraction.—In our determinations of the wave-lengths of the bright lines in the chromosphere we have used as standards of reduction the dark lines of the limb spectrum which appear upon the same photographs. Any differences of wave-length, therefore, are with reference to the lines at the sun's limb, and these, as is well known, are displaced slightly to the red when compared with the center of the sun. In the region covered by the observations this displacement amounts on the average to +0.008 A. Our chromospheric measures give for the average difference in wave-length between 512 identified bright lines and the corresponding dark lines at the limb −0.002 A. If we assume this small difference to be real, the bright lines in the chromosphere, therefore, will be displaced +0.006 A relative to the dark lines at the center of the sun.[1]

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[1] In the results given by Hale and Adams the same method was used, and the wave-lengths given were referred to the lines at the sun's limb. Apparently some

The ordinary absorption and emission theory of the origin of the solar lines would explain this result in a very simple way. The chromospheric bright lines are produced at nearly the same level and under nearly the same physical conditions as the dark lines at the sun's limb; hence their wave-lengths should be closely the same. On the whole we should perhaps expect the level of the chromospheric observations to be slightly above that of observations at the sun's limb. If such is the case the cause which produces the displacements of the lines at the limb relative to the center of the sun (which we are still inclined to consider in large part as pressure, in spite of Evershed's recent work on the subject) would operate to a less extent on the bright lines; and we might expect some such small negative difference as is found between their wave-lengths and those of the dark lines at the limb.

In the course of an extensive series of articles Julius has endeavored among other solar phenomena to account for the wave-lengths of the chromospheric lines and for the displacements of the dark lines at the sun's limb on the basis of anomalous refraction and dispersion in the solar atmosphere.[1] The principal feature of the explanation which he adopts is the assumption that the solar atmosphere is honeycombed with irregular density gradients, of which a sufficient number show increasing density outward to overcome wholly or mainly the effect of the well-known regular radial gradient. Most solar observers would desire some evidence tending to indicate the existence of such gradients apart from the necessity of postulating them in order to support the anomalous refraction hypothesis, more especially as they must be essentially permanent in character. If they fluctuated to any considerable

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misapprehension has arisen in regard to this matter, as both Julius and Brunt (*Monthly Notices*, **73**, June 1913) appear to consider the wave-lengths as referred to the center of the sun. No explicit statement was made because at the date the results were published the amount of the hypothetical anomalous dispersion effect, due to the regular density gradient, was supposed to be very large, amounting to one-half the width of the lines. The value given by Hale and Adams for chromosphere-limb wave-lengths was +0.002 A; the result found here is −0.002 A. We consider this agreement as quite satisfactory in view of the difficulty of the measurements.

[1] "On the Origin of the Chromospheric Light," *Proceedings Amsterdam Academy*, December 23, 1909; "Regular Consequences of Irregular Refraction in the Sun," *Proceedings Amsterdam Academy*, October 28, 1909.

extent, both the displacements at the sun's limb and the wave-lengths of the chromospheric bright lines should show marked systematic variations, and such has not been found to be the case. Assuming the existence of such irregular gradients, however, Julius accounts for the displacements of the dark lines at the limb as follows:

> Indeed rays coming from the limb have, as a rule, accomplished a longer distance through the solar gases than rays coming from the center, and, therefore, were more subject to loss of intensity by the process of incurvation towards the photosphere. The amount of the irregular ray-curving depends on the absolute magnitude of Rm Δm (the refracting power of the mixture of gases), which near the weaker lines of the solar spectrum is sensibly greater with red than with violet light. So the lines must chiefly widen at their red-facing side, in proportion as the opportunity for losing light increases.

That is, the displacements are due to the widening of the lines toward the red owing to the relatively greater loss of red light by anomalous refraction. In this explanation the effect of the regular radial gradient is neglected completely, and in fact in another place[1] Julius states expressly that it would produce displacements toward the violet. Accordingly he is obliged tacitly to assume that the irregular gradients are sufficient to counteract this effect and to give in addition the observed displacements toward the red.

When he passes to the discussion of the chromospheric spectrum Julius again makes use of the irregular density gradients, but in this case he considers they will produce essentially symmetrical bright lines, violet and red light being refracted in nearly equal proportions. As already stated, Julius has not understood the fact that the chromospheric lines are displaced to the red relative to those at the center of the sun by about 0.006 A, since his explanation is intended to account for the absence of any displacement. The photospheric light which is refracted to form the chromospheric lines is supposed to come from near and just beyond the sun's visible edge.

It seems to us very doubtful whether the results found on our photographs can be accounted for by this hypothesis. Bright and dark lines appear on the same negatives and the character of the path through the irregular density gradients must be nearly the same in the two cases. If the displacements of the dark lines at the sun's limb are due to irregular density gradients which tend to produce a loss of light on the red side of the absorption lines and thus widen them toward the red, we must conclude that these gradients would have a marked effect upon the light which forms the chromospheric lines. Accordingly if the photospheric light which is refracted comes from beyond the sun's limb, we should expect a relative loss either of red or of violet light. If the former the chromospheric lines should be displaced to the violet with regard to the dark line at the limb; if the latter it should be displaced to the red. Neither is the case to any marked degree. Apparently the lack of symmetry in refraction which is introduced by Julius to account for the dark line displacements is directly opposed to the chromospheric results. A resort to the regular radial gradient to explain the displacements of the chromospheric lines with reference to the lines at the center of the sun could not be made unless the explanation of the limb displacements were abandoned, since this would require displacements to the violet.

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[1] *Astrophysical Journal*, **31**, 427, 1910.

The results of a comparison of the wave-lengths of the chromospheric lines with the corresponding dark lines at the sun's limb are given in Table II. Only such lines have been included as can be identified with certainty, and whose wave-lengths have been measured directly.

TABLE II

Element	No. Lines	Chromosphere—Limb
Fe.	157	−0.001 A
C.	90	−0.002
Ti.	87	−0.004
Co.	42	−0.008
Cr.	35	+0.002
Ni.	32	−0.001
V.	27	−0.003
Ca.	16	0.000
Sc.	13	−0.001
Y.	8	−0.004
La.	5	+0.006
Mean.	512	−0.002

The preponderance of the negative sign in these residuals is rather striking, the only important exception being chromium. Since the average deviation for all of the lines measured amounts to 0.012 A, however, we do not feel justified in considering the slight mean residual of −0.002 A as certainly real.

One other characteristic of the chromospheric lines should be considered in connection with the anomalous dispersion theory. This is the phenomenon of double reversal. Although Julius admits that a portion of this effect may be due to common reversal by absorption, he introduces anomalous refraction and scattering of the light very close to the absorption lines to increase its amount. Two comments may be made in this connection. First, it would seem hardly necessary to bring anomalous refraction into question if the existence of absorption has to be assumed, since laboratory results have shown that it is easy to produce in the spectra of luminous gases just such reversals as are found in the chromospheric spectrum under conditions where only emission and absorption are involved. Second, it is difficult to understand why the double reversals should nearly always be symmetrical, although the refracting power of the solar atmosphere is greater on the red side of an absorption line.

It is a peculiar fact that the advocates of the anomalous dispersion hypothesis admit the existence of essentially all of the phenomena in the sun which are required by those who use the more usual explanation. Thus Julius is willing to admit the presence of ordinary selective absorption and emission to form the central portions of the Fraunhofer lines; double reversal may help to form the double lines in the chromosphere; the Doppler effect is essential in the convection currents required to produce his hypothetical irregular gradients; the existence of pressure can hardly be denied in any gaseous atmosphere; and variations in density are the fundamental basis of anomalous refraction theory. Apparently variations in temperature have not as yet been admitted by Julius, but it is certainly impossible to explain on the anomalous refraction theory the presence in sun-spots of a band spectrum not found at all in the solar spectrum. If the reduction in temperature is once admitted, the spectral characteristics of sun-spots find not only a ready but a necessary explanation in this fact, as laboratory investigations have shown. Many of the solar applications of anomalous dispersion share this characteristic of being quite superfluous.

Some features of the absorption spectrum on the chromosphere photographs.—The dark lines which appear upon our chromospheric photographs are due to the light from just within the sun's limb and represent the limb spectrum in its most extreme form. In general the changes in the character and intensity of the lines as compared with the center of the sun are those described by Hale and Adams in their publication on the subject[1] some years ago. They are, however, somewhat more pronounced, a result to be expected because of the closer proximity of the point under observation to the sun's limb. In one region of the spectrum, at least, its characteristics deserve a few words of comment.

Between the limits λ 5025 and λ 5150 there are some especially remarkable changes in the spectrum, consisting in the great intensification of certain very faint solar lines, the great weakening of some strong solar lines, and the appearance of several fairly strong lines which apparently are not present in the solar spectrum at all. The changes are such as to modify the spectrum completely and to render its comparison with the usual solar spectrum difficult.

The results of some measurements in this region are given in Table III.

There are some indications of the presence of a banded spectrum in certain portions of this region, especially near λ 5065, λ 5080, and λ 5100. It is possible that this is a trace of the band spectrum of magnesium hydride found by Fowler to be so prominent in the spectrum of sun-spots.

SUMMARY

1. Photographs of the flash spectrum taken with the 60-foot tower telescope show a slightly greater number of bright lines than the eclipse photographs of Mitchell in the same region.

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[1] *Mt. Wilson Contr.*, No. 17; *Astrophysical Journal*, **25**, 300, 1907.

2. The degree of accuracy in the determination of wave-lengths is between two and three times as high for the photographs taken without an eclipse.

3. The level of the observations is decidedly lower in the case of the tower telescope results.

4. Double reversal is probably a universal characteristic of the bright lines.

TABLE III

Wave-Lengths		Intensity		Character at Limb
Limb	Sun	Limb	Sun	
5034.259	....	1	....	
5034.737	....	2	....	Double
5035.506	.542	3	5	Double
5036.105	.115	2	5	Blend in sun
5037.458	.436	2	000	Blend in sun
5037.869	.885	2	000	
5040.429	.422	1	000	
5043.055	....	1	....	Narrow
5043.218	....	2	....	Narrow
5043.758	.761	1	00	
5044.107	....	1	....	
5056.608	.617	1	000	Narrow
5057.745	.665	2	0 ...	Broad
5061.287	....	1	....	
5076.432	.450	1	3	
5076.595	.504 .666	2	000 0000	
5086.791	.794	1	0000	
5094.584	.594	1	0	Broad
5097.669	.668	1	0	
5103.148	.142	2	1	
5149.037	.013	2	000	

5. The lines of the elements of high atomic weight are relatively stronger on the photographs taken without an eclipse. This indicates a low level for these elements in the solar atmosphere.

6. The average difference in wave-length between the bright lines of the chromospheric spectrum and the dark lines at the sun's limb is −0.002 A.

7. This close agreement in wave-length does not appear to be in accordance with the anomalous refraction theory of the chromosphere as developed by Julius.

8. Some remarkable differences have been found in the dark line spectrum at the sun's limb as compared with the ordinary solar spectrum. Several new lines appear in addition to great changes in intensity of certain solar lines. There is considerable evidence of the presence of a band spectrum in a portion of the green region of the spectrum.

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LIST OF STARS WITH PROPER MOTION EXCEEDING 0".50 ANNUALLY

By ADRIAAN VAN MAANEN

Since the publication, by Porter[1] and Kobold,[2] of the lists of stars having proper motions greater than 0".50 annually, and since the appearance of Bossert's[3] catalogue of proper motions of 2641 stars, there have been published several notes on stars of large proper motion. The following list (Table I), which has been based on all these sources, is, I hope, complete up to 1914.

In the first column is given the current number; in the second and third columns the name and number in the *B.D.* or *C.P.D.* The fourth column contains the magnitude. To have these as homogeneous as possible it seemed best to give the magnitudes of the *B.D.* and *C.P.D.* For the companions of double stars, which are not given in these sources, the difference as given in Burnham's *General Catalogue* is added to the magnitude of the principal stars. For stars not contained in any of these catalogues I have given the magnitudes as given by the authors of the proper motions. These are marked by an asterisk.

In the fifth column is given the spectrum as determined by W. S. Adams (in an unpublished list, for the use of which I feel greatly indebted to Mr. Adams) or by Miss Cannon.[4] The sixth and seventh columns contain α and δ for the equinox of 1900.0; and the eighth and ninth columns the amount and the direction of the proper motion; while the tenth column gives the authority. The last column finally gives some remarks, regarding uncertainties in the proper motions, and, in the case of double stars, the number in Burnham's *General Catalogue*.

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[1] *Astronomical Journal*, **12**, 25, 1892.

[2] *Der Bau des Fixsternsystems*. Vieweg und Sohn, 1906.

[3] *Catalogue d. mouv. propres de 2641 étoiles*, 1896.

[4] *Harvard Annals*, **50**, 1908.

TABLE I

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
1	Gi.Z. 16730	−68° 3597	9.0	....	0^h 0^m16^s	−68° 22′	0″.53	204°.9	Rist.	
2	Lal. 47231¹	+45 4408	8.3	K5:	0 0 24	+45 16	0.90	98.3	Po.	β 12740
3	Lal. 47231²	....	8.3	....	0 0 24	+45 16	0.89	102.9	β	
4	Lal. 9721	−49 11858	6.2	G	0 1 8	−49 38	0.53	94.8	Boss	
5	β Cass.	+58 3	2.2	F5	0 3 50	+58 36	0.54	110.4	Au.	
6	A. G. Hels. 38	+64 3	7.0	....	0 3 57	+64 31	0.88	86.9	Schulhof	
7	A. G. Chris. 15	+66 7	9.0	....	0 5 22	+66 35	0.91	84.0	Schulhof	
8	Lal. 248	+43 44	8.1	Ma	0 12 40	+43 27	2.82	81.8	Ps.	
9	ζ Toucani	−65 13	5.1	F8	0 14 52	−65 28	2.05	55.9	Po.	
10	A.W. 120	−27 23	8.4	....	0 17 0	−27 16	0.52	74.7	Rist.	
11	Lal. 475	−27 25	8.1	....	0 19 20	−27 35	0.70	86.7	Po.	
12	Lac. 75	−51 48	7.2	....	0 19 48	−51 35	0.57	122.0	Po.	
13	β Hydrae	−77 16	3.3	G	0 20 30	−77 49	2.27	81.9	Po.	
14	Lac. 127	−35 60	7.0	....	0 28 50	−35 32	0.53	180.0	Po.	
15	Pi. 0^h130	−25 64	6.5	G5	0 32 13	−25 19	1.41	91.2	Po.	
16	Lal. 999	+ 2 84	7.5	G1	0 34 0	+ 2 35	0.79	68.3	Po.	
17	54 Piscium	+20 85	6.2	K1	0 34 10	+20 43	0.61	231.6	Ps.	
18	Lal. 1045	+39 154	7.0	K3	0 35 20	+39 39	0.81	152.1	Po.	
19	Lal. 1065	−24 60	6.9	K	0 35 31	−24 21	0.79	114.0	Po.	
20	Lac. 172	−60 46	6.1	K	0 35 45	−60 1	1.00	64.5	Po.	
21	Lal. 1198	+ 1 131	8.2	K4	0 39 57	+ 1 15	0.63	180.9	Po.	
22	η Cass.	+57 150	3.8	F8	0 43 3	+57 17	1.19	113.8	Au.	β 426
23	Comp. to η Cass.	....	7.4	....	0 43 3	+57 17	1.19	113.8	Au.	
24	Lal. 1299	+ 4 123	6.0	K1	0 43 8	+ 4 46	1.37	146.9	Au.	
25	Groom. 145	+69 45	7.8	K0	0 43 17	+69 54	0.54	113.4	Greenwich	
26	Lal. 1353	−23 95	7.4	....	0 44 27	−23 46	0.55	76.2	Po.	
27	Lac. 239	−31 92	7.4	....	0 48 6	−30 54	0.71	84.3	M.N.	
28	A. G. Chris. 174	+68 60	9.1	....	0 50 21	+68 31	0.73	111.0	Schulhof	
29	Lal. 1799	+ 4 158	8.3	K6	0 57 12	+ 4 31	0.50	59.6	Po.	
30	A. G. Hels. 914	+63 137	8.7	....	1 0 18	+63 24	1.62	80.1	Bellamy†	

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† Miss E. F. Bellamy, *Astronomische Nachrichten*, **195**, 359, 1913.

31	μ Cass.	+54	223	5.7	G3	$1^h\ 1^m 37^s$	+54° 26′	3″.73	114°.9	Au.
32	Lal. 1964	+22	176	8.4	K5	1 2 14	+22 26	0.52	167.9	Po.
33	Cp. St. St. 352	−51	162	8.6	....	1 2 21	−51 31	0.50	92.1	Cape
34	Lal. 1966	+60	170	7.8	F3	1 3 18	+61 1	0.69	90.0	Po.
35	Anonymous	....		13.0*	....	1 6 1	−72 46	0.73	54.8	Pickering
36	ν Phoenicis	−46	127	6.1	G	1 10 40	−46 4	0.71	76.2	Po.
37	W.B. 1^h161	− 1	167	8.0	G8	1 13 32	− 1 23	0.54	123.7	Po.
38	La. 2387	− 9	256	8.5	....	1 14 1	− 9 27	0.57	208.3	Po.
39	W.B. 1^h262	+30	206	8.8	....	1 15 50	+30 49	0.52	98.0	Rist.
40	Lal. 2450	+17	197	7.9	G0	1 16 52	+18 10	0.57	90.0	Po.
41	W.B. 1^h356	+17	202	8.8	....	1 19 26	+17 59	0.67	106.0	Berlin A
42	Lal. 2682	+20	226	7.8	K1	1 23 35	+21 13	0.53	111.2	Po.
43	Lal. 2975	+28	271	8.5	....	1 32 43	+29 4	0.54	118.6	Po.
44	Lal. 3022	+27	262	7.8	G7	1 33 56	+27 36	0.53	75.7	Po.
45	Lal. 2966	+66	145	7.4	G4	1 34 11	+66 25	0.76	107.7	Po.
46	Lal. 3054	+41	328	5.3	F	1 35 42	+42 7	0.82	100.5	Po.
47	A.Oe. 1867–8	+63	229	8.2	....	1 36 46	+63 20	0.64	206.9	Rist.
48	Lal. 3128	+19	279	5.3	F	1 37 4	+19 47	0.73	204.1	Ps.
49	Lal. 3153	−18	287	7.2	....	1 37 22	−18 24	0.54	93.8	R3
50	τ Ceti	−16	295	3	K	1 39 25	−16 28	1.95	296.1	Ps.
51	Pi. 1^h159	+63	238	6.1	K1	1 40 31	+63 22	0.62	112.8	Po.
52	A.Oe. 1137–8	−23	215	9.0	....	1 48 2	−22 56	0.92	90.0	Po.
53	χ Eridani	−52	241	5.8	G5	1 52 4	−52 6	0.72	65.5	Po.
54	Cp. St. St. 647	−41	183	7.4	....	1 56 18	−41 12	0.58	138.7	Cape
55	Lal. 3922	− 1	293	7.5	G0	2 2 30	− 1 5	0.50	220.9	Po.
56	Lac. 661	−51	282	7.1	G	2 6 24	−51 19	2.28	70.5	Po.
57	Lal. 3987	+67	191	7.8	....	2 7 31	+67 13	0.56	122.6	Ps.
58	W.B. 2^h95	− 1	306	8.3	F5	2 9 28	− 1 40	0.97	95.3	Po.
59	Lal. 4141	+23	303	6.5	G4	2 9 41	+23 49	0.60	106.3	Ps.
60	δ Trianguli	+33	395	5.0	G	2 10 57	+33 46	1.15	101.5	Ps.
61	Lal. 4268	+ 1	410	5.8	F5	2 12 50	+ 1 17	0.49	44.8	Po.
62	A.Oe. 2618	+70	169	8.5	....	2 14 3	+70 43	0.58	69.6	Greenwich
63	A. G. Christ. 416	+68	166	7.5	....	2 14 30	+68 18	0.87	92.3	Schulhof‡

‡ Schulhof marks the P.M. as uncertain, *Bulletin Astronomique*, **27**, 206, 1910.
The Greenwich second *9Y Cat.* gives μ = 0″.12, *p* = 60°.9.

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
64	Lal. 4338	−26° 214	7.2	G	$2^h14^m30^s$	−26° 25′	$0''.50$	330°.9	Po.	
65	A.W. 1327	−17 484	9.4	......	2 26 40	−17 26	0.56	90.0	Po.	
66	Pi. 2^h123	+ 6 398	6.5	K6	2 30 36	+ 6 25	2.34	51.8	Po.	
67	Lal. 4855	+30 421	7.3	G0	2 32 35	+30 24	0.60	228.4	Po.	
68	A.Oe. 1739.41	−30 324	7.8	......	2 36 17	−30 34	0.66	90.0	Po.	
69	A.Oe. 1753	−30 327	9.0	......	2 37 22	−30 44	0.51	127.4	Po.	
70	A. G. Bonn 2435	+45 669	9.0	......	2 44 54	+45 35	0.64	119.2	Ling	
71	A. G. Berlin A 767	+15 395	9.0	......	2 45 2	+15 19	0.53	141.1	Berlin A	
72	Comp. to Anon. $2^h45^m54^s+22°37'$	......	12.1*	......	2 45 44	+22 38	0.87	355.6	β	
73	Anonymous	......	10.4*	......	2 45 54	+22 37	0.71	241.7	Puiseux	
74	W.B. 2^h927	+ 5 435	8.2	G8	2 55 15	+ 5 36	0.68	102.8	Po.	
75	Lal. 5490	+61 513	6.7	G3	2 55 58	+61 20	1.01	131.4	Po.	
76	ε Fornacis	−28 302	6.6	G	2 57 19	−28 28	0.50	145.7	Po.	
77	ι Persei	+49 857	4.3	G	3 1 51	+49 14	1.25	92.7	Au.	
78	Lal. 5761	+25 495	7.9	A3p	3 2 32	+25 58	0.90	191.6	Po.	
79	Lal. 5986	−29 362	4.2	F	3 7 49	−29 23	0.73	25.2	Au.	
80	W.B. 3^h113	+ 8 482	7.8	K0	3 9 22	+ 8 37	0.62	131.7	Po.	
81	ζ_1 Reticuli	−63 217	6.4	G	3 15 36	−62 57	1.48	65.2	Po.	
82	Lac. 1060	−43 354	5.6	G5	3 15 56	−43 27	3.05	75.5	Po.	
83	ζ_2 Reticuli	−62 265	5.6	F8	3 16 2	−62 53	1.44	63.6	Po.	
84	Lal. 6320	− 5 642	7.9	K6	3 20 4	− 5 42	0.86	198.2	Po.	
85	Lal. 6429	−20 643	8.0	......	3 23 19	−20 10	0.61	61.8	Po.	
86	Lac. 1143	−63 234	5.4	F5	3 27 38	−63 17	0.50	43.4	Po.	
87	ε Eridani	− 9 697	3.3	K	3 28 13	− 9 48	0.99	270.7	Au.	
88	Lal. 6665	− 0 572	4.2	G5	3 31 46	+ 0 5	0.55	205.6	Ps.	
89	W.B. 3^h617	− 3 592	7.2	F5	3 35 17	− 3 32	0.79	107.7	Po.	
90	δ Eridani	−10 728	3.3	K	3 38 27	−10 6	0.75	350.8	Au.	
91	Cp. St. St. 1228	−51 443	7.2	......	3 40 11	−50 58	0.51	12.3	Cape	
92	Lal. 6888	+41 750	8.2	G6	3 40 12	+41 9	1.38	154.2	Po.	β 1854
93	Comp. to Lal. 6888	......	8.8	......	3 40 12	+41 9	1.38	154.2	Po.	
94	τ^6 Eridani	−23 414	4.5	F8	3 42 33	−23 33	0.56	198.8	Po.	

95	A. G. Nicolajew 817	+ 0°	659	8.3	....	$3^h44^m24^s$	+ 1° 3′	0″.68	161°.4	Nicolajew	
96	Lal. 7036	+60	762	7.8	....	3 46 27	+60 53	0.50	117.6	Po.	
97	Cp. St. St. 1272	−42	366	8.4	....	3 47 10	−42 53	0.61	18.1	Cape	
98	Groom. 745	+75	154	8.2	K4	3 48 24	+75 53	0.64	144.6	Po.	
99	Lal. 7443	+34	796	8.5	G5	3 56 32	+35 2	2.19	128.1	Po.	
100	A.W. 2324	−21	784	9.1	....	4 2 12	−21 6	0.71	177.4	Rist.	
101	A. G. Berlin B 1366	+21	607	8.9	....	4 8 36	+22 7	0.51	130.9	Berlin B	
102	o^2 Eridani	− 7	780	4.7	G5	4 10 40	− 7 49	4.11	212.7	Po.	β 2109
103	W.B. 4^h166	− 7	781	9.1	A0	4 10 45	− 7 49	4.11	212.7	β	
104	Comp. to W.B. 4^h166	....	....	10.8	....	4 10 45	− 7 49	4.11	212.7	β	
105	Lal. 8735	+41	931	7.1	G0	4 34 31	+41 56	0.73	127.2	Po.	
106	Cp. St. St. 1594	−50	637	7.6	....	4 43 7	−50 15	0.59	234.4	Cape	
107	Lal. 9012	+45	992	6.5	F8	4 44 22	+45 41	0.68	145.1	Po.	
108	A. G. Leiden 1812	+34	927	8.0	....	4 51 19	+34 7	0.76	114.6	Wilkens	
109	W.B. 4^h1189	− 5	1123	6.3	K9	4 55 51	− 5 52	1.25	151.4	Po.	
110	Lal. 9599	+18	779	5.2	G	5 1 32	+18 31	0.53	86.8	Ps.	
111	W.B. 5^h87	− 9	1094	8.0	....	5 7 8	− 9 13	0.58	185.0	Rist.	
112	C.Z. 5^h243	−44	612	9.2	G–K	5 7 42	−44 59	8.71	131.0	Kapteyn	
113	λ Aurigae	+39	1248	4.9	G1	5 12 6	+40 1	0.84	141.7	Au.	
114	Lal. 9960	− 3	1061	8.5	....	5 14 8	− 3 11	0.73	81.3	Po.	β 2654
115	Comp. to Lal. 9960	....	....	12.5	....	5 14 8	− 3 11	0.73	81.3	β	
116	Lal. 10299	− 3	1110	8.5	K5	5 23 29	− 3 34	0.89	198.4	Po.	
117	W.B. 5^h592	− 3	1123	8.7	....	5 26 23	− 3 42	2.23	161.7	Po.	
118	B.D. −0° 981	− 0	981	9.0	....	5 26 35	+ 0 2	0.52	165.0	Roy	
119	Groom. 990	+51	1094	7.9	K0	5 30 24	+51 23	0.56	279.3	Po.	
120	Pi. 5^h146	+53	934	6.3	K0	5 33 14	+53 26	0.55	179.0	Po.	
121	Lal. 10797	+37	1312	7.1	K2	5 39 9	+37 15	0.72	136.1	Po.	
122	π Mensae	−80	161	6.5	G5	5 45 8	−80 33	1.28	10.8	Po.	
123	Cordoba 6886	−70	447	8.6	....	5 45 42	−70 13	1.29	343.1	Rist.	
124	δ Leporis	−20	1211	4.3	K	5 47 1	−20 53	0.69	161.3	Ps.	
125	Lal. 11196	+13	1036	7.0	G2	5 50 21	+13 55	0.69	136.8	Po.	
126	Lac. 2106	−63	498	6.2	K	5 53 20	−63 7	0.50	17.4	Po.	
127	A. G. Berlin A 1866	+19	1185	9.0	....	5 57 12	+19 23	0.76	128.7	Wolf	
128	B.D. +26° 1067	+26	1067	8.9	....	5 59 43	+26 35	0.54	227.1	Wolf	
129	B.D. −21° 1377	−21	1377	8.7	....	6 6 22	−21 49	0.73	186.0	Po.	
130	Comp. to 71 Orionis	....	....	11.2*	....	6 8 32	+19 11	0.98	194.9	Lau†	

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† Lau probably used Burnham's companion C in 1886 and Burnham's companion D in 1909. According to Burnham both companions have no large P.M.

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
131	Cp. St. St. 2181	−42° 915	7.4	....	6ʰ21ᵐ21ˢ	−42°49′	0″.75	346°.6	Cape	
132	Groom. 1159	+79 212	5.5	A2	6 29 10	+79 40	0.66	186.1	Po.	
133	Gi.Z. 4158	−68 536	8.9	....	6 32 29	−68 37	0.50	195.0	Rist.	
134	α Canis Majoris	−16 1591	1.0	A	6 40 45	−16 35	1.31	203.8	Au.	β 3596
135	Comp. to α Canis Maj	....	9.0†	....	6 40 45	−16 35	1.31	203.8	Au.	
136	Lal. 13284	− 5 1844	6.9	K9	6 47 26	− 5 3	0.57	270.0	Po.	
137	A. G. Bonn 5621	+40 1758	8.3	....	6 49 30	+40 13	0.57	155.7	Bn.	
138	Lac. 2501	−28 1525	7.2	F	6 49 35	−28 24	0.58	145.5	Po.	
139	W.B. 6ʰ1500	+ 1 1600	8.0	G3	6 51 24	+ 1 18	0.57	169.9	Po.	
140	Lal. 13427	+48 1469	8.2	K1	6 54 0	+48 32	0.71	124.1	Po.	
141	Lal. 13576	+29 1441	6.3	F9	6 57 9	+29 30	0.82	168.0	Po.	
142	Lal. 13849	+21 1528	7.0	G6	7 4 11	+21 25	0.52	196.7	Po.	
143	Cp. St. St. 2483	−49 1163	7.6	....	7 8 53	−49 17	0.77	359.3	Cape	
144	Lal. 14146	−12 1871	7.3	F9	7 11 16	−12 53	0.55	283.8	Po.	
145	Lac. 2740	−46 1360	7.5	....	7 14 37	−46 49	0.60	342.5	Po.	
146	α Canis Minoris	+ 5 1739	1.0	F5	7 34 4	+ 5 29	1.25	215.0	Au.	β 4187
147	Comp. to α Canis Min	....	13.0†	....	7 34 4	+ 5 29	1.25	215.0	Au.	
148	W.B. 7ʰ1029	+39 1998	6.8	....	7 38 9	+39 49	0.52	174.8	Rist.	
149	β Geminorum	+28 1463	1.3	K	7 39 12	+28 16	0.63	265.5	Au.	
150	Groom. 1339	+80 238	6.5	G6	7 39 45	+80 31	0.50	272.3	Po.	
151	T Puppis	−44 1829	6.6	F8	7 39 52	−44 55	0.55	183.1	Po.	
152	Lac. 2957	−33 1748	7.1	....	7 41 51	−33 59	1.68	351.1	Po.	
153	Lal. 15290	+31 1684	8.2	F7	7 47 10	+30 55	1.97	158.3	Po.	
154	Lal. 15394	+19 1869	8.2	....	7 49 5	+19 31	0.50	170.4	β	
155	Lal. 15547	+21 1731	8.5	....	7 53 41	+21 8	0.59	159.1	Po.	
156	A.Oe. 7734	−25 3148	9.0	....	7 53 50	−25 21	0.52	122.5	Po.	
157	Lal. 15565	+29 1664	7.5	G7	7 54 21	+29 31	1.19	186.8	Po.	
158	Lac. 3122	−59 944	6.6	F2	7 55 56	−60 2	0.56	77.7	Po.	
159	Fed. 1221	+72 395	8.0	....	7 57 13	+72 13	0.50	333.7	Greenwich	
160	Lac. 3152	−29 2316	7.8	....	8 2 55	−29 6	0.56	141.0	St.	
161	Lal. 15950	+32 1695	6.7	G2	8 5 23	+32 46	0.82	213.5	Po.	

† Magnitude uncertain.

162	Fed. 1266	+73°	407	8.5	....	8^h 8^m 47^s	+73° 44′	0″.58	330°.7	Greenwich	
163	W.B. 8^{h}224	−11	2296	9.3	....	8 11 32	−11 27	0.72	59.0	Rist.	
164	W.B. 8^{h}181	+31	1781	8.3	K5	8 12 0	+30 56	0.90	200.3	Po.	
165	Lal. 16304	−12	2449	6.3	G9	8 13 39	−12 18	1.03	166.5	Po.	
166	A.Oe. 8869	+66	550	8.8	....	8 16 21	+66 48	0.52	1.0	Greenwich	
167	A. G. Cambridge 3181	+54	1244	8.6	....	8 28 12	+54 4	0.93	92.6	Cambridge	
168	W.B. 8^{h}596	+42	1899	8.5	....	8 28 48	+42 6	0.76	202.1	Rist.	
169	Lac. 3386	−31	2448	7.6	G2	8 28 57	−31 11	1.32	301.4	Po.	
170	Lal. 17046	+12	1888	8.1	....	8 34 23	+11 53	0.52	194.6	Po.	
171	Lal. 17161	+42	1922	8.2	K0	8 38 35	+42 3	0.69	204.0	Po.	
172	Fed. 1384^1	+71	482	8.5	Ma	8 45 59	+71 11	1.40	255.6	Po.	β 4815
173	Fed. 1384^2	....		8.6	....	8 45 59	+71 11	1.38	256.9	β	
174	ρ^1 Cancri	+28	1660	6.0	K0	8 46 39	+28 43	0.55	243.9	Po.	
175	ι Ursae Majoris	+48	1707	3.1	A5	8 52 22	+48 26	0.51	240.4	Au.	β 4866
176	Comp. B. to ι Ursae Maj.	....		9.5	....	8 52 22	+48 26	0.51	240.4	β	
177	Comp. C. to ι Ursae Maj.	....		9.8	....	8 52 22	+48 26	0.51	240.4	β	
178	Lal. 17697	+42	1956	4.1	F5	8 54 9	+42 11	0.52	240.0	Ps.	
179	C. du C. +26° 25798	....		11.2	....	8 59 53	+25 42	0.50	229.8	Turner	
180	Lal. 18067	−14	2757	7.2	....	9 3 46	−14 44	0.54	244.8	Po.	
181	Lal. 18137	+15	2003	7.0	G	9 6 49	+15 24	0.60	294.5	Po.	
182	Lal. 18115^1	+53	1320	8.0	K8	9 7 35	+53 7	1.69	248.5	Po.	β 4972
183	Lal. 18115^2	+53	1321	8.0	K8	9 7 36	+53 7	1.69	248.5	Po.	
184	Lal. 18286	+29	1883	7.0	K3	9 11 57	+29 0	0.52	173.4	Po.	Sp. Bin.
185	Lal. 18397	+40	2197	7.5	K7p	9 16 8	+40 38	0.55	223.5	Po.	
186	Lal. 18672	+ 6	2182	7.7	....	9 24 43	+ 6 5	0.54	282.3	Po.	
187	θ Ursae Majoris	+52	1401	3.0	F8	9 26 10	+52 8	1.11	239.7	Au.	Sp. Bin. β 5123
188	Comp. to θ Ursae Maj.	....		13.7	....	9 26 10	+52 8	1.11	239.7	β	
189	Lal. 18812	+36	1979	5.5	G7	9 29 40	+36 16	0.77	249.4	Po.	
190	Lal. 19022	+43	1953	8.0	K5	9 37 7	+43 10	0.80	180.0	Po.	
191	Lal. 19229	+14	2151	8.2	A3p	9 43 28	+14 14	0.83	156.6	Po.	
192	W.B. 9^{h}954	−11	2741	9.3	....	9 46 10	−11 49	1.95	140.2	Po.	
193	Groom. 1596	+56	1421	8.2	G6	9 54 53	+56 5	0.49	205.0	Po.	
194	Lal. 19545	+32	1964	5.8	G1	9 55 15	+32 25	0.69	230.3	Ps.	
195	Groom. 1618	+50	1725	6.8	K7	10 5 15	+49 58	1.45	248.9	Po.	
196	A. G. Cambridge 3591	+51	1585	6.5	....	10 7 21	+50 59	0.89	271.2	Cambridge	

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
197	A.Oe. 10640	+53° 1395	9.2	......	$10^h\ 7^m 28^s$	+53° 1′	0″.70	174°.3	Rist.	
198	W.B. 10^h234	+20 2465	9.2	......	10 14 11	+20 22	0.51	270.0	Po.	
199	Munich II 5178	− 0 2326	8.8	......	10 15 42	− 0 58	0.70	255.1	Nicolajew	
200	Groom. 1646	+49 1961	6.2	F9	10 21 54	+49 19	0.90	173.0	Po.	
201	W.B. 10^h366	− 5 3071	8.0	......	10 23 13	− 6 5	0.50	232.2	Po.	
202	W.B. 10^h520	−11 2918	5.7	F1	10 31 34	−11 42	0.74	156.9	Po.	
203	Lal. 20881	+21 2247	7.9	......	10 46 7	+20 49	0.50	210.2	Po.	
204	Lal. 21008	+28 1952	8.3	......	10 50 52	+28 17	0.50	254.8	Po.	
205	A. G. Berlin C 1500	+70 639	9.5	......	10 50 59	+70 8	0.66	270.0	Berlin C.	
206	Lal. 21120	−17 3273	4.0	K	10 54 54	−17 46	0.50	286.3	Ps.	
207	Lal. 21185	+36 2147	7.3	Ma	10 57 53	+36 38	4.77	186.4	Po.	
208	Lal. 21258	+44 2051	8.5	Ma	11 0 31	+44 2	4.47	282.8	Po.	
209	Lac. 4606	−29 3437	7.6	G	11 3 9	−29 38	0.51	252.1	Po.	
210	Comp. to Lal. 21368	....	9.5	......	11 5 32	+31 0	0.58	110.3	Po.	β 5695
211	Lal. 21368	+31 2240	8.5	......	11 5 35	+31 0	0.58	110.3	Po.	
212	Anonymous	....	9.5*	......	11 5 54	+ 6 59	0.97	226.4	Wolf	
213	A. G. Washington 4420	−14 3277	9.0	......	11 6 28	−14 26	0.90	130.7	Washington	
214	A. G. Nicolajew 3220	− 1 2505	9.4	......	11 12 11	− 1 26	0.62	268.3	Nicolajew	
215	Lal. 21554	+32 2132	3.7	G	11 12 51	+32 6	0.74	219.5	Au.	Sp. Bin.
216	Comp. to Lal. 21554	....	4.6	G	11 12 51	+32 6	0.74	219.5	Au.	β 5734
217	Lal. 21565	− 4 3049	7.2	G6	11 13 12	− 4 31	0.79	100.9	Po.	
218	A.Oe. 11677	+66 717	9.0	......	11 14 50	+66 23	3.03	274.5	Po.	
219	83^1 Leonis	+ 3 2502	7.5	G8	11 21 42	+ 3 33	0.75	283.1	Po.	β 5779
220	83^2 Leonis	+ 3 2503	8.0	K8	11 21 43	+ 3 33	0.68	283.8	Ps.	
221	Anonymous	....	9.7	......	11 23 20	+ 8 6	1.23	190.4	β	
222	Br. 1584	−32 3122	6.7	G	11 29 38	−32 18	1.08	321.0	Po.	
223	Lal. 22069^2	....	8.6	......	11 33 28	+45 40	0.63	272.7	Po.	β 5858
224	Lal. 22069^1	+45 1947	6.5	F9	11 33 29	+45 40	0.63	272.7	Po.	
225	Cp. St. St. 4223	−43 5524	8.3	......	11 36 14	−43 51	0.72	284.8	Cape	
226	A. G. Chris. 1806	+68 658	8.8	......	11 36 48	+68 44	0.67	270.0	Schulhof†	

† Schulhof marks the P.M. as uncertain, *loc. cit.*

227	Groom. 1822	+48°	1964	7.8	G0	$11^h40^m18^s$	+48° 14′	0″.66	245°.0	Po.	
228	Lac. 4887	−39	5265	5.8	G	11 41 45	−39 57	1.58	284.3	Po.	
229	β Leonis	+15	2383	2.0	A2	11 43 58	+15 8	0.53	259.1	Au.	
230	β Virginis	+ 2	2489	3.3	F8	11 45 29	+ 2 20	0.77	109.9	Au.	
231	Groom. 1830	+38	2285	6.5	G5	11 47 13	+38 26	7.06	145.3	Po.	
232	W.B. 11^h920	+20	2658	8.2	....	11 49 27	+19 58	0.50	272.3	Berlin B	β 5960
233	W.B. 11^h921	+20	2659	8.2	....	11 49 30	+19 59	0.50	272.3	β	
234	Lac. 4955	−26	4626	7.5	....	11 52 59	−27 8	1.21	234.8	Po.	
235	Lal. 22585	− 9	3413	6.4	G6	11 55 36	− 9 53	0.50	175.4	Po.	
236	Lal. 22632	+43	2182	6.7	G6	11 57 25	+43 39	0.67	213.5	Po.	
237	Lal. 22667	+ 4	2568	9.0	....	11 58 58	+ 3 55	0.55	173.8	Po.	
238	Lal. 22701	− 0	2532	8.4	....	12 0 9	− 0 57	0.51	270.0	Po.	
239	W.B. 12^h69	− 2	3481	7.3	G4	12 7 25	− 2 32	0.73	304.4	Po.	
240	Lal. 22901	+10	2391	8.1	....	12 7 50	+10 36	0.52	35.0	Ps.	
241	Lal. 22908	+11	2439	8.0	G3	12 8 7	+11 24	0.59	177.1	Po.	
242	Lal. 22954	− 9	3468	6.2	F6	12 10 2	− 9 44	1.03	176.1	Po.	
243	B.D. +29° 2279	+29	2279	9.5	....	12 14 28	+28 56	0.66	291.7	Wolf	
244	W.B. 12^h309	+42	2296	8.8	....	12 16 56	+42 42	0.65	149.8	Rist.	
245	Anonymous	......	......	10.7	....	12 17 25	+25 44	0.79	293.0	Wolf	
246	Lac. 5123	−66	1752	7.6	K	12 17 51	−67 5	0.82	281.3	Po.	
247	A.Oe. 12584	+74	493	8.0	....	12 17 54	+73 48	0.50	248.4	Greenwich	
248	A. G. Lund 5422	+39	2519	7.5	....	12 19 55	+38 52	0.61	272.9	Lund	
249	Lal. 23243	+32	2244	8.9	....	12 20 22	+32 25	0.50	239.2	Po.	
250	Cp. St. St. 4496	−48	4799	7.6	G	12 21 31	−48 21	0.63	261.7	Cape	
251	Lal. 23361	− 2	3528	8.5	....	12 24 37	− 2 46	0.69	205.8	Po.	
252	Lal. 23498	+42	2321	4.5	G	12 29 0	+41 54	0.78	291.8	Ps.	
253	Gi.Z. 8755	−77	859	9.3	....	12 33 33	−77 18	0.83	294.0	Rist.	
254	γ Virginis^1	− 0	2601	2.8	F	12 36 36	− 0 54	0.57	272.0	Ps.	β 6243
255	γ Virginis^2	......	......	2.8	F	12 36 36	− 0 54	0.57	272.0	Ps.	
256	C.P.D. −37° 5339	−37	5339	7.9	....	12 38 24	−37 9	0.69	250.9	Kapteyn	
257	Lal. 23808	+10	2468	6.4	K1	12 41 18	+10 6	0.52	149.0	Ps.	
258	Lal. 23917	+ 2	2585	8.2	....	12 44 39	+ 1 45	0.68	182.5	Po.	
259	Lal. 23995	−17	3723	8.2	....	12 47 56	−17 57	0.89	159.5	Po.	
260	δ Virginis	+ 4	2669	3.0	Ma	12 50 34	+ 3 56	0.50	264.3	Po.	
261	Lal. 24168	− 9	3595	7.7	G9	12 53 54	− 9 18	0.82	282.0	Po.	

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
262	A. G. Nicolajew 3508	− 1° 2754	9.1	......	$12^h55^m14^s$	− 2° 10′	0″.90	270°.6	Nicolajew	
263	A.Oe. 12585	−26 4871	7.9	......	12 56 5	−26 50	0.54	240.1	Po.	
264	W.B. 12^h920	− 7 3525	8.5	......	12 56 10	− 7 54	0.54	256.0	Po.	
265	Cp. St. St. 4746	−40 6050	9.2	......	13 2 58	−41 6	0.69	268.1	Cape	
266	Lal. 24414	+ 6 2697	6.8	G3	13 3 47	+ 5 46	0.72	175.2	Po.	
267	A. G. Washington 5002	−15 3605	9.1	......	13 3 50	−16 13	0.50	180.0	Washington	Only μ δ
268	Lal. 24504	+10 2519	8.5	......	13 6 25	+10 9	0.54	297.5	Po.	
269	A.Oe. 13407	+68 714	8.5	......	13 7 2	+68 2	0.74	272.0	Greenwich	
270	Lal. 24522	+28 2193	5.0	G	13 7 12	+28 23	1.20	318.7	Po.	
271	A. G. Berlin A 4855	+18 2700	7.8	......	13 7 31	+18 3	0.60	273.4	Berlin A	
272	Lal. 24652	+17 2611	7.0	......	13 11 52	+17 35	0.70	113.6	Krüger	
273	Comp. to Lal. 24652	....	10.1	......	13 11 52	+17 35	0.70	113.6	Krüger	β 6442
274	Lal. 24680	−17 3813	5.0	K	13 13 10	−17 45	1.51	225.5	Po.	
275	W.B. 13^h241	+35 2436	9.0	......	13 14 55	+35 39	0.92	154.3	Po.	
276	Comp. to W.B. 13^h241	....	11.5	......	13 14 56	+35 39	0.92	154.3	β	β 6452
277	W.B. 13^h216	+ 4 2729	8.5	......	13 15 43	+ 4 39	0.58	292.2	Po.	
278	Anonymous	....	11.0*	......	13 18 31	−13 31	0.61	232.8	Wolf	
279	Lal. 24838	+29 2405	8.6	......	13 18 55	+29 45	0.54	299.1	Po.	
280	Lal. 24915	− 0 2691	8.0	......	13 23 9	− 0 19	0.50	149.3	Nicolajew	
281	Lal. 24926	+14 2621	5.5	F	13 23 32	+14 19	0.64	204.2	Po.	
282	B.D. −7° 3632	− 7 3632	9.5	......	13 25 7	− 8 3	1.20	250.7	Wolf	
283	Lal. 25012	− 1 2832	7.5	G6	13 26 36	− 1 49	0.91	284.7	Po.	
284	ι Centauri	−32 3479	4.9	F5	13 40 0	−32 32	0.51	251.6	Po.	
285	A. G. Berlin A 4999	+18 2776	9.2	......	13 40 13	+18 21	1.92	169.6	Berlin A	
286	Lal. 25372	+15 2620	8.5	Ma5	13 40 40	+15 26	2.32	129.2	Po.	
287	Lal. 25404	+ 7 2690	6.5	F5	13 42 0	+ 6 51	0.50	254.8	Po.	
288	Lal. 25426	+18 2782	4.6	F5	13 42 31	+17 57	0.50	274.7	Po.	
289	Lal. 25484	−23 5802	6.8	F8	13 45 50	−23 53	0.68	238.9	Po.	
290	Cp. St. St. 5010	−50 6387	7.8	......	13 46 20	−50 26	0.65	266.8	Cape	
291	C. du C. +25° 86067	....	10.4*	......	13 54 49	+25 44	0.54	262.4	Bellamy	
292	θ Centauri	−35 6109	3.4	K	14 0 48	−35 53	0.78	225.0	Po.	

293	α Boötis	+19° 2777	1.0	K	$14^h 11^m 6^s$	+19° 42′	2″.28	209°.6	Au.	
294	C.P.D. −58° 5467	−58 5467	8.0		14 12 0	−58 54	0.93	214.5	Kapteyn	
295	Lal. 26196	− 4 3665	7.6	K1	14 14 25	− 4 41	0.65	259.4	Po.	
296	Lal. 26294	+30 2512	8.3		14 17 40	+30 6	0.56	296.3	Ps.	
297	Lal. 26289	+ 1 2920	6.5	G0	14 18 8	+ 1 43	0.55	156.3	Po.	
298	A. G. Berlin B 5072	+24 2733	9.0		14 21 5	+24 6	1.38	143.5	Po.	β 6869
299	A. G. Berlin B 5073		9.0		14 21 9	+24 6	1.38	143.5	Po.	
300	C. du C. +25° 42937	+24 2735	9.5		14 23 17	+24 17	0.50	273.1	Bellamy	
301	B.D. −7° 3856	− 7 3856	9.0		14 25 38	− 8 12	1.23	260.0	Hayden	
302	B.D. +10° 2703	+10 2703	8.4		14 28 40	+ 9 47	0.53	162.1	Po.	
303	A. G. Leiden 5231	+34 2541	9.0		14 30 54	+34 10	0.74	342.7	Kapteyn	
304	Lal. 26630	−11 3770	6.0	F5	14 31 40	−11 53	0.96	292.0	Po.	
305	α^2 Centauri	−60 5483	2.1	G	14 32 48	−60 25	3.68	281.9	Po.	
306	α^1 Centauri		3.0*	K5	14 32 49	−60 25	3.68	281.9	St.	
307	W.B. 14^h847	+17 2785	8.8		14 41 43	+16 56	0.94	185.4	Kobold	
308	W.B. 14^h810	+ 7 2850	9.1		14 45 29	+ 7 14	0.62	263.6	Po.	
309	Lal. 27026	−23 5999	8.0		14 46 0	−23 53	1.02	241.9	Po.	
310	Lal. 27137	+19 2881	6.3	K	14 48 51	+19 33	0.53	242.4	Ps.	
311	Lal. 27155	+23 2751	8.5		14 49 20	+23 45	0.75	275.0	Ps.	
312	Yarnall 6246	−20 4123	8.7		14 51 36	−20 58	2.01	151.4	Gr.	β 7060
313	Lal. 27173	−20 4125	6.0	K6	14 51 37	−20 58	2.06	150.7	Po.	
314	Lal. 27298	+54 1716	7.7	K0	14 52 21	+54 4	1.08	296.3	Po.	
315	Lal. 27274	−21 4009	8.3		14 54 9	−21 36	0.80	227.6	Po.	
316	W.B. 14^h989	−10 4011	9.3		14 55 18	−10 44	0.54	180.0	Po.	
317	Cp. St. St. 5447	−48 7086	8.7		14 59 14	−48 53	0.55	186.7	Cape	
318	Lal. 27531	− 7 3963	8.0		15 2 53	− 7 31	0.50	198.8	Po.	
319	Lal. 27558	+ 9 3001	8.7		15 2 58	+ 9 16	0.54	258.2	Ps.	
320	B.D. +25° 2874	+25 2874	9.2		15 3 8	+25 18	0.92	298.5	Po.	
321	A.Oe. 14318	−15 4041	9.2		15 4 44	−15 59	3.76	194.8	Po.	
322	A.Oe. 14320	−15 4042	9.0	G0	15 4 45	−15 54	3.75	194.7	Po.	
323	Lal. 27742	+19 2939	6.7	G4	15 8 15	+19 39	0.68	296.2	Po.	β 7162
324	Lal. 27743		7.6	G6	15 8 15	+19 40	0.62	295.3	Ps.	
325	Lal. 27744	− 0 2944	7.0	G7	15 8 50	− 0 58	1.38	247.4	Po.	
326	Lal. 27917	+ 2 2944	5.0	G	15 14 12	+ 2 9	0.65	146.3	Ps.	β 7213
327	Comp. to Lal. 27917		10.0		15 14 12	+ 2 9	0.65	146.3	β	

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
328	Lal. 27958	+26° 2677	8.0	G2	15h 14m 45s	+26° 4′	0″.51	257°.6	Po.	
329	Cp. St. St. 5545	−47 7075	7.2	G	15 15 4	−47 57	1.67	260.8	Cape	
330	Comp. to A. G. Leiden 5456	....	9.4	....	15 16 5	+31 3	1.04	113.6	Wilkens	β 7226
331	A. G. Leiden 5456	+31 2722	9.2	....	15 16 6	+31 3	1.04	113.6	Wilkens	
332	W.B. 15h268	+ 1 3071	8.7	....	15 17 40	+ 1 47	0.52	230.5	Po.	
333	W.B. 15h716	+40 2903	7.8	K6	15 32 23	+40 10	0.50	278.1	β	
334	W.B. 15h720	+40 2905	6.8	K1	15 32 29	+40 8	0.50	278.1	Po.	β 7332
335	Comp. to W.B. 15h720	....	7.1	....	15 32 29	+40 8	0.50	278.1	Po.	
336	Lal. 28607	−10 4149	7.0	A2p	15 37 42	−10 36	1.18	253.3	Po.	
337	Lac. 6521	−37 6571	6.9	G	15 40 59	−37 36	0.59	240.4	Po.	
338	Lal. 28935	+13 3024	6.3	G	15 48 33	+13 31	0.61	197.2	Po.	
339	Lal. 28992	+42 2648	4.3	F	15 49 13	+42 44	0.75	35.2	Po.	
340	Cp. St. St. 5767	−47 7499	9.0	....	15 50 15	−47 21	0.86	262.9	Cape	
341	γ Serpentis	+16 2849	3.8	F8	15 51 50	+15 59	1.32	168.2	Au.	
342	W.B. 15h1323	+28 2503	8.0	....	15 54 32	+28 1	0.85	291.5	Bur. des Long.1913	
343	Lal. 29085	−16 4196	6.2	F	15 54 43	−16 14	0.77	239.4	Po.	
344	Lal. 29213	+33 2663	5.7	F	15 57 13	+33 36	0.81	194.2	Po.	
345	Lal. 29307	+25 3020	7.5	....	15 59 56	+25 31	0.87	211.1	Ps.	
346	Lal. 29330	+11 2910	8.5	....	16 1 12	+10 57	0.50	256.2	Po.	
347	Lal. 29381	+39 2947	6.5	G5	16 1 30	+39 26	0.55	275.2	Po.	
348	Lal. 29439	+39 2950	8.4	....	16 2 54	+38 55	0.60	159.4	Po.	
349	Lal. 29437	+ 6 3169	6.5	K1	16 4 16	+ 6 40	0.79	162.3	Po.	
350	Lal. 29617	− 7 4242	5.9	G1	16 10 11	− 8 6	0.55	157.7	Po.	
351	Lal. 29917	+67 935	8.6	....	16 16 32	+67 29	0.50	281.9	Po.	
352	Comp. to Lal. 29917	....	10.6	....	16 16 34	+67 30	0.50	281.9	β	
353	Anonymous	....	10.7*	....	16 21 7	+48 36	1.24	111.4	Kapteyn	
354	W.B. 16h400	+ 3 3203	8.9	....	16 23 36	+ 3 29	0.51	180.0	Po.	
355	Lal. 30024¹	+18 3182	7.0	K1	16 24 29	+18 37	0.53	319.7	Po.	β 7642
356	Lal. 30024²	....	7.0	....	16 24 29	+18 37	0.53	319.7	β	
357	Cp. St. St. 5969	−40 7356	9.5	....	16 25 32	−40 6	1.06	317.1	Cape	

358	Lal. 30044	+4° 3195	7.6	F6	$16^h25^m33^s$	+4°26′	1″.46	197°.9	Po.	
359	Lal. 30191	−1 3220	5.8	F	16 31 6	−2 7	0.55	126.1	Boss	
360	Lal. 30271	+31 2873	7.2	F2	16 32 34	+31 9	0.49	177.2	Po.	
361	W.B. 16^h964	+31 2875	9.1	....	16 32 55	+31 19	0.60	141.0	Rist.	
362	Lal. 30433	+31 2884	3.0	G	16 37 31	+31 47	0.62	311.7	Au.	Sp. Bin.
363	Comp. to Lal. 30433	....	6.5	....	16 37 31	+31 47	0.62	311.7	Au.	β 7717
364	A. G. Chris. 2517	+66 969	9.0	....	16 38 44	+66 7	0.64	269.7	Schulhof†	
365	Lal. 30699	+68 883	7.6	G7	16 42 51	+68 16	0.52	235.3	Greenwich	
366	ε Scorpii	−34 6635	5.0	K	16 43 41	−34 7	0.68	245.7	Po.	
367	Lal. 30694	+0 3593	7.0	G6	16 47 56	+0 11	1.66	206.1	Po.	
368	W.B. 16^h906	−8 4352	8.8	Mb	16 50 8	−8 9	1.27	226.8	Po.	
369	C. du C. +25° 48511	+25 3173	9.4	....	16 54 4	+25 55	0.52	188.0	Bellamy	
370	Lal. 31132	+47 2420	6.6	G5	16 59 47	+47 12	0.83	9.0	Po.	
371	Lal. 31055	−4 4225	7.5	K5	16 59 51	−4 54	1.48	218.9	Po.	
372	C.P.D. −75° 1368	−75 1368	8.0	....	17 9 0	−75 14	1.03	265.3	Kapteyn	
373	Lal. 31311^1	−26 5858	5.5	K	17 9 12	−26 27	1.29	203.8	Po.	β 7905
374	Lal. 31311^2	....	5.5	K	17 9 12	−26 27	1.29	203.8	β	
375	B.D. +42° 2810	+42 2810	9.4	....	17 10 0	+42 28	1.11	250.5	Kostinsky	
376	Br. 2179	−26 5863	8.0	K3	17 10 4	−26 24	1.25	204.1	Po.	
377	Cp. St. St. 6241	−46 8513	7.3	K	17 11 28	−46 32	0.95	72.1	Cape	
378	Lac. 7215	−34 6803	7.2	K5	17 12 9	−34 53	1.20	100.1	Po.	
379	Lal. 31618	+32 2896	5.7	Go	17 16 55	+32 36	1.06	172.9	Po.	
380	Cp. St. St. 6277	−41 7979	8.2	....	17 17 47	−41 24	0.58	82.9	Cape	
381	W.B. 17^h322	+2 3312	8.0	K9	17 20 47	+2 14	1.36	206.2	Po.	
382	A. G. Leiden 6197	+31 3025	9.1	....	17 22 57	+31 8	0.52	285.6	Wilkens	
383	Lal. 32047	+67 1014	6.5	K1	17 25 19	+67 23	0.52	273.3	Po.	
384	A. G. Chris. 2661	+68 930	8.8	....	17 26 14	+68 27	0.80	88.0	Schulhof†	
385	W.B. 17^h514	+6 3455	8.5	F1	17 29 53	+6 4	0.58	308.7	Po.	
386	Anonymous	....	9.1*	....	17 33 27	+18 37	1.29	40.0	Slocum	
387	Lal. 32324	+61 1678	6.0	F	17 33 57	+61 57	0.56	153.4	Po.	β 8099
388	Comp. to Lal. 32324	....	10.6	....	17 33 57	+61 57	0.56	153.4	β	
389	B.D. +18° 3424	+18 3424	9.2	....	17 34 20	+18 37	1.20	311.6	Berlin A‡	
390	A. G. Chris. 2703	+68 941	8.3	....	17 35 47	+68 52	0.83	279.2	Schulhof	

† Schulhof marks the P.M. as uncertain, *loc. cit.*

‡ Berlin A has a small weight for the P.M. of this star. Slocum gives $\mu=0''.09$, $p=144^\circ.7$, *Astrophysical Journal*, **38**, 15, 1913

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
391	Lal. 32322	+37° 2926	8.4	....	17ʰ36ᵐ15ˢ	+37°16′	0″.88	204°.2	Ps.	
392	A.Oe. 17415	+68 946	9.1	Mb	17 37 0	+68 26	1.30	196.5	Po.	
393	Lal. 32390	+21 3198	7.5	....	17 39 2	+21 40	0.61	175.3	Ps.	
394	Comp. B. to Lal. 32519	....	10.5	....	17 42 31	+27 47	0.81	203.4	β	β 8162
395	Comp. C. to Lal. 32519	....	11.0	Ma	17 42 31	+27 47	0.81	203.4	β	
396	Lal. 32519	+27 2888	3.5	G5	17 42 33	+27 47	0.81	203.4	Po.	
397	B.D. +26° 3151	+26 3151	7.3	....	17 58 26	+26 20	0.66	148.0	Po.	
398	Lal. 33169	+ 2 3482	4.0	K	18 0 24	+ 2 31	1.15	167.4	Po.	
399	Comp. to Lal. 33169	....	6.0	....	18 0 24	+ 2 31	1.15	167.4	Po.	Sp. Bin.
400	A. G. Washington 6520	−16 4706	9.1	....	18 0 31	−16 42	0.55	90.0	Washington	β 8340
401	Lal. 33439	+38 3095	6.7	K2	18 6 19	+38 27	0.62	217.5	Rist.	Only μ α
402	Lal. 33802	− 2 4599	3	K	18 16 8	− 2 55	0.91	221.4	Au.	
403	B.D. +8° 3689	+ 8 3689	8.0	G1	18 21 25	+ 8 44	0.49	200.4	Kapteyn	
404	B.D. +8° 3692	+ 8 3692	8.5	G5	18 21 37	+ 8 34	0.50	202.7	Kapteyn	
405	Lal. 34392	+72 839	3.8	F8	18 22 52	+72 41	0.63	125.9	Au.	
406	A. G. Leiden 6797	+31 3330	8.7	....	18 37 7	+31 28	0.82	174.8	Wilkens	
407	Pos. Med. 2164¹	+59 1915	8.9	Mb	18 41 40	+59 29	2.28	325.0	Po.	β 8798
408	Pos. Med. 2164²	....	9.4	Ma	18 41 41	+59 28	2.28	325.0	Po.	
409	Lal. 34986	+10 3665	8.1	K5	18 43 46	+10 39	0.51	168.3	Comstock	
410	W.B. 18ʰ1295	+17 3729	9.0	....	18 44 29	+17 20	0.60	221.5	Po.	
411	Munich I 18180	+ 5 3993	9.3	Ma	18 53 7	+ 5 48	1.24	191.1	Po.	
412	A. G. Berlin A 7153	+18 3911	9.5	....	18 56 23	+18 57	0.58	209.7	Berlin A	
413	L.Bo. 2844	−20 5385	9.0	....	18 58 3	−20 35	0.72	204.6	Rist.	
414	Munich I 18816	+ 7 3967	9.2	K5:	19 2 14	+ 7 29	0.80	199.8	Po.	
415	Comp. C. to 17 Lyrae	....	11.3*	Ma	19 3 40	+32 21	1.75	49.5	β	
416	A.Oe. 19156	−21 5273	8.5	....	19 3 43	−21 37	0.50	205.0	Po.	
417	B.D. −0° 3676	− 0 3676	9.1	....	19 8 14	− 0 45	0.62	217.2	Roy	
418	Lal. 36245	+49 2959	6.2	G3	19 9 30	+49 40	0.62	348.0	Po.	β 9137
419	Lal. 36249	....	6.7	G2	19 9 31	+49 40	0.62	348.0	Po.	
420	Lal. 36647	+11 3833	5.5	G7	19 20 12	+11 44	0.96	48.8	Au.	
421	Lal. 36717	+24 3737	6.1	F4	19 21 17	+24 44	0.66	197.6	Au.	

422	Lal. 36872	−28°	6957	7.8	....	$19^h26^m26^s$	−28° 13′	0″.76	177°.7	Po.	
423	Groom. 2875	+58	1929	6.4	K1	19 29 28	+58 23	0.66	232.6	Po.	
424	Lal. 37120	+32	3474	7.0	G2	19 29 44	+32 59	0.52	296.1	Po.	
425	σ Draconis	+69	1053	5.0	G5	19 32 33	+69 29	1.83	163.8	Po.	
426	α Aquilae	+ 8	4236	1.2	A5	19 45 54	+ 8 36	0.64	53.3	Au.	
427	Lal. 38100	−10	5238	6.3	F7	19 54 21	−10 13	0.51	217.1	Po.	
428	Lal. 38139	−12	5613	7.3	....	19 55 30	−12 31	0.50	225.0	Po.	
429	Lac. 8267	−67	3703	7.1	G5	19 55 34	−67 35	1.25	122.3	Po.	
430	Lal. 38287	+15	4026	7.0	G6	19 58 0	+15 20	0.60	198.4	Po.	
431	δ Pavonis	−66	3474	4.3	G5	19 58 55	−66 26	1.65	135.5	Po.	
432	Lal. 38380	+29	3872	6.0	K0	19 59 31	+29 38	0.86	127.4	Po.	
433	15 Sagittae	+16	4121	5.5	G0	19 59 37	+16 48	0.58	223.6	Po.	
434	Lal. 38383	+22	3908	7.0	K1	19 59 41	+23 5	1.39	227.4	Po.	
435	b^1 Cygni	+35	3959	5.5	K	20 2 39	+35 42	0.50	207.8	Boss	
436	Lac. 8362	−36	9037	6.6	K5	20 4 38	−36 21	1.70	164.7	Po.	
437	Lal. 38683	+15	4074	7.0	G9	20 6 34	+15 53	0.55	310.6	Po.	
438	Cp. St. St. 7242	−45	9899	8.8	....	20 6 45	−45 28	0.75	104.6	Cape	
439	Lal. 38711	−27	6972	7.2	G9	20 9 3	−27 20	1.28	100.8	Po.	
440	B.D. +76° 785	+76	785	9.0	....	20 13 51	+76 55	0.53	168.3	Greenwich	
441	A. G. Cambridge 6486	....		7.4	....	20 15 56	+55 5	0.81	270.0	Cambridge	β 10135
442	A. G. Cambridge 6487	+54	2329	6.0	A	20 15 57	+55 5	0.81	270.0	Cambridge	
443	Groom. 3150	+66	1281	6.2	F8	20 16 32	+66 32	0.57	57.2	Po.	
444	A.Oe. 20452	−21	5703	8.2	F5	20 17 42	−21 40	1.22	154.7	Po.	
445	Lal. 39704	+41	3799	6.7	G6	20 29 20	+41 33	0.50	337.9	Bonn	
446	B.D. +5° 4556	+ 5	4556	8.7	....	20 30 13	+ 5 47	0.51	135.3	β	
447	Lac. 8490	−60	7419	6.5	F8	20 31 46	−60 53	0.56	150.2	St.	
448	W.B. 20^h802	+ 5	4570	9.0	....	20 33 40	+ 5 18	0.90	261.6	Sj.	
449	Lal. 39816	−24	7050	6.9	G	20 34 15	−24 8	0.62	48.9	Po.	
450	Lal. 39866	+ 4	4510	8.4	K5	20 34 33	+ 4 37	0.85	86.6	Po.	
451	Lal. 39854	−19	5889	8.2	....	20 35 6	−19 8	0.50	181.1	Po.	
452	Fed. 3562	+75	752	7.2	....	20 38 42	+75 14	0.64	33.0	Po.	
453	ε Cygni	+33	4018	2.6	K	20 42 10	+33 36	0.51	48.4	β	Sp. Bin.
454	η Cephei	+61	2050	3.7	K0	20 43 15	+61 27	0.82	6.3	Au.	
455	Lac. 8620	−44	9918	6.8	G	20 51 3	−44 29	1.12	208.1	Po.	
456	B.D. +61° 2068	+61	2068	8.4	....	20 51 16	+61 48	0.80	192.9	β	

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
457	Fed. 3638	+74° 889	7.8	G4	$20^{h}52^{m}22^{s}$	+74° 23′	0″.70	38°.0	Po.	
458	W.Mu.Z. 53.22	−34 8843	8.4		20 55 56	−34 27	1.31	172.6	Rist.	
459	Lac. 8625	−73 2192	6.6	F8	20 58 54	−73 34	0.60	129.6	Po.	
460	W.B. $20^{h}1454$	+ 2 4295	8.2	F3	20 59 6	+ 2 36	0.55	222.0	Po.	
461	Lal. 40844	+ 6 4741	8.8	K6	21 0 23	+ 6 41	0.57	177.0	Po.	
462	61^{1} Cygni	+38 4343	5.0	K8	21 2 25	+38 15	5.18	51.4	Au.	β 10732
463	61^{2} Cygni	+38 4344	5.3	K8	21 2 26	+38 15	5.12	53.7	Au.	
464	W.B. $21^{h}97$	+17 4519	7.3	F5	21 7 21	+17 21	0.91	188.2	Po.	
465	Fed. 3738	+73 925	8.6		21 8 48	+73 18	0.52	222.0	Küstner	
466	Lac. 8733	−61 6537	7.4		21 10 47	−61 46	0.67	130.7	Po.	
467	Lac. 8760	−39 8920	7.4		21 11 24	−39 15	3.46	249.4	Po.	
468	Lal. 41348	− 0 4195	8.5		21 12 52	− 0 15	0.50	112.4	β	
469	Lal. 41363	−26 7193	7.4	G5	21 13 59	−26 46	0.67	237.7	Po.	β 10881
470	Comp. to Lal. 41363		9.9		21 13 59	−26 46	0.67	237.7	Po.	
471	A.Oe. 21308	−20 6185	9.0		21 14 36	−20 15	0.80	192.3	Po.	
472	γ Pavonis	−65 3918	4.1	F8	21 18 11	−65 49	0.80	5.7	Po.	
473	W.B. $21^{h}502$	−13 5945	9.1		21 24 30	−12 56	1.06	105.3	Po.	
474	W.B. $21^{h}594$	+45 3561	7.5		21 26 0	+45 27	0.51	46.0	Rist.	
475	Cp. St. St. 7704	−50 11576	8.0		21 28 14	−50 14	0.62	259.8	Cape	
476	Lal. 42128	− 2 5588	8.7		21 33 2	− 2 45	0.50	303.0	Bossert	
477	A. G. Berlin B 8366	+24 4460	8.8		21 39 42	+24 53	0.68	213.3	Bellamy	
478	Lal. 42883	+29 4550	7.4	F7	21 54 15	+29 21	0.56	222.8	Po.	
479	ε Indi	−57 10015	6.7	K5	21 55 43	−57 12	4.71	123.9	Po.	
480	Lal. 43065	+ 4 4800	5.0	Kp	22 0 38	+ 4 34	0.50	263.6	Ps.†	
481	Lal. 43205	+52 3112	7.9	G8	22 3 5	+52 39	0.61	235.0	Po.	
482	W.B. $22^{h}82$	+22 4567	8.8		22 5 55	+22 15	0.60	264.2	Po.	
483	Lac. 9061	−41 9759	6.4	G	22 8 32	−41 51	0.92	142.5	Po.	
484	Lac. 9076	−54 10055	6.2	G	22 11 42	−54 7	0.80	147.3	Po.	
485	Lal. 43492	+12 4797	7.0	F9	22 12 15	+12 24	0.84	83.9	Po.	
486	ν Indi	−72 2690	6.0	F	22 16 2	−72 45	1.45	120.6	Po.	

† The P.M. given in Paris IV seems well founded. Still Bossert gives $\mu=0''.13$, $p=34°.1$, Boss $\mu=0''.14$, $p=52°.0$.

487	Cp. St. St. 7975	−51° 11806	8.6		$22^h16^m17^s$	−51° 17′	1″.58	178°.9	Cape	
488	C.P.D. −30° 6641	−30 6641	7.9		22 23 35	−30 30	0.83	163.7	Union Obs.	
489	Krüger 60	+56 2783	9.0	Mb	22 24 28	+57 12	0.94	245.2	β	β 11761
490	Comp. B. to Krüger 60		12.0		22 24 28	+57 12	0.94	245.2	β	
491	B.D. +53° 2911	+53 2911	9.5		22 28 43	+53 16	1.23	83.1	Balanowsky	
492	Kli. 6256	+ 4 4879	8.3		22 30 32	+ 4 52	0.54	140.0	Rist.	
493	Münich I 31343	+ 1 4637	9.4		22 33 52	+ 2 0	0.55	99.5	Po.	
494	Lal. 44552	+11 4875	4.8	F5	22 41 42	+11 40	0.54	155.1	Po.	β 11957
495	Comp. to Lal. 44552		12.8		22 41 43	+11 40	0.54	155.1	β	
496	Cp. St. St. 8141	−42 9492	7.9		22 44 29	−42 1	0.51	240.0	Cape	
497	Lal. 44739	+ 9 5122	5.0	F	22 47 20	+ 9 18	0.50	85.4	Po.	
498	Lal. 44964	−23 8259	8.1		22 55 0	−23 4	0.92	270.4	Po.	
499	B.D. +68° 1345	+68 1345	8.3		22 55 9	+68 29	0.63	118.1	Greenwich	
500	Lal. 45028	− 4 5804	7.8	K2	22 56 39	− 4 23	0.54	122.8	Po.	
501	Lac. 9352	−36 9694	8.2	K	22 59 25	−36 26	7.00	80.5	Po.	
502	Fed. 4371	+67 1498	7.5	G3	23 1 10	+67 52	0.60	75.5	Po.	
503	Lal. 45292	− 3 5577	8.2	K5:	23 3 58	− 2 48	0.59	102.6	Po.	
504	Lac. 9396	−63 4862	7.1	G	23 7 57	−63 14	0.63	129.2	Po.	
505	Br. 3077	+56 2966	6.0	K7	23 8 28	+56 37	2.08	82.3	Au.	
506	Lal. 45455	− 9 6149	8.2	F0	23 8 52	− 9 28	0.56	98.3	Po.	β 12234
507	Lal. 45456	− 9 6150	9.0		23 8 52	− 9 29	0.55	95.2	Po.	
508	W.B. 23^h175	−14 6437	8.2	A2	23 11 54	−14 22	1.31	202.9	Po.	
509	Lal. 45565	+ 2 4648	3.8	K	23 11 59	+ 2 44	0.73	88.4	Au.	
510	Lal. 45638	+ 4 4994	8.5	G8	23 13 46	+ 4 52	0.52	103.6	Ps.	β 12290
511	Comp. to Lal. 45638		9.4		23 13 46	+ 4 52	0.52	103.6	β	
512	W.B. 23^h257	+28 4562	8.5		23 14 59	+28 19	0.57	94.0	Po.	
513	Lal. 45755	+43 4445	7.3	G8	23 16 46	+43 33	0.67	70.8	Po.	
514	A. G. Cambridge 8232	+52 3440	7.0		23 20 57	+52 26	1.59	90.6	Cambridge	
515	A.Oe. 25685	+58 2605	7.5	K1	23 26 30	+58 37	1.08	85.2	Rist.	
516	d'Agelet 6348	+17 4946	8.0		23 31 0	+17 53	0.68	73.8	Po.	
517	Lac. 9537	−73 2299	7.4		23 33 42	−73 15	0.74	170.7	Po.	
518	Lal. 46351	+ 4 5035	4.2	F5	23 34 48	+ 5 5	0.56	141.5	Au.	
519	Lal. 46495	+57 2787	7.5	F8	23 38 32	+57 31	0.62	37.8	Po.	
520	W.B. 23^h819	+28 4634	8.6		23 40 0	+29 0	0.85	86.7	Po.	
521	Lac. 9585	−42 9607	7.3		23 41 15	−42 7	0.86	172.6	Po.	

TABLE I—*Continued*

No.	Name	*B.D.* or *C.P.D.*	Mag.	Spec.	α 1900	δ 1900	μ	φ	Authority	Remarks
522	Cp. St. St. 8461	−48° 10958	7.4	....	$23^h42^m\ 7^s$	−48° 50′	0″.50	242°.1	Cape	
523	Lal. 46650	+ 1 4774	8.7	Ma	23 43 59	+ 1 52	1.39	135.9	Po.	
524	Munich I 32805	+ 2 4723	8.3	G7	23 44 56	+ 2 19	0.50	62.8	Po.	
525	Lal. 46867	+27 4642	7.0	....	23 49 55	+28 5	0.56	88.0	Po.	
526	Lal. 46972	+49 4291	6.7	....	23 53 3	+49 53	0.61	293.7	Bonn†	
527	B.D. +45° 4378	+45 4378	9.2	....	23 53 32	+46 10	0.53	93.4	Bonn	
528	A.Oe. 23166	−20 6684	7.3	....	23 54 17	−20 35	0.63	116.6	Po.	
529	Lal. 47118	+26 4734	6.1	G	23 56 57	+26 33	1.29	140.1	Po.	} β 12701
530	Comp. to Lal. 47118	....	12.6	....	23 56 57	+26 33	1.29	140.1	β	
531	Gould 32416	−37 9435	8.8	....	23 59 31	−37 51	6.29	114.2	Po.	
532	Lal. 47207	+33 4828	6.3	Go	23 59 39	+34 6	0.78	84.1	Po.	
533	Anonymous	....	9.5*	....	23 59 54	+45 14	0.86	100.0	Furuhjelm	

† Porter gives for this star $\mu = 0''.26$, $p = 349°.1$, *Cin. Publ.*, **13**, 1893.

After this paper had been sent to the printer, *Circular No. 19 of the Union Observatory* appeared with the list by Innes of proper motion stars south of −19°. The following numbers of Innes' list should be included here, although for some of the stars the proper motion needs confirmation:

1[a] 33 42 73 74 80 99 137 173
177 224 227 296 357 397 415 416 417
427 429 430 452 542[a] 548[a] 638 653 690
744 748

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PHOTOGRAPHIC AND PHOTO-VISUAL MAGNITUDES OF STARS NEAR THE NORTH POLE

By FREDERICK H. SEARES

I. INTRODUCTION

The results here presented summarize an extended investigation of the photographic and photo-visual magnitude scales made with the 60-inch reflector, which is soon to be published in detail as Vol. 3 of *Papers from the Mount Wilson Solar Observatory.*[1]

The investigation was begun in 1910, but various difficulties encountered in devising satisfactory methods of observation and reduction had first to be overcome, and it was not until 1912 that preliminary results were obtained. These were photographic magnitudes for the interval 10.5–17.5,[2] and, for the most part, were in excellent agreement with the Harvard results which, in the meantime, had been published in *H.C.*, No. 170.

The next step was the extension of the photographic scale in the direction of the brighter stars. The provisional magnitudes[3] were in good agreement with independent determinations by Parkhurst[4] and Schwarzschild,[5] but presented a serious divergence from the Harvard results, which thus far has not been satisfactorily explained.

For the extension of the scale to the fainter stars certain long-exposure plates, some of them taken with diaphragms and screens, were next reduced. The comparison of these results with those previously found in the region 10–15 gave material for the determination and discussion of systematic errors and led eventually to the scale finally adopted for the interval from the tenth to the twentieth magnitude.

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[1] *Publications of the Carnegie Institution of Washington.*

[2] "Magnitude Scale of the Polar Sequence" (read at the fourteenth meeting of the Astronomical and Astrophysical Society of America), *Science*, N.S., **37**, 34, 1913.

[3] *Mt. Wilson Contr.*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

[4] "Yerkes Actinometry," *Astrophysical Journal*, **36**, 169, 1912.

[5] "Göttingen Aktinometrie," *Astron. Mitt. d. Kgl. Sternwarte zu Göttingen*, No. 14 B, p. 42, 1912.

Finally, the extension of the scale in the direction of the bright stars was thoroughly revised and strengthened by the inclusion of a large number of additional photographs.

The procedure for the determination of the photo-visual scale has been similar, but much more direct.

Although the methods used for the observations and reductions have been fully described in *Mt. Wilson Contr.*, No. 80,[1] it may be recalled that circular diaphragms and screens of wire gauze are generally employed for the reduced-intensity exposures; further, that the objects observed are grouped into (*a*) intermediate stars (magnitudes 10–18), (*b*) bright stars (brighter than *a*), (*c*) faint stars (fainter than *a*), for each of which the method of treatment is somewhat different. Finally, it has been found convenient to separate the observations of the intermediate stars into those of short and long exposure.

The distribution of the observational data is shown by Table I. The procedure followed in securing the photographs is described in *Mt. Wilson Contr.*, No. 70, p. 5, and No. 80, p. 16.[2]

TABLE I

SUMMARY OF PLATES USED

STAR GROUP	PHOTOGRAPHIC SCALE			PHOTO-VISUAL SCALE		
	Exposure	No. Plates	Apertures*	Exposure	No. Plates	Apertures
Intermediate—						
Short exposure.	1^m to 11^m	15	60, 32, 14, 8, *G*, *g*, *S*	5^m and 20^m	2	60, 32, 14
Long exposure.	30 to 60	14	60, 32, 14	45^m	2	60, 32
Bright..........	1 or 2	213	See Table V	3^m or 4^m	122	See Table VIII
Faint...........	3^h to 5^h	5	60	4^h	2	60

* *G* and *g* are screens of wire gauze, the former placed over the end of the tube, the latter in front of the plate; *S* is a 60° sector diaphragm. For further details of constants, etc., see *Mt. Wilson Contr.*, No. 80, p. 11.

II. PHOTOGRAPHIC SCALE FOR INTERMEDIATE STARS

As stated above, the reduction began with the short-exposure (SE) photographs of the intermediate stars. Magnitudes on an absolute scale were derived for each combination of apertures by the method described in *Mt. Wilson Contr.*, No. 80, p. 23.[1] The zero point was in each case determined by the magnitudes of *H.C.*, No. 170. The results for individual stars were combined into means, and the residuals were discussed for systematic errors depending upon the apertures. The degree of accordance for the different apertures and a comparison with *H.C.*, No. 170, are shown in Table IX, *Mt. Wilson Contr.*, No. 80, p. 31.[2]

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[1] "Photographic Photometry with the 60-Inch Reflector of the Mount Wilson Solar Observatory," *Astrophysical Journal*, **39**, 307, 1914.

[2] *Astrophysical Journal*, **38**, 245, 1913; **39**, 322, 1914.

The next step was the determination of magnitudes from the long-exposure (LE) photographs, the method of reduction being the same as before, except in the case of three plates which were reduced by a method described on p. 9.

All of the plates were measured with photometric scale III (*Mt. Wilson Contr.*, No. 80, p. 19[3]), which is approximately a uniform scale of magnitudes; but, as is the case with all such scales, local irregularities affect the individual readings. The residuals from the LE and SE plates afforded abundant material for the determination of these errors; not only were the scale-readings different for a given star on the SE and LE plates, but further diversity was introduced by the numerous images of varying size obtained with the screens and diaphragms. The residuals for the LE and SE plates were first discussed separately for fear that small differences in the magnitude-scales for the two series might influence the results. As the correction-curves thus obtained were practically identical, they were combined into a mean curve based upon about 2800 residuals. No star with less than 10 residuals was included in the discussion, and even then it was not used unless the scale-readings were well distributed over the scale.

All of the residuals were then corrected with the aid of the final curve, and the mean magnitudes for the different stars were revised. A comparison of the SE and LE results for the 58 stars included in both series is shown in the fifth column of Table II, the unit being 0.01 magnitude. The number of individual magnitudes entering into each group from the SE and LE plates, respectively, appears in the eighth column. A comparison of the mean magnitudes from all the plates with those of *H.C.*, No. 170, is given in the next to the last column.

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[1] *Astrophysical Journal*, **39**, 329, 1914.

[2] *Ibid.*, p. 337, 1914.

[3] *Ibid.*, p. 325, 1914.

TABLE II

RESULTS FROM SE AND LE PLATES

GROUP	MEAN MAG.	SCALE-READING		LE−SE	REL. WEIGHT	NO. STARS	NO. MAGS.	COMP. WITH *H.C.*, NO. 170	
		SE	LE					MW−HC	No. Stars
1......	10.6	......	......	+6	23	4	136, 29	+2	3
2......	12.6	......	......	0	81	6	282,115	−2	8
3......	13.4	7.8	2.8	+1	123	12	355,202	+1	5
4......	14.3	10.7	6.0	0	101	12	219,203	+3	3
5......	14.8	12.5	7.9	+2	104	9	228,210	−4	5
6......	15.5	14.6	10.4	+5	50	7	83,142	−6	5
7......	16.6	17.9	14.5	+10	43	8	74,112	−22	8
Totals						58	1377,1013		37

The weight of the first LE−SE difference is small, and the difference itself is not surprisingly large. The next four differences show a close accordance in the two series of results. The last two, however, require further examination. Turning to the Harvard comparison, we find the same systematic divergence for the faint stars that characterized the preliminary results from the SE plates.[1]

It was suspected that the origin of the small systematic difference in the SE and LE results would be found in the corrections applied to the original scale-readings to reduce them to the center of the plate. With the 60-inch reflector these corrections are large, and their direct determination, especially for the faint stars, is difficult. As the distance error for the diaphragm apertures is negligible, or at least very small, it is evident that incorrect values for the full aperture must systematically affect the calculated scale of magnitudes.

The corrections which had been used were those resulting from the provisional investigation described in *Mt. Wilson Contr.*, No. 80, p. 20.[2] This gave a linear relation between the correction and the distance from the center of the plate, and further, for the full aperture, a relation between error and size of image defined by the curve on p. 22 of that article. Fortunately, the data from the photographs here under discussion were sufficient to permit a thorough examination of the question. The number of separate results for each star was large, and, in general, the range in the distance from the center of the plate was considerable, for the centering of the field was different on different photographs.

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[1] *Mt. Wilson Contr.*, No. 70, p. 4; No. 80, p. 31; *Astrophysical Journal*, **38**, 244, 1913; **39**, 337, 1914.

[2] *Astrophysical Journal*, **39**, 326, 1914.

The residuals for the full-aperture results from about a hundred stars were arranged in groups, each covering one interval of scale-reading and 2 mm of distance. About 2000 residuals were used although only those from stars having numerous and well-distributed observations were included. In order to reduce accidental irregularities, overlapping means were formed for the groups. The results were then plotted, those relating to a single scale-reading defining a curve whose abscissae were distances from the center of the plate, and whose ordinates were corrections which, applied to the individual magnitudes, would remove any systematic dependence upon brightness and distance; in other words, would reduce the individual results to the mean scale. The mean scale, however, is not the true scale, for, as just remarked, the use of an erroneous correction for distance has affected the result.

Two corrections were therefore applied to the calculated magnitudes: (*a*) the ordinate δD of the correction-curve, corresponding to the position and size of the image, which reduced the individual results to the mean scale; (*b*) a scale correction, δm, to reduce from the mean scale to that which would have been obtained had the true distance-correction been used. The final solution could also have been obtained by repeating the calculation from the beginning with the revised distance-correction, but this would have been a very laborious operation.

An examination of the curves giving the values of δD shows that the original distance-correction was substantially correct, except for very large and very small images for which the results have been overcorrected. The curves further show that for all the intermediate and fainter images the assumption of a linear relation between distance-correction and distance is permissible. For the brighter images, however, an appreciable deviation from this relation is now apparent.

To determine the extent to which the scale has been affected by the use of the original distance-correction, it is necessary to know how much the correction was in error. This may be determined by displacing the δD-curves parallel to themselves in a vertical direction until they pass through the origin, for the correction must be zero at the center of the plate.[1] The ordinates of the displaced curves are the required errors.

The determination of the influence of these errors upon the magnitude-scale is now a simple matter. We have only to assume a series of hypothetical observations s and s_1 representing the scale-readings for a full aperture and a diaphragm exposure, respectively. These we transform into magnitudes by the usual process. We then correct the full-aperture readings s by the amount of the error in the original distance-correction and repeat the calculation. The difference in the two series of magnitudes is the required scale correction δm, which is conveniently expressed as a function of the scale-reading.

Two points, however, must be noted: First, in reducing the hypothetical observations, we must use for the constant of the diaphragm a value corresponding to the real observations, taking care, further, to select for the scale-readings values such that the differences $\Delta s = s_1 - s$ agree approximately with the observations. Secondly, the change in the distance-correction by which the values of s are modified before the repetition of the reduction to magnitudes must correspond to the mean distance of the stars actually observed.

It is further tacitly assumed that the distance-correction for the reduced aperture is negligible, or at least requires no revision. As a matter of fact, the discussion of about a thousand residuals for the 32- and 14-inch diaphragms showed only minute deviations, which, although systematic, have been disregarded because they are without influence upon the scale.

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[1] At first sight, errors of focus would seem to require a modification of this statement; but for a reflector, images off the axis are always larger than central images. In other words, the increase in the images due to the aberrations of the mirror is large as compared with that due to curvature of the field. Moreover, in the present case, errors of focus play a very minor part, for with the diaphragm and screen plates the exposures were short, or moderate at most, and the focus was always carefully watched.

With regard to the first of the foregoing points, it may be remarked that several different apertures, with correspondingly different reduction constants, have been employed in reducing the plates. To take this circumstance into account, it is only necessary to reduce the hypothetical observations with the mean of the reduction constants actually used.

It was thus found that for scale-readings less than 12 no correction of the magnitude scale was required. For larger scale-readings the corrections are given in the second column of Table III. These results apply only to the plates taken with diaphragms. Those with which the wire screens were used require no scale correction whatever. Since the distance-correction with the screen is the same as that without the screen,[1] an erroneous correction, used for both full and reduced intensity images, can have no influence upon the *mean* scale, that is, the scale established for a large number of stars of each degree of brightness uniformly scattered over the plate. Individual magnitudes will be in error, but these compensate each other in the mean. The matter is easily tested with the aid of hypothetical observations as above. For this purpose the reduced intensity scale-readings, before the repetition of the calculation, should receive the same modification as those of the full aperture. The two series of magnitudes will then differ only by a constant.

It has been assumed that these remarks also apply to the three plates taken with the 60° sector diaphragm. As this method of reducing the intensity has been but little used, its distance-correction has not been investigated. It seems probable that the correction would not be unlike that for the full aperture, and, in the absence of definite information, this assumption has been accepted as valid.

Before correcting the several thousand individual magnitudes in the manner indicated, information was desired as to whether this procedure would account for the systematic difference between the SE and LE plates shown in Table II. This information was, however, already available, for an examination of the results on the assumption that the scale error depended only on the size of the image had shown what that error must be. Its value was derived by a step-by-step process based on the data in Table II.

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[1] *Mt. Wilson Contr.*, No. 80, p. 21; *Astrophysical Journal*, **39**, 327, 1914.

Since for scale-readings less than 12 the differences LE−SE are zero, it follows from the foregoing assumption that, as far as this limit, both scales are correct. The differences for Groups 5 and 6 of Table II are therefore the errors of the SE magnitude-scale at points whose photometric scale-readings are 12.5 and 14.6. We thus know the error of the LE scale for reading 14.5 (Group 7), and with the aid of the corresponding LE−SE difference find the error for reading 17.9.

The results of the calculation thus outlined are shown in the third column of Table III alongside those derived from the revision of the distance-correction. As the two series of values are practically identical, we infer that the difference between the scales from the SE and LE plates may be attributed entirely to errors in the original distance-correction.

TABLE III

ERROR OF PROVISIONAL SCALE

SCALE-READING	δm	
	DC	SE and LE
	mag.	mag.
12	+0.02	+0.01
13	.03	.02
14	.04	.04
15	.05	.05
16	.06	.07
17	.10	.10
18	.16	.14
19	0.26	0.23

Proceeding to the revision of the individual magnitudes, the two corrections δD and δm were applied to each residual. The mean values were then revised and the results for the SE and LE plates again compared. The accordance of the two scales (Table IV) is now all that can be desired and indicates that there are no remaining errors of any consequence which depend upon the exposure time or the size of the images. The comparison with *H.C.*, No. 170, in the last column of Table IV is less satisfactory, however. The differences MW−HC are but little changed as compared with those in Table II, and the results for the faint stars which now appear in the reduction show that the divergence beginning at magnitude 15 continues to increase and becomes ultimately very large.

It has been remarked (p. 3) that three of the LE plates received special treatment. As the exposure time for these was one hour, they are of great importance in establishing the scale for the fainter stars and a more detailed account of their reduction is desirable.

The necessity for special treatment arose from the fact that the reduced intensity exposures were obtained with a 6-inch diaphragm whose constant is 4.81 mag. With constants as large as this, the usual method[1] of deriving the scale cannot safely be applied. The range of scale-reading for the secondary images of stars with measurable full-intensity images is too small to permit the use of the ordinary interpolation process.

TABLE IV

FINAL RESULTS FOR SE AND LE PLATES

MW Mean Mag.	LE−SE	Rel. Weight	MW−HC
	mag.		mag.
10.6.........	+0.018	36	−0.05
11.7.........	+ .007	49	− .05
12.8.........	− .007	76	+ .03
13.4.........	.000	100	+ .02
14.0.........	+ .023	74	+ .03
14.3.........	− .026	71	+ .03
14.8.........	+ .004	98	− .03
15.5.........	+ .034	77	− .06
16.8.........	+0.017	34	− .17
17.9.........	....	....	− .39
18.5.........	....	....	− .53
19.1.........	....	....	−0.87

On this account, the scale, as far as the fifteenth magnitude, was assumed to be known. The adopted magnitudes were plotted against the scale-readings of the primary images, and at the same time, the adopted magnitudes, less the amount of the reduction constant, were plotted against the readings of the corresponding secondary images. Smooth curves were drawn, from which were read magnitudes for all of the images shown. The scale for the faint stars thus found is not, of course, independently derived, for the method merely transfers an adopted scale to the region of the fainter objects. The results, however, are in close agreement[1] with those from other LE plates, which, because of smaller constants, could be independently reduced. The zero point is of course that of the adopted scale.

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[1] *Mt. Wilson Contr.*, No. 80, p. 23; *Astrophysical Journal*, **39**, 329, 1914.

III. PHOTOGRAPHIC SCALE FOR FAINT STARS

The magnitudes derived from the three LE plates just discussed carry the scale somewhat below the limit (magnitude 18) which is regarded as separating the intermediate from the faint stars, but as they depend upon diaphragm exposures, it has been convenient to group them with the intermediate stars. On the other hand, one plate of moderately short exposure, because of the manner in which it was taken, appears among the faint-star plates. We now turn to the discussion of this latter portion of the data.

The plates for the faint stars usually received two exposures, both with the full aperture, one long and one short. The method of reduction, which involves an extrapolation, though a fairly reliable one, is described in *Mt. Wilson Contr.*, No. 80, pp. 5, 26.[2] The scale for the intermediate stars must be known. As this was the case for the objects brighter than the eighteenth magnitude, mean results for about 160 stars, none including less than 8 or 10 individual values, were available for the reduction. As each of the five plates was reduced with the revised distance-correction no subsequent corrections were required.

The results extend the photographic scale to the twentieth magnitude and include values for the brightness of a large number of objects not shown by the other plates. They also afford additional values for many of the intermediate stars, which have been combined with the original magnitudes to reduce the effect of accidental errors. The revised mean magnitudes, together with the magnitudes of the additional stars, have been entered in the catalogue.[3]

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[1] See residuals for Plates 196, 200, and 204 in Table XII.

[2] *Astrophysical Journal*, **39**, 311, 332, 1914.

[3] With a few exceptions no results appear in the catalogue given in Table IX unless both photographic and photo-visual magnitudes have been determined.

IV. PHOTOGRAPHIC SCALE FOR BRIGHT STARS

The method here applied has been fully described and illustrated in *Mt. Wilson Contr.*, No. 70,[1] and little in the way of detail need be added. It involves a transference of the scale for the intermediate stars to the region of the brighter objects. The operation is repeated with a number of different apertures, and the results for each give a scale for the bright stars which is homogeneous with the adopted scale unless there is some unsuspected form of error; the slope of the scale found with each aperture, independently of error in the reduction constant, is the same as that of the adopted scale. An error in the constant affects the calculated magnitudes by a fixed amount; hence their relative values should be correct. Further, since the separate scales overlap, a comparison of results for the various apertures controls the errors of the constants.

As stated above, the data for the bright stars in *Mt. Wilson Contr.*, No. 70,[2] have been strengthened by the inclusion of additional plates. Further, six stars fainter than the tenth magnitude were added to the list in order that the results by the method used for the bright stars might overlap considerably those derived from the diaphragm and screen plates for the intermediate stars. Finally, the earlier plates were entirely re-reduced, using the revised distance-correction and the magnitudes whose derivation has been described in Section II.

The results, now based upon 662 individual magnitudes as against 295 in the original investigation, are given in Table V which corresponds to Table II in *Mt. Wilson Contr.*, No. 70. The second column gives the final magnitudes referred to the zero point defined by HC magnitudes between 10 and 15, which thus far has been consistently used. As before, the headings indicate the apertures and the approximate reduction constants used in establishing the separate scales. "Various" includes several different apertures which were only occasionally employed. The tabular values are the residuals, referred to the means in the second column, of the results for each star as found with each aperture. The number of values included in each residual is indicated by the attached number

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[1] *Astrophysical Journal*, **38**, 241, 1913.

[2] *Loc. cit.*

TABLE V

PHOTOGRAPHIC MAGNITUDES OF BRIGHT STARS

No.	MW Pg. Mag.	40 1.1	32 2.0	14 3.0	*G* 3.1	9 3.9	6 4.8	32+*G* 5.0	14+*G* 6.0	G_2 6.1	9+*G* 7.0	6+*G* 7.9	14+2*G* 9.2	Various	MW *minus* HC
1*s*...	2.14	...	...	...	...	...	...	...	...	...	...	...	−7 (8)	+2 (32)	−57
1....	3.99	...	...	...	...	...	...	...	...	...	− 9 (4)	− 2 (4)	+3 (6)	+6 (6)	−48
2....	4.90	...	...	...	...	...	...	...	−15 (5)	+ 3 (3)	+ 4 (4)	+18 (3)	...	...	−34
3....	5.43	...	...	...	...	...	...	...	− 3 (6)	− 5 (5)	+ 2 (6)	+ 8 (3)	+6 (2)	...	−31
4....	5.51	...	...	...	...	...	...	...	− 4 (6)	−16 (4)	+ 5 (7)	+14 (4)	+2 (2)	...	−40
5....	6.05	...	...	...	...	...	−16 (5)	+ 6 (5)	+ 5 (5)	0 (4)	+ 5 (6)	...	...	...	−34
2*s*...	6.05	...	...	...	...	−19 (3)	−13 (6)	− 2 (4)	+16 (6)	+ 2 (7)	+ 4 (7)	...	...	...	−33
1*r*...	6.21	...	...	...	...	...	− 4 (2)	...	+ 2 (9)	−13 (8)	+ 8 (6)	+16 (2)	...	...	−48
3*s*...	6.24	...	...	...	...	−26 (1)	−13 (4)	− 5 (3)	+ 7 (5)	− 8 (3)	+13 (6)	...	...	...	−30
6....	6.71	...	...	...	...	+ 1 (3)	+ 5 (4)	+10 (2)	− 2 (3)	− 8 (4)	...	...	...	...	−27
7....	7.01	...	...	...	...	−14 (5)	− 3 (5)	+10 (4)	+ 5 (7)	+ 3 (5)	...	...	...	...	−17
2*r*...	7.54	...	+21 (1)	−12 (2)	−11 (4)	− 4 (3)	+ 1 (3)	−13 (3)	+19 (3)	+ 9 (3)	...	...	...	...	−19
8....	7.92	...	...	−13 (5)	− 1 (5)	+ 3 (5)	− 3 (6)	+18 (3)	+11 (6)	− 6 (6)	...	...	...	...	−18
9....	8.48	...	+22 (4)	+ 2 (2)	− 3 (6)	+ 7 (4)	− 2 (6)	−20 (4)	...	...	...	...	...	...	−22
3*r*...	8.56	...	−22 (2)	− 1 (4)	+12 (4)	− 1 (4)	0 (2)	+ 4 (2)	...	...	...	...	...	+1 (1)	− 6
10....	8.72	...	+ 1 (5)	+ 9 (3)	+ 2 (3)	− 9 (4)	+ 3 (3)	− 3 (3)	...	...	...	...	...	...	−17
4*r*...	8.82	...	−20 (2)	+ 6 (5)	+ 8 (3)	+ 5 (5)	− 1 (3)	−20 (2)	...	...	...	...	...	...	−15
11....	9.33	...	− 4 (6)	− 6 (7)	+ 7 (7)	+ 2 (6)	...	+ 2 (2)	...	...	...	...	...	...	− 9
12....	9.69	−10 (4)	− 8 (5)	− 7 (5)	+ 7 (5)	+11 (6)	...	...	...	...	...	...	...	...	− 4
5*r*...	9.73	0 (4)	−13 (6)	− 8 (6)	+11 (5)	+21 (3)	...	...	...	...	...	...	...	...	− 5
4*s*...	9.85	+ 1 (3)	0 (4)	− 4 (4)	+ 4 (4)	+ 2 (4)	...	...	...	...	...	...	...	...	−14
6*r*...	10.06	−10 (6)	0 (4)	− 1 (6)	+10 (6)	+ 4 (4)	...	...	...	...	...	...	...	...	−11
13....	10.13	0 (4)	0 (6)	− 8 (5)	+ 8 (4)	...	...	...	...	...	...	...	...	...	− 4
7*r*...	10.51	+ 6 (2)	− 1 (2)	+ 4 (1)	−14 (1)	...	...	...	...	...	...	...	...	...	− 4
14....	10.52	− 4 (2)	− 2 (3)	+ 4 (3)	+ 2 (2)	...	...	...	...	...	...	...	...	...	− 6
15....	10.90	−14 (4)	− 8 (4)	+14 (4)	+ 8 (4)	...	...	...	...	...	...	...	...	...	0
6*s*...	10.98	−10 (3)	− 4 (3)	+14 (1)	+16 (2)	...	...	...	...	...	...	...	...	...	− 4
8*r*...	11.12	−11 (5)	+ 2 (5)	+14 (5)	+ 8 (2)	...	...	...	...	...	...	...	...	−12 (3)	+ 6
16....	11.21	0 (6)	− 1 (7)	...	...	...	...	...	...	...	...	...	...	+ 6 (2)	− 5
17....	11.53	− 8 (8)	+ 8 (8)	...	...	...	...	...	...	...	...	...	...	...	+ 4
18....	11.90	− 4 (8)	+ 5 (8)	...	...	...	...	...	...	...	...	...	...	...	− 2
Systematic deviations ..		−6 (59)	−1 (85)	−1 (68)	+5 (67)	0 (60)	−5 (49)	−1 (37)	+4 (61)	−4 (52)	+5 (46)	+10 (16)	−1 (18)	+ 2 (44)	

in parentheses.[1] The weighted mean residuals appearing at the bottom are the systematic deviations of the results for each aperture from the mean of all. These quantities are closely related to the errors of the reduction constants.

It is unnecessary to repeat here the details of the discussion undertaken in *Mt. Wilson Contr.*, No. 70, as the results are essentially the same. The following, however, may be noted:

1. No marked progression in the residuals appears in any of the columns. The parallelism of the constituent scales with the mean scale is therefore close.

2. The systematic deviations at the bottom of the table are small; hence the errors in the reduction constants are also small and probably have had but little influence upon the mean scale.

3. The re-reduction and the inclusion of new data have diminished the accidental errors, but have not materially altered the scale. The divergence from *H.C.*, No. 170, remains substantially as before.

4. Since the constituent scales are sensibly parallel, they must all show the same divergence from the results of *H.C.*, No. 170.

A comparison of the overlapping portion of the scale here established with that for the intermediate stars is shown in the first half of Table VI. In view of the fact that the IS results are here subject to some uncertainty because of large distance error, the agreement is as good as can be expected.

There remains to be considered the influence of the apparent magnitude to which the bright stars have been reduced in determining their real magnitudes.[2] This is shown in the first part of Table VII. The second column gives the mean residual for all the results derived from apparent magnitudes within the limits shown in the first column. Thus the 69 values for bright stars found by reducing to apparent magnitudes between 9.5 and 11.0 are fainter by 0.08 magnitude, on the average, than the final magnitudes in the second column of Table V. This difference is the zero-point error of this particular portion of the results relative to the mean of them all. Its origin will be discussed at a later time. Now it need only be noted that, while obviously systematic, the dependence upon apparent magnitude is small and probably without influence upon the mean scale for the bright stars.

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[1] Note that the algebraic signs of the residuals are here reversed as compared with those of *Mt. Wilson Contr.*, No. 70. Thus for star 12, the residual −10 for the 40-inch diaphragm indicates that the mean of the four values found with this aperture is 9.79.

[2] *Mt. Wilson Contr.*, No. 70, pp. 16–22; *Astrophysical Journal*, **38**, 256–262, 1913.

TABLE VI

Comparison of Overlapping Scales for Bright and Intermediate Stars

Star No.	Photographic				Photo-visual			
	BS	IS	No.	B−I	BS	IS	No.	B−I
13........	10.13	....	19, 0	....	10.37	10.46	11, 5	− 9
14........	10.52	10.59	10, 47	− 7	10.52	10.68	12, 6	−16
15........	10.90	10.76	16, 14	+14	10.92	10.97	9, 1	− 5
16........	11.21	11.22	15, 28	− 1	11.19	11.40	11, 5	−21
17........	11.53	11.43	16, 27	+10	11.27	11.37	8, 5	−10
18........	11.90	11.85	16, 33	+ 5	11.92	11.92	8, 6	0
7r.......	10.51	10.55	6, 28	− 4	9.91	9.96	18, 10	− 5
8r.......	11.12	10.92	20, 12	+20	10.44	10.54	12, 5	−10
5s.......	....	10.69	0, 60	....	10.04	10.16	17, 11	−12
6s.......	10.98	....	9, 0	....	10.78	10.72	9, 4	+ 6
Means..				+ 5				− 8

TABLE VII

Dependence upon Apparent Magnitude

Range of Apparent Mag.	Photographic		Photo-visual	
	Residual	No.	Residual	No.
9.5–11.0........	−8	69	−2*	19
11.0–11.5........	−2	72	−6	68
11.5–12.0........	−1	92	−1	55
12.0–12.5........	0	132	+3	76
12.5–13.0........	−1	83	0	62
13.0–13.5........	+3	125	+4	60
13.5–14.0........	+5	66	+2	27
14.0–14.7........	+6	23	....	....
Totals.........		662		367

* The range of apparent magnitude for this value is 10.1–10.8.

V. THE PHOTO-VISUAL SCALE

The photo-visual scale has been determined in precisely the same manner as the photographic; but the operations have been simpler and the labor much less, for fewer plates were used and

TABLE VIII

PHOTO-VISUAL MAGNITUDES OF BRIGHT STARS

No.	MW Pv. Mag.	60 0	40 1.1	32 2.0	14 3.0	*G* 3.1	9 3.9	40+*G* 4.2	6 4.8	14+*G* 6.0	*G*₂ 6.1	9+*G* 7.0	40+*G*₂ 7.2	Various	MW *minus* HC
1*s*......	2.11	......	......	......	......	......	......	......	......	......	......	......	......	0 (23)	− 1
1......	4.40	......	......	......	......	......	......	......	......	......	......	− 2 (2)	− 2 (2)	+ 1 (5)	− 4
1*r*......	5.12	......	......	......	......	......	......	......	......	−14 (2)	− 8 (3)	+ 8 (2)	......	+13 (3)	−14
2......	5.31	......	......	......	......	......	......	......	......	0 (2)	−17 (3)	+10 (1)	0 (2)	+14 (3)	− 7
3......	5.59	......	......	......	......	......	......	......	− 7 (2)	− 4 (2)	− 1 (3)	+ 5 (4)	......	......	− 3
4......	5.87	......	......	......	......	......	......	......	......	−23 (1)	+ 9 (3)	+ 9 (2)	− 4 (2)	−14 (1)	+ 1
2*s*......	6.36	......	......	......	......	......	......	......	−18 (3)	+15 (3)	− 3 (3)	+20 (1)	......	......	+ 8
3*s*......	6.38	......	......	......	......	......	......	......	−18 (2)	+ 5 (2)	− 9 (3)	+18 (3)	......	......	+ 5
2*r*......	6.38	......	......	......	......	......	......	......	− 9 (3)	+14 (2)	− 3 (3)	+13 (1)	− 2 (3)	......	−17
5......	6.50	......	......	......	......	......	......	......	−15 (2)	+10 (2)	− 1 (4)	+ 5 (2)	......	......	+ 7
6......	7.08	......	......	......	......	......	− 6 (2)	......	− 2 (3)	− 1 (3)	+12 (2)	......	......	......	+ 7
7......	7.55	......	......	......	......	......	......	......	− 2 (5)	+ 8 (4)	− 8 (3)	......	......	......	+11
3*r*......	7.59	......	......	......	− 4 (1)	......	− 4 (2)	+ 4 (2)	+ 4 (2)	−13 (1)	+19 (1)	......	......	− 2 (2)	+ 2
8......	8.16	......	......	− 9 (4)	− 7 (3)	+13 (3)	+ 6 (4)	......	......	......	......	......	......	......	+19
4*r*......	8.29	......	......	......	− 4 (1)	+18 (2)	− 4 (1)	0 (2)	+ 4 (1)	......	......	......	......	−16 (2)	+ 3
5*r*......	8.68	......	......	0 (4)	− 7 (2)	+10 (2)	+ 4 (2)	− 6 (2)	......	......	......	......	......	......	+ 1
9......	8.84	......	......	......	+ 4 (2)	− 2 (2)	− 1 (2)	− 8 (2)	+10 (2)	......	......	......	......	0 (2)	+23
10......	9.10	......	......	−16 (2)	+ 9 (3)	+ 2 (3)	− 8 (2)	+11 (2)	......	......	......	......	......	......	+16
6*r*......	9.24	......	......	−21 (3)	+ 6 (3)	+ 9 (3)	− 7 (1)	+10 (2)	......	......	......	......	......	......	− 1
11......	9.56	......	......	− 2 (2)	− 4 (2)	− 1 (3)	+20 (1)	− 4 (2)	......	......	......	......	......	......	+26
12......	9.83	......	......	− 4 (4)	− 1 (4)	+ 4 (4)	......	......	......	......	......	......	......	......	+29
4*s*......	9.86	......	......	− 4 (3)	− 2 (4)	− 3 (2)	+ 3 (1)	+24 (1)	......	......	......	......	......	......	+28
7*r*......	9.91	......	−13 (4)	+ 1 (6)	+ 5 (6)	+ 6 (2)	......	......	......	......	......	......	......	......	+ 7
5*s*......	10.04	......	−12 (4)	+ 2 (5)	+ 5 (6)	+ 6 (2)	......	......	......	......	......	......	......	......	+18
13......	10.37	......	− 7 (4)	+ 6 (2)	−10 (2)	+13 (3)	......	......	......	......	......	......	......	......	+12
8*r*......	10.44	0 (3)	− 1 (4)	− 4 (4)	+23 (1)	......	......	......	......	......	......	......	......	......	+ 5
14......	10.52	+ 2 (2)	+ 1 (3)	−13 (3)	+15 (2)	+ 4 (2)	......	......	......	......	......	......	......	......	+ 8
6*s*......	10.78	+ 2 (3)	0 (3)	− 1 (3)	......	......	......	......	......	......	......	......	......	......	+ 3
15......	10.92	......	+ 2 (4)	− 5 (4)	+18 (1)	......	......	......	......	......	......	......	......	......	+14
16......	11.19	− 6 (3)	+ 3 (4)	+ 2 (4)	......	......	......	......	......	......	......	......	......	......	+12
17......	11.27	+12 (4)	− 5 (3)	−31 (1)	......	......	......	......	......	......	......	......	......	......	+ 9
18......	11.92	+ 1 (4)	+ 5 (3)	−16 (1)	......	......	......	......	......	......	......	......	......	......	− 1
Systematic deviations.....		+2 (19)	−3 (36)	−5 (55)	+2 (43)	+6 (33)	0 (18)	+2 (15)	−6 (25)	+3 (24)	−3 (31)	+9 (18)	− 2 (9)	+1 (41)	

the irregularities of the photometric scale and the errors of the distance-correction tables had previously been investigated.

The intermediate stars require no comment beyond the statement that the provisional zero point was that defined by the visual magnitudes of *H.C.*, No. 170, which are fainter than 9.8. The results for the bright stars are given in Table IX, which is similar in arrangement to Table V. The mean magnitudes in the second column depend upon 367 individual values.

Although the visual scale of *H.C.*, No. 170, and the Mount Wilson photo-visual scale coincide at the sixth and the twelfth magnitudes, there are important differences at intervening points. An appreciable color equation also makes its appearance, for it will be noted that the differences MW−HC are systematically smaller for the red stars than for the white.

The accordance of the overlapping portions of the scales for the bright and the intermediate stars is shown by the second half of Table VI. The systematic difference of 0.08 magnitude is probably due to an error in the IS scale, which for this region is somewhat uncertain.

Finally, the dependence of the scale for the bright stars upon the apparent magnitudes from which the individual values have been derived appears in Table VII. Here the residuals are again slightly systematic, but satisfactory.

VI. DETERMINATION OF THE ZERO POINTS

The zero-point corrections have been determined in such a way that both photographic and photo-visual magnitudes of the white stars near the sixth magnitude have the same mean value as the corresponding visual magnitudes of *H.C.*, No. 170. The objects actually used in determining the corrections were Nos. 2–6 and 2*s* and 3*s*. As only two of these are of spectrum A0, an allowance for color was made in the case of the photographic scale. The resulting corrections are +0.40 magnitude for the photographic scale, and −0.03 magnitude for the photo-visual.

VII. THE CATALOGUE

The complete catalogue includes 645 objects. Photographic magnitudes have been derived for 617 of these and photo-visual magnitudes for 339. The catalogue in Table IX, however, contains only 329, for with the exception of Polar Sequence stars no object has been entered unless both photographic and photo-visual magnitudes have been determined.

For convenience the Polar Sequence stars appear at the beginning of the table, arranged in the order of their numbers as given in *H.C.*, No. 170. The numbers of the remaining stars, unless a letter is attached, are those of *Harvard Annals*, **48**, Part I. Those with letters are faint objects near the *H.A.* star of the same number. Their rectangular co-ordinates for 1900 are given in Table X.

The magnitudes in the second and third columns of Table IX were obtained by applying to the preliminary values the zero-point corrections given above. For the stars appearing in Table VI, the weighted means of the IS and BS results were the values thus corrected and entered in the catalogue. The number of separate determinations entering into each photographic and photo-visual magnitude, respectively, is shown in the fifth column. The remaining columns contain comparisons of the Mount Wilson photographic scale with that of Harvard[1] and of Greenwich[2] and with the results of Dziewulski[3] obtained with the 80-cm refractor of the Potsdam Observatory.

VIII. SYSTEMATIC DIFFERENCES

The systematic differences affecting the results for the bright stars have already been shown in Tables V and VIII. Those for the photographic magnitudes of the intermediate stars which depend on the method of reducing the intensity are given in Table XI. The agreement is much better than it was before the revision of the distance-correction.[4] The circular diaphragms give practically identical results, but the screens still show divergences of a few hundredths of a magnitude. The last two columns of the table

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[1] *H.C.*, No. 170, 1912.

[2] *Monthly Notices*, R.A.S., **74**, 40, 1913.

[3] *Astronomische Nachrichten*, **198**, 65, 1914.

[4] *Mt. Wilson Contr.*, No. 80, p. 31, Table IX; *Astrophysical Journal*, **39**, 337, 1914. Note that the signs of the residuals must be reversed to make them comparable with Table XI of this paper.

TABLE IX

CATALOGUE OF MAGNITUDES

No.	MAGNITUDE		COLOR INDEX	NO. OBS.	MW *minus* HC	MW *minus* GR	MW *minus* DZ	MW *minus* HH_W	MW *minus* GR_1	MW *minus* DZ_1
	Pg	Pv								
1.....	4.39	4.37	0.02	20, 9	− 8	......	......	−16	......	......
2.....	5.30	5.28	0.02	15, 11	+ 6	......	......	− 1	......	......
3.....	5.83	5.56	0.27	22, 11	+ 9	......	......	− 1	......	......
4.....	5.91	5.84	0.07	23, 9	0	......	......	− 8	......	......
5.....	6.45	6.47	−0.02	25, 10	+ 6	......	......	− 2	......	......
6.....	7.11	7.05	0.06	16, 10	+13	......	......	+ 4	......	......
7.....	7.41	7.52	−0.11	26, 12	+23	......	......	+16	......	......
8.....	8.32	8.13	0.19	36, 14	+22	......	......	+11	......	......
9.....	8.88	8.81	0.07	26, 12	+18	......	......	+ 9	......	......
10.....	9.12	9.07	0.05	21, 12	+23	+29	+27	+15	+ 5	− 4
11.....	9.73	9.53	0.20	28, 10	+31	......	......	+20	......	......
12.....	10.09	9.80	0.29	25, 12	+36	......	......	+24	......	......
13.....	10.53	10.37	0.16	19, 16	+36	+27	......	+26	0	......
14.....	10.98	10.54	0.44	57, 18	+46	+36	+38	+33	+ 1	+10
15.....	11.22	10.89	0.33	30, 10	+28	+27	+19	+17	− 5	−10
16.....	11.62	11.23	0.39	43, 16	+36	+36	+38	+25	+ 2	+ 9
17.....	11.87	11.28	0.59	43, 13	+38	+40	+27	+22	+ 1	0
18.....	12.27	11.89	0.38	49, 14	+35	+30	+22	+22	− 3	− 7
19.....	12.69	12.28	0.41	78, 12	+41	+37	+37	+28	+ 3	+ 8
20.....	13.02	12.51	0.51	86, 12	+43	+41	+37	+29	+ 4	+ 9
21.....	13.33	12.48	0.85	37, 12	+51	+47	+16	+29	+ 1	−10
22.....	13.44	12.81	0.63	45, 11	+39	+42	+24	+22	+ 2	− 3
23.....	13.60	13.10	0.50	80, 12	+40	+40	+32	+25	+ 3	+ 4
24.....	13.93	13.33	0.60	71, 12	+43	+40	+33	+28	+ 1	+ 6
25.....	14.08	13.60	0.48	78, 12	+35	+33	+25	+22	− 3	− 3
26.....	14.64	13.72	0.92	74, 12	+45	+49	+28	+27	+ 1	+ 3
27.....	14.91	14.33	0.58	61, 7	+36	+35	+33	+22	− 4	+ 5
28.....	15.27	14.54	0.73	40, 7	+30	+34	+22	+15	− 9	− 5
29.....	15.82	15.21	0.61	40, 6	+30	+32	+ 8	+16	− 7	−19
30.....	16.18	15.44	0.74	34, 5	+28	......	+12	+14	......	−15
31.....	16.41	15.62	0.79	45, 6	+33	......	+34	+18	......	+ 8
32.....	16.76	15.58	1.18	33, 3	+28	......	+46	+12	......	+22
33.....	17.06	15.97	1.09	32, 3	+24	......	......	+ 7	......	......
34.....	17.24	16.29	0.95	24, 2	+13	......	......	− 1	......	......
35.....	17.63	16.94	0.69	20, 2	+27	......	......	+13	......	......
36.....	17.78	16.80	0.98	18, 2	+17	......	......	+ 4	......	......
37.....	18.01	16.81	1.20	16, 2	+21	......	......	+ 7	......	......
38.....	18.20	17.05	1.15	15, 2	+ 3	......	......	−10	......	......
39.....	18.58	17.13	1.45	14, 2	+ 5	......	......	− 9	......	......
40.....	18.87	17.29	1.58	11, 2	− 2	......	......	−15	......	......
41.....	19.02	17.47	1.55	8, 2	−11	......	......	−24	......	......
42.....	19.18	....	......	8	−36	......	......	−44	......	......
43.....	19.53	....	......	4	−55	......	......	−63	......	......
44.....	19.59	....	......	3	−71	......	......	−79	......	......
45.....	19.80	....	......	1	−80	......	......	−88	......	......
46.....	19.82	....	......	2	−114	......	......	−122	......	......
1r....	6.61	5.09	1.52	27, 10	− 8	......	......	−26	......	......
2r....	7.94	6.35	1.59	22, 12	+21	......	......	+ 3	......	......
3r....	8.96	7.56	1.40	19, 11	+34	......	......	+17	......	......
4r....	9.22	8.26	0.96	20, 9	+25	......	......	+11	......	......

TABLE IX—*Continued*

No.	Magnitude		Color Index	No. Obs.	MW *minus* HC	MW *minus* Gr	MW *minus* Dz	MW *minus* HH_W	MW *minus* Gr_1	MW *minus* Dz_1
	Pg	Pv								
5*r*...	10.13	8.65	1.48	24, 12	+35	......	......	+16	......	......
6*r*...	10.46	9.21	1.25	26, 12	+29	......	......	+11	......	......
7*r*...	10.94	9.90	1.04	34, 28	+37	+40	+ 9	+22	−11	−16
8*r*...	11.44	10.44	1.00	32, 17	+38	+50	......	+24	0	......
10*r*...	......	12.03	......	5	......	......	......	......	......	......
11*r*...	13.22	12.06	1.16	79, 12	+39	+56	+21	+25	+ 2	− 3
12*r*...	13.84	12.46	1.38	82, 12	+50	+65	+26	+31	+ 5	+ 3
1*s*...	2.54	2.08	0.46	40, 23	−17	......	......	−25	......	......
2*s*...	6.45	6.33	0.12	33, 10	+ 7	......	......	− 2	......	......
3*s*...	6.64	6.35	0.29	22, 10	+10	......	......	− 1	......	......
4*s*...	10.25	9.83	0.42	19, 11	+26	......	......	+17	......	......
5*s*...	11.09	10.06	1.03	60, 28	+41	+45	+22	+26	− 6	− 3
6*s*...	11.38	10.73	0.65	9, 13	+41	+48	+10	+26	+ 7	−17
7*s*...	12.61	12.10	0.51	70, 12	+30	+36	+36	+16	− 1	+ 8
8*s*...	14.49	13.78	0.71	50, 12	+43	+37	+27	+27	− 5	0
9*s*...	14.75	13.85	0.90	66, 12	+42	+42	+29	+25	− 5	+ 3
10*s*...	15.29	14.49	0.80	45, 7	+41	+42	+31	+25	− 3	+ 5
11*s*...	15.31	14.35	0.96	63, 10	+40	+37	+27	+22	−12	+ 2
12*s*...	15.33	14.69	0.64	61, 7	+36	+31	+21	+21	− 9	− 6
13*s*...	15.54	14.54	1.00	43, 3	+45	+55	+25	+27	+ 5	0
14*s*...	15.99	15.07	0.92	47, 7	+38	+36	+ 3	+21	−12	−22
15*s*...	16.57	15.71	0.86	41, 5	+29	......	......	+15	......	......
16*s*...	16.86	15.50	1.36	36, 4	+28	......	......	+11	......	......
17*s*...	17.19	15.89	1.30	27, 2	+22	......	......	+ 5	......	......
18*s*...	17.94	16.91	1.03	19, 2	+ 8	......	......	+ 5	......	......
19*s*...	18.16	16.95	1.21	15, 2	−14	......	......	−32	......	......
20*s*...	18.60	17.19	1.41	13, 2	+ 4	......	......	− 9	......	......
21*s*...	18.65	17.33	1.32	12, 2	− 1	......	......	−20	......	......
22*s*...	18.75	17.13	1.62	11, 2	0	......	......	−18	......	......
23*s*...	18.70	17.41	1.29	12, 2	−18	......	......	−38	......	......
24*s*...	18.88	17.34	1.54	12, 2	0	......	......	− 9	......	......
25*s*...	18.84	17.38	1.46	10, 2	−18	......	......	−31	......	......
26*s*...	18.99	......	......	9	−30	......	......	−38	......	......
27*s*...	19.08	17.43	1.65	9, 2	−22	......	......	−30	......	......
28*s*...	19.23	......	......	8	−33	......	......	−41	......	......
29*s*...	19.28	......	......	6	−76	......	......	−84	......	......
30*s*...	19.52	......	......	3	−66	......	......	−74	......	......
31*s*...	19.49	......	......	3	−69	......	......	−77	......	......
32*s*...	19.56	......	......	4	−62	......	......	−70	......	......
33*s*...	19.68	......	......	4	−56	......	......	−64	......	......
34*s*...	19.70	......	......	2	−60	......	......	−68	......	......
35*s*...	19.86	......	......	3	−60	......	......	−68	......	......
36*s*...	19.48	......	......	3	−108	......	......	−116	......	......
37*s*...	19.65	......	......	2	−93	......	......	−101	......	......
38*s*...	20.10	......	......	1	−90	......	......	−98	......	......
119....	17.38	15.97	1.41	2, 1	......	......	......	......	......	......
119*a*...	17.88	16.40	1.48	1, 1	......	......	......	......	......	......
124....	16.50	15.09	1.41	4, 3	......	......	......	......	......	......
125....	15.12	14.37	0.75	2, 5	......	+70	+76	......	+27	+50
125*a*...	16.90	15.91	0.99	1, 2	......	......	......	......	......	......
126....	15.58	14.65	0.93	2, 4	......	+29	+23	......	−19	− 2
127....	14.35	13.31	1.04	6, 11	......	+65	+51	......	+14	+26

TABLE IX—*Continued*

No.	MAGNITUDE		COLOR INDEX	NO. OBS.	MW *minus* HC	MW *minus* GR	MW *minus* Dz	MW *minus* HH_W	MW *minus* GR_1	MW *minus* Dz_1
	Pg	Pv								
127*a*...	16.78	15.81	0.97	4, 1	......	......	......	......	......	......
127*b*...	18.40	16.96	1.44	3, 1	......	......	......	......	......	......
127*c*...	17.01	15.50	1.51	8, 2	......	......	......	......	......	......
127*d*...	18.29	17.05	1.24	2, 2	......	......	......	......	......	......
127*e*...	18.24	16.97	1.27	2, 2	......	......	......	......	......	......
128....	15.88	13.87	2.01	1, 2	......	+61	+48	......	−16	+29
130....	15.26	14.21	1.05	2, 9	......	+63	+57	......	+12	+32
134....	14.43	13.54	0.89	6, 10	......	+64	+54	......	+17	+28
134*a*...	17.52	16.22	1.30	4, 2	......	......	......	......	......	......
134*b*...	17.96	16.72	1.24	2, 2	......	......	......	......	......	......
139....	15.30	13.96	1.34	2, 7	......	+48	+26	......	−11	+ 3
142....	14.60	13.71	0.89	2, 11	......	+56	+45	......	+ 9	+19
143....	16.35	15.51	0.84	7, 2	......	......	......	......	......	......
146....	14.52	13.56	0.96	2, 10	......	+70	+56	......	+21	+31
151....	13.72	12.58	1.14	29, 11	......	+53	+22	......	− 1	− 2
151*a*...	18.66	17.21	1.45	3, 2	......	......	......	......	......	......
151*b*...	18.55	17.15	1.40	3, 2	......	......	......	......	......	......
151*c*...	18.10	16.88	1.22	3, 2	......	......	......	......	......	......
151*d*...	18.21	16.93	1.28	3, 2	......	......	......	......	......	......
151*e*...	18.45	17.08	1.37	3, 2	......	......	......	......	......	......
152....	14.56	13.76	0.80	21, 12	......	+45	+31	......	0	+ 5
152*a*...	18.38	16.63	1.75	5, 2	......	......	......	......	......	......
152*b*...	17.56	16.74	0.82	7, 2	......	......	......	......	......	......
152*c*...	17.15	16.09	1.06	10, 2	......	......	......	......	......	......
152*d*...	18.66	17.38	1.28	2, 1	......	......	......	......	......	......
152*e*...	18.52	17.12	1.40	2, 2	......	......	......	......	......	......
154....	14.95	14.36	0.59	8, 9	......	+29	+39	......	−10	+12
154*a*...	17.78	16.59	1.19	3, 2	......	......	......	......	......	......
154*b*...	18.11	17.04	1.07	4, 1	......	......	......	......	......	......
154*c*...	18.40	17.13	1.27	3, 1	......	......	......	......	......	......
158....	15.10	14.57	0.53	16, 5	......	+14	+12	......	−23	−16
158*a*...	18.77	17.09	1.68	3, 2	......	......	......	......	......	......
158*b*...	18.88	17.20	1.68	3, 1	......	......	......	......	......	......
159....	15.20	14.50	0.70	16, 4	......	+24	+32	......	−18	+ 5
159*a*...	17.80	16.34	1.46	5, 1	......	......	......	......	......	......
159*b*...	18.13	16.64	1.49	5, 2	......	......	......	......	......	......
159*g*...	19.03	17.10	1.93	2, 1	......	......	......	......	......	......
160....	16.68	15.54	1.14	11, 2	......	......	......	......	......	......
160*a*...	18.73	16.95	1.78	3, 2	......	......	......	......	......	......
163....	15.13	13.93	1.20	1, 3	......	+16	− 7	......	−39	−31
164....	16.02	15.03	0.99	18, 5	......	......	+11	......	......	−14
164*a*...	18.73	17.13	1.60	3, 1	......	......	......	......	......	......
169....	16.85	16.02	0.83	13, 2	......	......	......	......	......	......
169*a*...	18.46	16.95	1.51	6, 2	......	......	......	......	......	......
172....	13.85	12.24	1.61	2, 1	......	+71	+28	......	+ 5	+ 7
176....	14.33	13.81	0.52	36, 12	......	+26	+26	......	−11	− 2
176*a*...	16.87	16.07	0.80	12, 2	......	......	......	......	......	......
176*c*...	18.22	17.15	1.07	3, 2	......	......	......	......	......	......
183....	16.53	15.64	0.89	21, 3	......	......	......	......	......	......
184*a*...	17.18	16.03	1.15	3, 2	......	......	......	......	......	......
184*b*...	17.14	16.14	1.00	3, 2	......	......	......	......	......	......
184*c*...	18.52	16.82	1.70	3, 2	......	......	......	......	......	......
186....	16.34	15.54	0.80	22, 3	......	......	......	......	......	......

TABLE IX—*Continued*

No.	Magnitude		Color Index	No. Obs.	MW *minus* HC	MW *minus* Gr	MW *minus* Dz	MW *minus* HH_W	MW *minus* Gr_1	MW *minus* Dz_1
	Pg	Pv								
188....	16.29	15.27	1.02	24, 4	......	+67	+31	......	+16	+ 6
190*a*...	17.56	16.57	0.99	15, 2	......	......	......	......	......	......
190*b*...	18.10	16.78	1.32	16, 2	......	......	......	......	......	......
190*c*...	17.87	16.95	0.92	12, 2	......	......	......	......	......	......
190*d*...	17.99	16.73	1.26	11, 2	......	......	......	......	......	......
190*f*...	19.19	17.21	1.98	3, 1	......	......	......	......	......	......
191*a*...	19.17	17.53	1.64	3, 1	......	......	......	......	......	......
202....	16.12	15.41	0.71	27, 4	......	......	+21	......	......	− 6
202*a*...	18.80	17.21	1.59	11, 2	......	......	......	......	......	......
205....	15.97	15.08	0.89	11, 4	......	+53	+28	......	+ 6	+ 2
207....	16.69	15.73	0.96	10, 2	......	......	......	......	......	......
208....	16.67	15.93	0.74	12, 2	......	......	......	......	......	......
208*a*...	18.65	17.07	1.58	3, 2	......	......	......	......	......	......
208*b*...	18.69	17.17	1.52	3, 2	......	......	......	......	......	......
212*d*...	17.81	16.88	0.93	17, 2	......	......	......	......	......	......
212*e*...	17.04	16.31	0.73	18, 2	......	......	......	......	......	......
213....	16.81	15.75	1.06	20, 3	......	......	......	......	......	......
214....	15.39	14.66	0.73	9, 7	......	+22	+23	......	−21	− 4
214*c*...	18.53	16.79	1.74	3, 2	......	......	......	......	......	......
214*d*...	17.59	16.23	1.36	3, 2	......	......	......	......	......	......
214*e*...	17.88	16.73	1.15	3, 2	......	......	......	......	......	......
214*f*...	18.15	16.47	1.68	3, 2	......	......	......	......	......	......
214*g*...	18.06	16.69	1.37	3, 2	......	......	......	......	......	......
215....	15.47	13.69	1.78	10, 3	......	+60	+30	......	−11	+10
217....	16.55	15.51	1.04	10, 4	......	......	......	......	......	......
218....	15.47	14.11	1.36	17, 8	......	+47	+27	......	−13	+ 4
218*a*...	16.82	15.59	1.23	12, 4	......	......	......	......	......	......
218*b*...	17.20	16.02	1.18	11, 2	......	......	......	......	......	......
219*a*...	17.61	16.63	0.98	17, 2	......	......	......	......	......	......
219*b*...	17.53	16.20	1.33	17, 2	......	......	......	......	......	......
219*c*...	17.85	16.81	1.04	13, 2	......	......	......	......	......	......
219*d*...	18.69	16.95	1.74	12, 2	......	......	......	......	......	......
219*e*...	19.03	17.50	1.53	9, 1	......	......	......	......	......	......
219*f*...	18.26	16.93	1.33	13, 2	......	......	......	......	......	......
219*g*...	18.53	16.99	1.54	11, 2	......	......	......	......	......	......
219*h*...	19.11	17.50	1.61	3, 1	......	......	......	......	......	......
224....	16.49	15.48	1.01	22, 4	......	......	......	......	......	......
226....	14.93	14.23	0.70	46, 7	......	+35	+30	......	− 7	+ 3
226*a*...	18.45	16.79	1.66	10, 2	......	......	......	......	......	......
226*b*...	18.30	16.92	1.38	11, 2	......	......	......	......	......	......
226*c*...	18.73	16.93	1.80	10, 2	......	......	......	......	......	......
226*d*...	18.79	17.11	1.68	11, 1	......	......	......	......	......	......
226*e*...	18.81	17.02	1.79	9, 1	......	......	......	......	......	......
227*a*...	17.69	16.49	1.20	14, 2	......	......	......	......	......	......
227*b*...	18.15	16.99	1.16	12, 2	......	......	......	......	......	......
227*c*...	18.51	17.29	1.22	10, 2	......	......	......	......	......	......
227*d*...	17.93	16.67	1.26	17, 2	......	......	......	......	......	......
229....	16.34	15.57	0.77	24, 3	......	......	......	......	......	......
229*a*...	18.23	16.99	1.24	16, 2	......	......	......	......	......	......
229*b*...	17.80	16.69	1.11	16, 2	......	......	......	......	......	......
244....	16.22	15.47	0.75	13, 4	......	......	......	......	......	......
244*a*...	17.93	16.68	1.25	8, 2	......	......	......	......	......	......
244*b*...	18.75	16.83	1.92	1, 1	......	......	......	......	......	......

TABLE IX—*Continued*

No.	Magnitude		Color Index	No. Obs.	MW *minus* HC	MW *minus* GR	MW *minus* Dz	MW *minus* HH$_W$	MW *minus* GR$_1$	MW *minus* Dz$_1$
	Pg	Pv								
247	16.43	15.89	0.54	21, 2	......	......	......	......	......	......
248*a*	16.30	15.23	1.07	4, 3	......	......	......	......	......	......
254*a*	18.71	17.64	1.07	8, 1	......	......	......	......	......	......
256	14.67	13.86	0.81	50, 11	......	+56	+29	......	+11	+ 3
258	16.07	15.22	0.85	6, 4	......	+58	+28	......	+12	+ 2
258*a*	17.45	16.17	1.28	5, 2	......	......	......	......	......	......
277	15.30	14.61	0.69	62, 7	......	+21	+22	......	−21	− 5
277*a*	18.54	17.04	1.50	10, 1	......	......	......	......	......	......
279*a*	17.98	16.87	1.11	7, 2	......	......	......	......	......	......
279*b*	18.47	17.07	1.40	8, 2	......	......	......	......	......	......
279*c*	17.73	16.27	1.46	7, 2	......	......	......	......	......	......
279*d*	18.44	17.21	1.23	6, 2	......	......	......	......	......	......
279*e*	18.60	16.88	1.72	6, 2	......	......	......	......	......	......
279*f*	17.80	16.75	1.05	7, 2	......	......	......	......	......	......
283	14.87	13.50	1.37	28, 3	......	+54	+21	......	− 6	− 2
286	15.97	14.96	1.01	10, 4	......	+48	......	......	− 2	......
288*a*	16.63	15.29	1.34	23, 5	......	......	......	......	......	......
288*b*	18.53	17.45	1.08	13, 2	......	......	......	......	......	......
288*c*	18.10	16.81	1.29	13, 2	......	......	......	......	......	......
288*d*	19.06	17.53	1.53	8, 1	......	......	......	......	......	......
289*a*	17.91	16.84	1.07	12, 2	......	......	......	......	......	......
289*b*	18.77	17.25	1.52	11, 1	......	......	......	......	......	......
289*c*	18.76	17.21	1.55	11, 1	......	......	......	......	......	......
289*d*	17.44	16.29	1.15	14, 2	......	......	......	......	......	......
289*e*	17.52	16.55	0.97	12, 2	......	......	......	......	......	......
292*a*	17.96	16.82	1.14	5, 2	......	......	......	......	......	......
292*b*	17.27	16.16	1.11	5, 2	......	......	......	......	......	......
292*c*	17.49	16.77	0.72	3, 1	......	......	......	......	......	......
292*d*	18.82	16.46	2.36	2, 1	......	......	......	......	......	......
295	15.57	14.93	0.64	32, 5	......	+31	+23	......	− 9	− 4
295*a*	17.97	16.95	1.02	14, 2	......	......	......	......	......	......
295*b*	18.09	16.88	1.21	13, 2	......	......	......	......	......	......
295*c*	18.85	17.53	1.32	10, 1	......	......	......	......	......	......
295*d*	19.26	17.68	1.58	3, 1	......	......	......	......	......	......
297	15.81	14.79	1.02	5, 3	......	+41	+17	......	−10	− 8
297*a*	18.56	17.11	1.45	2, 1	......	......	......	......	......	......
297*b*	18.10	16.93	1.17	3, 1	......	......	......	......	......	......
299	16.59	15.83	0.76	22, 2	......	......	......	......	......	......
301	16.23	15.57	0.66	26, 2	......	......	+12	......	......	−15
302*a*	16.91	15.97	0.94	21, 2	......	......	......	......	......	......
302*b*	18.24	16.89	1.35	14, 2	......	......	......	......	......	......
302*c*	19.02	17.50	1.52	9, 1	......	......	......	......	......	......
302*d*	18.97	17.33	1.64	6, 1	......	......	......	......	......	......
311	15.00	14.36	0.64	53, 7	......	+30	+32	......	−10	+ 5
311*a*	17.22	16.05	1.17	14, 2	......	......	......	......	......	......
311*b*	17.73	16.68	1.05	5, 2	......	......	......	......	......	......
319	14.44	13.55	0.89	40, 3	......	+49	+29	......	+ 2	+ 3
323*a*	17.55	16.55	1.00	17, 2	......	......	......	......	......	......
323*b*	18.96	17.17	1.79	8, 1	......	......	......	......	......	......
323*c*	18.66	17.01	1.65	10, 2	......	......	......	......	......	......
323*e*	18.81	17.18	1.63	10, 2	......	......	......	......	......	......
323*f*	18.90	17.05	1.85	10, 1	......	......	......	......	......	......

TABLE IX—*Continued*

No.	Magnitude		Color Index	No. Obs.	MW *minus* HC	MW *minus* Gr	MW *minus* Dz	MW *minus* HH$_W$	MW *minus* Gr$_1$	MW *minus* Dz$_1$
	Pg	Pv								
323*h*...	19.03	17.08	1.95	8, 1	......	......	......	......	......	......
324*a*...	17.95	16.72	1.23	17, 2	......	......	......	......	......	......
324*b*...	18.57	17.05	1.52	13, 2	......	......	......	......	......	......
329....	14.32	13.34	0.98	38, 3	......	+47	+31	......	− 2	+ 6
334....	15.20	14.42	0.78	70, 7	......	+30	+20	......	−14	− 6
335....	16.11	15.11	1.00	23, 4	......	+35	......	......	−15	......
335*a*...	17.69	16.67	1.02	13, 2	......	......	......	......	......	......
335*b*...	18.77	17.33	1.44	7, 1	......	......	......	......	......	......
336....	14.92	13.97	0.95	26, 5	......	+42	+27	......	− 7	+ 2
336*a*...	17.31	16.27	1.04	5, 2	......	......	......	......	......	......
336*b*...	17.27	16.41	0.86	5, 2	......	......	......	......	......	......
337*a*...	18.50	16.84	1.66	14, 2	......	......	......	......	......	......
337*c*...	18.82	17.38	1.44	11, 2	......	......	......	......	......	......
337*d*...	18.98	17.34	1.64	10, 1	......	......	......	......	......	......
338....	14.72	13.79	0.93	28, 3	......	+45	+28	......	− 3	+ 3
341....	15.16	13.92	1.24	26, 3	......	+36	+25	......	−20	+ 1
342....	14.86	14.09	0.77	26, 3	......	+24	+27	......	−20	+ 1
343....	16.44	15.01	1.43	23, 5	......	......	......	......	......	......
343*a*...	17.94	16.83	1.11	16, 2	......	......	......	......	......	......
343*b*...	18.86	17.27	1.59	9, 1	......	......	......	......	......	......
343*c*...	17.16	16.59	0.57	17, 2	......	......	......	......	......	......
343*d*...	18.46	16.89	1.57	10, 2	......	......	......	......	......	......
343*e*...	18.56	16.95	1.61	11, 2	......	......	......	......	......	......
345....	13.79	12.01	1.78	58, 11	......	+66	+17	......	− 5	− 3
350....	11.01	10.64	0.37	46, 12	......	+37	+40	......	+ 4	+11
350*a*...	17.39	16.59	0.80	8, 1	......	......	......	......	......	......
350*b*...	17.72	16.79	0.93	5, 2	......	......	......	......	......	......
350*c*...	18.19	16.64	1.55	5, 2	......	......	......	......	......	......
358....	15.22	14.11	1.11	26, 2	......	+23	+15	......	−30	− 9
362....	13.13	12.31	0.82	86, 12	......	+53	+31	......	+ 8	+ 5
362*a*...	16.39	15.61	0.78	19, 2	......	......	......	......	......	......
362*b*...	17.47	16.57	0.90	15, 2	......	......	......	......	......	......
362*c*...	18.15	16.48	1.67	13, 2	......	......	......	......	......	......
362*d*...	17.96	16.75	1.21	14, 2	......	......	......	......	......	......
362*e*...	18.47	16.99	1.48	11, 2	......	......	......	......	......	......
362*f*...	18.78	17.23	1.55	10, 2	......	......	......	......	......	......
362*h*...	18.74	17.29	1.45	10, 2	......	......	......	......	......	......
363*a*...	17.94	16.41	1.53	14, 2	......	......	......	......	......	......
363*b*...	18.36	17.05	1.31	13, 1	......	......	......	......	......	......
364....	16.44	15.49	0.95	22, 3	......	......	......	......	......	......
364*a*...	17.20	16.35	0.85	17, 2	......	......	......	......	......	......
364*b*...	18.01	16.99	1.02	12, 1	......	......	......	......	......	......
364*c*...	16.88	16.11	0.77	17, 1	......	......	......	......	......	......
364*d*...	17.55	16.71	0.84	13, 1	......	......	......	......	......	......
364*e*...	18.24	16.82	1.42	10, 1	......	......	......	......	......	......
364*f*...	18.54	17.24	1.30	10, 1	......	......	......	......	......	......
364*i*...	18.39	17.05	1.34	10, 1	......	......	......	......	......	......
365*c*...	18.85	17.11	1.74	11, 1	......	......	......	......	......	......
365*d*...	19.04	17.20	1.84	6, 1	......	......	......	......	......	......
365*e*...	18.97	17.34	1.63	9, 1	......	......	......	......	......	......
368....	15.78	14.48	1.30	2, 3	......	+46	+24	......	−12	+ 1
368*a*...	17.17	16.09	1.08	1, 1	......	......	......	......	......	......

TABLE IX—*Continued*

No.	Magnitude		Color Index	No. Obs.	MW *minus* HC	MW *minus* Gr	MW *minus* Dz	MW *minus* HH_W	MW *minus* Gr_1	MW *minus* Dz_1
	Pg	Pv								
370*c*...	16.80	15.73	1.07	22, 1	......	......	......	......	......	......
372....	16.09	15.52	0.57	10, 3	......	......	......	......	......	......
372*a*...	17.40	16.62	0.78	3, 1	......	......	......	......	......	......
372*b*...	17.82	16.63	1.19	5, 2	......	......	......	......	......	......
372*c*...	17.31	16.69	0.62	4, 1	......	......	......	......	......	......
378....	14.14	12.74	1.40	54, 5	......	+ 68	+20	......	+ 7	− 3
386*c*...	18.17	16.77	1.40	12, 1	......	......	......	......	......	......
386*e*...	17.72	16.85	0.87	15, 1	......	......	......	......	......	......
389....	13.57	12.83	0.74	43, 8	......	+ 47	+32	......	+ 4	+ 5
391....	14.63	13.85	0.78	38, 6	......	+ 48	+42	......	+ 4	+16
401....	14.53	13.61	0.92	36, 3	......	+ 43	+26	......	− 5	+ 1
403....	14.51	13.75	0.76	41, 3	......	+ 29	+25	......	−15	− 1
410*a*...	16.96	16.18	0.78	17, 1	......	......	......	......	......	......
424....	14.51	13.64	0.87	29, 3	......	+ 38	+38	......	− 8	+12
426....	13.83	13.33	0.50	28, 3	......	+ 47	+29	......	+10	+ 1
427....	13.27	12.07	1.20	33, 5	......	+ 62	+35	......	+ 7	+11
439....	15.29	13.49	1.80	1, 3	......	+118	+89	......	+46	+69

show the mean results for these two methods of reducing the intensity. The deviations are in all cases referred to the mean scale, which depends upon 42 separate determinations.

As the lower limit of Table XI is only 17.1, some indication of the accordance of the results for the faint stars is desirable. This is shown in Table XII, which gives the mean deviations of groups of stars for each of the long-exposure plates. In no case is there a serious divergence from the mean scale. Some of the average deviations at the bottom of the table appear to be large; but it must be recalled that they include the errors of the scales derived from the separate plates for a range of 9 or 10 magnitudes. The number of magnitudes from each plate and the diaphragms used for the reduced-intensity exposures are also shown at the bottom of the table.

Although the photo-visual results are based upon a much smaller number of plates than the photographic, their relative weights are more nearly equal than the number of observations would indicate. This arises from the higher precision of the individual magnitudes from the photo-visual plates. Thus the average deviation of a single photographic magnitude, based on the results for stars fainter than the tenth magnitude and including the effect of

TABLE X

RECTANGULAR CO-ORDINATES FOR 1900

No.	*X*	*Y*	No.	*X*	*Y*	No.	*X*	*Y*
119*a*	−755″	− 79″	219*a*	−329″	−181″	302*d*	+230″	−339″
125*a*	−750	−446	219*b*	−304	−156	311*a*	+193	−549
127*a*	−695	+ 36	219*c*	−332	−312	311*b*	+171	−703
127*b*	−664	− 76	219*d*	−270	−160	323*a*	+178	+242
127*c*	−651	− 70	219*e*	−254	−168	323*b*	+163	+214
127*d*	−670	−101	219*f*	−288	−214	323*c*	+118	+254
127*e*	−667	−112	219*g*	−322	−240	323*e*	+245	+194
134*a*	−649	−365	219*h*	−342	−285	323*f*	+268	+190
134*b*	−697	−280	226*a*	−311	+251	323*h*	+288	+228
151*a*	−544	− 93	226*b*	−357	+268	324*a*	+207	− 6
151*b*	−625	− 35	226*c*	−211	+196	324*b*	+162	+ 78
151*c*	−636	− 7	226*d*	−172	+174	335*a*	+247	−524
151*d*	−619	0	226*e*	−228	+286	335*b*	+296	−599
151*e*	−607	+ 17	227*a*	−268	−412	336*a*	+187	−820
152*a*	−608	−252	227*b*	−209	−394	336*b*	+210	−788
152*b*	−608	−321	227*c*	−178	−402	337*a*	+289	− 62
152*c*	−619	−234	227*d*	−206	−265	337*c*	+248	− 87
152*d*	−630	−258	229*a*	−243	− 73	337*d*	+248	− 31
152*e*	−622	−364	229*b*	−231	− 85	343*a*	+302	−465
154*a*	−538	−574	244*a*	−181	−599	343*b*	+301	−460
154*b*	−478	−530	244*b*	−200	−685	343*c*	+328	−468
154*c*	−536	−580	248*a*	− 80	−954	343*d*	+362	−506
158*a*	−524	−316	254*a*	− 79	−292	343*e*	+354	−455
158*b*	−556	−277	258*a*	−140	−791	350*a*	+335	−685
159*a*	−648	+145	277*a*	− 78	−571	350*b*	+240	−710
159*b*	−543	+171	279*a*	+ 45	−690	350*c*	+283	−755
159*g*	−505	+233	279*b*	− 94	−645	362*a*	+482	−352
160*a*	−536	+ 57	279*c*	− 57	−691	362*b*	+487	−194
164*a*	−531	−199	279*d*	− 39	−712	362*c*	+466	−182
169*a*	−520	− 31	279*e*	+ 49	−693	362*d*	+457	−154
176*a*	−487	−210	279*f*	+ 84	−682	362*e*	+455	−224
176*c*	−415	−333	288*a*	− 2	−240	362*f*	+486	−276
184*a*	−460	−649	288*b*	+ 27	−254	362*h*	+372	−310
184*b*	−422	−836	288*c*	+ 70	−288	363*a*	+351	+ 4
184*c*	−409	−633	288*d*	+ 89	−210	363*b*	+398	+101
190*a*	−432	+152	289*a*	+ 42	+186	364*a*	+458	−462
190*b*	−364	+137	289*b*	− 35	+193	364*b*	+503	−481
190*c*	−452	+169	289*c*	− 28	+224	364*c*	+542	−460
190*d*	−449	+261	289*d*	+ 2	+353	364*d*	+488	−554
190*f*	−376	+150	289*e*	+ 51	+311	364*e*	+548	−511
191*a*	−392	−146	292*a*	− 20	−854	364*f*	+452	−513
202*a*	−355	− 22	292*b*	+109	−780	364*i*	+540	−377
208*a*	−376	−434	292*c*	+ 52	−912	365*c*	+410	−115
208*b*	−346	−395	292*d*	+122	−836	365*d*	+384	−138
212*d*	−227	+130	295*a*	+ 68	−530	365*e*	+374	−185
212*e*	−333	+196	295*b*	+ 37	−489	368*a*	+413	−796
214*c*	−241	−833	295*c*	+ 26	−429	370*c*	+311	+215
214*d*	−310	−849	295*d*	+ 11	−406	372*a*	+443	−691
214*e*	−358	−830	297*a*	+100	−914	372*b*	+411	−692
214*f*	−345	−783	297*b*	+102	−905	372*c*	+416	−721
214*g*	−352	−768	302*a*	+162	−286	386*c*	+472	+ 52
218*a*	−278	−515	302*b*	+164	−270	386*e*	+531	− 56
218*b*	−320	−595	302*c*	+224	−264	410*a*	+577	−247

scale errors, is ±0.125 magnitude; the corresponding photo-visual result is ±0.087. From this it follows that the accordance of the separate photo-visual scales must be even better than that of the results in Table XII. A discussion of the residuals proves this to be the case.

IX. COMPARISONS WITH OTHER MAGNITUDES

The reference of the Mount Wilson photographic magnitudes to the international zero point introduces a large systematic difference into the comparison with *H.C.*, No. 170. Between the tenth and the sixteenth magnitudes this is nearly constant. Below this region there is a divergence which rapidly increases and, for the faintest stars given, amounts to about one magnitude. The faintest magnitude given in *H.C.*, No. 170, is 21.00; the faintest here shown is about 20.

TABLE XI

SYSTEMATIC DIFFERENCES FOR VARIOUS APERTURES (UNIT=0.01 MAG.)

MEAN MAG.	*G*	*g*	32	14	8, 6	*S*	MEANS	
							Screens	Dia-phragms
11.4......	+9.6	+8.4	−1.3	−1.6	−2.8	+11.1	+9.4	−1.6
12.7......	−0.6	+2.6	−0.8	+1.3	−0.8	+ 5.0	+0.3	−0.8
13.7......	−2.2	+6.2	−0.2	+0.7	−2.5	+ 0.9	+0.7	−1.2
14.2......	−2.4	+4.8	+0.6	+0.8	−0.3	− 6.4	−0.8	+0.6
14.8......	−5.3	−0.4	+0.7	+0.7	+1.9	− 5.9	−4.3	+0.8
15.2......	−5.2	−1.7	−0.2	+0.9	+3.0	− 4.8	−4.5	+0.6
15.5......	−7.5	−1.5	−0.8	+2.1	+2.5	− 8.5	−6.2	+0.7
16.4......	−8.2	−0.2	−0.4	+1.9	+5.1	−16.2	−5.4	+1.4
17.1......	−7.3	−0.2	+1.9	−0.4	+2.8	....	−4.5	+1.1
No. scales.	5	2	17	10	5	3	7	32
No. values.	423	156	2326	1654	507	227	579	4487

Although some modification of the scale of *H.C.*, No. 170, has been made by Miss Leavitt in her final discussion of the data,[1] the changes are not great. They affect the divergence for the bright stars only slightly, and that for the faint ones not at all. A detailed comparison with the Harvard scale is reserved for a later paper, but it may be remarked here that the divergence at the lower end of the scale is to be attributed to differences in the methods of reduction. The explanation of that for the bright stars is less obvious, although some information is afforded by a re-reduction of the Harvard plates by methods which could not well be employed in the initial discussion of the data.

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[1] *Harvard Annals*, **71**, Part III, 1914.

TABLE XII

SYSTEMATIC DEVIATIONS OF LONG-EXPOSURE PLATES (UNIT=0.01 MAG.)

Mean Mag.	769	788	795	808	196	200	204	230	232	235
11.1......	+21	−26	−11	−24	− 4	+11	+15	+ 4	− 9	+26
12.2......	+ 2	−22	−17	− 6	−23	+ 6	+10	− 4	0	+15
13.2......	+ 4	− 1	−10	+ 7	− 5	− 2	− 7	+ 2	− 1	+11
13.7......	− 7	+ 4	0	+10	−18	+ 4	−13	−11	+ 4	+12
14.1......	− 4	0	+ 1	+ 2	+ 6	+ 3	+ 3	−11	+ 4	− 2
14.6......	+ 1	+ 6	+ 1	− 6	− 2	+ 5	− 7	− 3	+ 4	−12
15.1......	− 2	+ 7	+ 2	+ 4	+ 7	− 2	+ 9	+ 4	+ 2	− 9
15.6......	+ 2	−12	+ 1	+16	+10	− 2	− 3	+ 3	+ 1	+ 9
16.1......	+ 5	−12	0	+ 7	+14	+ 7	− 1	+ 6	− 4	0
16.6......	+15	− 6	+ 2	+ 6	− 1	+12	− 7	− 2	−11	− 7
17.1......	+19	+ 5	+ 4	− 7	− 5	+ 8	−14	− 6	−19	− 4
17.6......	+15	− 4	+ 7	− 3	−11	+ 9	−12	− 4	−12	0
18.1......	+ 8	−12	+ 9	0	− 2	+ 2	+ 2	− 6	−18	+ 5
18.6......	+25	+27	......	......	+ 6	−10	+16	− 7	−10	+13
19.1......	......	......	......	......	+22	−13	......	− 4	− 7	+18
19.6......	......	......	......	......	......	−30	......	......	......	......
A.D......	14.7	15.6	10.6	15.0	14.6	12.7	16.0	11.0	13.5	12.3
No.......	181	168	179	154	168	300	170	309	292	294
Ap.......	32	32	32	14	6	6	6	14	14	14

Mean Mag.	310	312	313	314	225	229	1343	1347	1550	1561
11.1......	+ 2	− 4	−19	+26	......	+10	+ 3	......	......	......
12.2......	+ 1	+15	− 5	+16	......	− 1	−12	−12	......	......
13.2......	− 3	+ 6	+ 2	+ 5	......	− 1	− 2	+ 2	......	......
13.7......	−10	+11	+ 2	− 7	......	− 6	+ 9	+ 5	......	......
14.1......	0	− 4	+ 3	−11	......	− 2	− 6	− 9	......	......
14.6......	− 1	+ 7	+11	− 8	− 7	− 1	+ 6	− 8	......	......
15.1......	+ 5	− 6	+ 2	−15	− 5	+ 1	+ 2	+ 2	......	+13
15.6......	+ 4	− 5	−13	−13	+ 4	+ 9	+ 1	+ 4	+ 4	−14
16.1......	− 1	+ 4	− 3	−14	+ 8	+ 4	− 2	− 1	− 2	+ 3
16.6......	+ 3	+ 8	− 1	− 7	+ 2	− 5	0	+ 5	......	−22
17.1......	+ 5	+ 8	+ 5	+ 5	− 2	− 1	− 1	+ 3	0	−10
17.6......	+ 2	+ 4	+11	+ 9	− 5	+ 2	− 1	− 4	+15	− 7
18.1......	+ 8	+ 4	+ 9	+ 9	− 4	− 1	− 6	− 7	+ 9	− 2
18.6......	+14	+ 1	+ 4	+15	− 3	− 5	− 7	−10	+ 2	− 4
19.1......	+12	0	− 1	+10	−17	−14	+25	+ 2	+16	0
19.6......	......	......	......	......	−14	......	......	+32	+28	+ 8
A.D......	13.3	13.5	12.3	14.1	12.7	10.7	11.2	11.9	14.8	12.1
No.......	277	307	281	333	323	354	253	267	181	179
Ap.......	14	14	14	32	60	60	60	60	60	60

Since the zero points for the results by Chapman and Melotte and by Dziewulski depend upon the magnitudes of *H.C.*, No. 170, they, too, show large systematic differences when compared with the final Mount Wilson scale. Here again the differences are nearly constant, so that for the region in question the four scales are sensibly parallel; but the agreement becomes even better when it is noted that each of the three series of differences is affected by a more or less pronounced color equation.

By plotting the differences MW−HC, MW−Gr, and MW−Dz against color indices, the following relations were found:

$$\begin{aligned}\mathrm{MW-HC} &= +0.30+0.15\,C\\ \mathrm{MW-Gr} &= +0.23+0.27\,C\\ \mathrm{MW-Dz} &= +0.31-0.06\,C\end{aligned}$$

in which C represents the color index.

The first of these, which is derived from the values of MW−HC for magnitudes 10–15, is a mean result and does not necessarily apply to any given star. The magnitudes of *H.C.*, No. 170, are based upon data obtained with many different instruments, among them the 60-inch reflector at Mount Wilson, and the results enter in varying proportions into the final magnitudes. Although reductions to a standard system were made in the case of stars known to be red, the HC results are not altogether homogeneous, for the colors of the fainter stars, many of which are also red, were not then known. In consequence, each star requires a special correction for color. Moreover, the corrections decrease in amount with increasing magnitude, for with the fainter objects the influence of the Mount Wilson observations entering into the HC results becomes increasingly predominant. The detailed determination of these corrections which reduce the differences to the system of the reflector will be given later. The results, however, appear in Table IX under the heading $\mathrm{MW-HH_w}$.[1]

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[1] The symbol $\mathrm{HH_w}$ designates the magnitude system of *H.C.*, No. 170, after the application of the following corrections: (*a*) corrections to complete the reduction to the standard color system adopted by Miss Leavitt; (*b*) a zero-point correction of +0.08 mag. applied in her final discussion in *H.A.*, **71**, Part III; (*c*) a color correction which refers the results to the color system of the Mount Wilson reflector. The addition of (*a*) and (*b*) to the results of *H.C.*, No. 170, gives what may be called the Harvard Homogeneous Scale (HH). The further application of (*c*) gives the system $\mathrm{HH_w}$. Details of this part of the discussion will appear in *Mt. Wilson Contr.*, No. 98.

The corrected differences for Gr and Dz are similarly shown under the headings MW$-$Gr$_1$ and MW$-$Dz$_1$. For these the color equations given above, including the constant term, have been used as they stand.

With the exception of the beginning and ending of the MW$-$HH$_w$ series, the residuals are very satisfactory, at least for all cases including any considerable number of observations. The parallelism of all four scales is now very good between the ninth and the sixteenth magnitudes.

TABLE XIII

COMPARISON OF SCALES FOR BRIGHT STARS

Star No.	Photographic				Photo-visual			
	MW	MW$-$G	MW$-$Y	MW$-$HH$_w$	MW	MW$-$P	MW$-$Y	MW$-$HC
1*s*....	2.54	....	($-$31)	$-$25	2.08	$-$ 4	$-$12	$-$ 4
1.....	4.39	$-$ 1	$-$ 8	$-$16	4.37	$-$13	$-$13	$-$ 7
2.....	5.30	+10	+ 7	$-$ 1	5.28	+10	$-$ 5	$-$10
3.....	5.83	+ 7	+ 8	$-$ 1	5.56	$-$ 7	$-$ 7	$-$ 6
4.....	5.91	$-$ 7	$-$12	$-$ 8	5.84	$-$ 7	+ 2	$-$ 2
5.....	6.45	$-$ 5	$-$ 6	$-$ 2	6.47	$-$ 4	$-$ 7	+ 4
2*s*....	6.45	0	0	$-$ 2	6.33	+ 2	+17	+ 5
1*r*....	6.61	$-$11	$-$16	$-$26	5.09	+10	+ 5	$-$17
3*s*....	6.64	$-$ 1	+ 2	$-$ 1	6.35	+ 6	+14	+ 2
6.....	7.11	$-$ 9	$-$ 5	+ 4	7.05	$-$ 4	0	+ 4
7.....	7.41	+ 7	+11	+16	7.52	+ 1	$-$ 2	+ 8
2*r*....	7.94	+ 9	$-$13	+ 3	6.35	+ 7	+ 3	$-$20
Z.P. correction .		+ 6	+47			$-$19	+ 9	

Since the zero-point reduction for the Mount Wilson photo-visual magnitudes is only 0.03, the differences in the last column of Table VIII show with sufficient exactness the relation of the final values to the visual magnitudes of *H.C.*, No. 170. As already remarked, there is here also an appreciable color equation, the agreement for the red stars being noticeably better than for the white.

There remains to be noted a comparison of the Mount Wilson results with those of Parkhurst, Müller and Kempf, and Schwarzschild. The differences are shown in Table XIII alongside the corresponding values of MW$-$HH$_w$. To facilitate the comparison, the Göttingen, Yerkes, and Potsdam magnitudes have received the zero-point corrections which appear at the bottom of the table.

In addition, the Yerkes photographic magnitudes have been multiplied by 0.94.[1] There is some evidence that the Yerkes photo-visual magnitudes require a similar correction, but as the case is not altogether clear, no change has been made in the slope of the scale.

X. COLOR OF THE STARS

One of the most interesting results of the investigation is the series of color indices given in Table IX. Attention has already been called to the gradual change in the minimum color index with increasing magnitude.[2] The additional data now available only confirm in all essential particulars the results previously found. At the seventeenth magnitude (photo-visual) the smallest color index appears to be 0.7 or 0.8 mag. The change with increasing magnitude is approximately linear.

SUMMARY

The paper deals with the determination of absolute scales of photographic and photo-visual magnitude within the intervals 2.5–20.0 and 2.0–17.5, respectively. With diaphragms and screens scales were first established for the stars of intermediate brightness. These were then extended in both directions to include both bright and faint stars. The results give photographic and photo-visual magnitudes for 617 and 339 objects, respectively, of which 311 appear in both groups. For these color indices are therefore available. The relation between color index and magnitude previously announced is confirmed.

The internal accordance of the results is satisfactory. With due allowance for color, the photographic scale agrees well from the ninth to the sixteenth magnitude with results found at Harvard, Greenwich, and Potsdam. It also agrees well with the Yerkes and Göttingen magnitudes of bright stars, for which values between 4.4 and 7.9 are available. The divergence from the scale of *H.C.*, No. 170, below the sixteenth magnitude is due to differences of reduction. That for the bright stars has been decreased by the

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[1] *Vierteljahrsschrift der Astron. Gesellschaft*, **47**, 356, 1913.

[2] *Mt. Wilson Contr.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

correction for color, but there are still important differences which will be considered in *Mt. Wilson Contr.*, No. 98.

The photo-visual scale agrees well with other determinations in the region of the bright stars, and coincides with the visual scale of *H.C.*, No. 170, at the twelfth magnitude. For intermediate points, the comparison with the latter scale is less satisfactory and shows the influence of color.

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A COMPARISON OF THE HARVARD AND MOUNT WILSON SCALES OF PHOTOGRAPHIC MAGNITUDE

By FREDERICK H. SEARES

I. INTRODUCTION

For several years the question of standard photographic magnitudes has received much attention at the Harvard Observatory. Announcements of results have been made from time to time,[1] and recently the details and the final magnitudes derived from a long and painstaking investigation, which has been in the hands of Miss Leavitt, have come from the press.[2] In the meantime, the desirability of applying the 60-inch reflector of the Solar Observatory to problems in photographic photometry led to the formulation of methods of observation and reduction adapted for use with that instrument. In order that there might be some control over the results, the first observations were of the stars of the Polar Sequence, which had already been selected by Professor Pickering. Little by little the investigation broadened, and eventually it developed into an independent determination of the photographic and photo-visual scales, not only of the Polar Sequence, but of numerous other stars near the Pole. This investigation is now finished and has been summarized in a previous paper.[3]

A comparison of the Harvard and Mount Wilson results[4] reveals a very satisfactory agreement between the tenth and the fifteenth magnitudes so far as parallelism of the scales is concerned. For both the brighter and the fainter stars there are differences, which, although not greater than might have been anticipated, are sufficient to introduce an uncomfortable degree of uncertainty into the statistical investigations for which precise magnitudes are required. To avoid this difficulty Miss Leavitt has combined the magnitudes of *H.C.*, No. 170, with preliminary results from Mount Wilson,[1] and with various other determinations covering a limited portion of the scale, thus forming a system of somewhat greater weight than that based upon the Harvard data alone. But this procedure can be regarded only as a temporary expedient, for the outstanding differences are so large as to indicate serious systematic errors, and until these can be traced to their origin and eliminated no sense of security can be attained.

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[1] *Harvard Circulars*, Nos. 125, 150, 160, 170 (1907, 1909, 1910, 1912).

[2] *Harvard Annals*, **71**, No. 3, 1914.

[3] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

[4] *Loc. cit.* The Harvard results referred to are those of *H.C.*, No. 170, or of the twelfth column of Table LXVII of Miss Leavitt's memoir. They define the scale derived from the photographs measured and reduced at the Harvard Observatory.

Now that the details of Miss Leavitt's reduction are in print, it is possible to test various hypotheses which might account for the differences between Mount Wilson and the scale of *H.C.*, No. 170; and it is the purpose of this paper to examine critically the data and the results which she has given. A circumstance of great assistance is the fact that we may certainly regard the Harvard scale between the tenth and the fifteenth magnitudes as substantially correct in slope. Not only is the parallelism with Mount Wilson close, but the agreement with the scales of Chapman and Melotte[2] and of Dziewulski[3] is equally good. It is therefore possible to apply to the discussion methods which could not be used for the initial reduction of the data. For example, with the magnitudes between ten and fifteen as an adopted scale, we can in a number of cases derive results for the brighter stars without making any assumption as to the values of the reduction-constants, since these are determined directly from the data, thus eliminating a considerable portion of the errors affecting the individual plates. And by starting from a series of known magnitudes, we can in other cases, namely, those involving exceptionally large values of the constants, avoid certain difficulties which are inherent in the usual process of reduction.

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[1] *Mt. Wilson Contr.*, No. 70; *Astrophysical Journal*, **38**, 241, 1913. In combining the results of *H.C.*, No. 170, with those of Mount Wilson an arbitrary change has been made in the latter between the ninth and tenth magnitudes. Miss Leavitt has interpreted as a break in the scale what appears to be only a consequence of the accidental errors affecting the preliminary magnitudes. As a result, the Mount Wilson determination of the magnitudes of stars fainter than ten, relatively to the international zero-point defined by those of the sixth magnitude, is disregarded, and Miss Leavitt's modified scale below this limit is substantially that of *H.C.*, No. 170.

[2] *Monthly Notices*, **74**, 40, 1913. These results are not independent of the Harvard scale, as they were obtained with the aid of Harvard standard magnitudes. An independent test by means of gratings gave closely accordant results, however.

[3] *Astronomische Nachrichten*, **198**, 65, 1914.

It is thus possible to re-reduce by independent methods a part of the data upon which the Harvard magnitudes are based. An account of this re-reduction will be found in the following pages. The least satisfactory in its results is that part of the present discussion relating to the stars brighter than the tenth magnitude. The deviation for the faint stars, on the other hand, is easily explained. Since the Harvard scale in this region depends exclusively upon Mount Wilson plates which have also been used in determining the Mount Wilson magnitudes, the divergence must be due to differences in measurement and reduction. Such is found to be the case, and the exact point at which they enter is easily specified.

Before proceeding to the detailed discussion of these two divergent regions of the scale, the effect of color upon the Harvard results must be briefly considered.

II. COLOR-CORRECTIONS AND THE ADOPTED SCALE

The Harvard results are based upon 20 groups of plates for each of which magnitudes on an absolute scale were derived by the methods and instruments listed in Table I. These constitute Section A of the results. Four other sections, namely, B, C, D, E, include results from plates which were reduced to the scale of Section A. Section A therefore determines the scale, while the remaining sections afford material for the reduction of the accidental errors affecting individual stars. The mean of the five sections[1] is the scale of *H.C.*, No. 170. A zero-point correction of +0.08 magnitude was subsequently applied and the results were combined with those of other observers to form the final scale of *H.A.*, **71**. It is, however, the mean result from Sections A to E of Miss Leavitt's investigation plus the zero-point correction of 0.08 magnitude, in which we are at present interested. For convenience this series of magnitudes will be referred to as the Harvard Scale (HS). Hence,

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[1] Excepting small differences for a few of the faint stars.

HS=System A to E+0.08=System *H.C.*, No. 170 +0.08

As stated in *Mt. Wilson Contr.*, No. 97, this scale is not altogether homogeneous with respect to the influence of color. For all cases of known color Miss Leavitt has applied corrections which reduce the results to the system of the AC (Cooke 1-inch) and I (Draper 8-inch) instruments. But most of the faint stars, whose colors were then unknown, are red or reddish and also require appreciable corrections. The completion of the reduction to a homogeneous color-system therefore first requires our attention. Although a tedious operation, this is easily accomplished with the color-indices now available.

TABLE I

HARVARD OBSERVATIONS—SECTION A

Group	Mag. Range	Method	Instrument	Δm	Aperture
					mm
1......	2.7– 6.7	Different foci	11-in. Draper	....	....
2......	2.7– 8.7	Iceland spar	8-in. Draper	2.50	....
3......	4.5– 8.9	Pleiades comp.	Various	....	....
4......	6.4–10.6	Praesepe comp.	1-in. Cooke	....	....
5......	2.7–11.3	Screen B2	1-in. Cooke	2.50	....
6......	8.9–15.5	Screen C	16-in. Metcalf	1.55	....
7......	8.9–13.7	Screens C, D, E, B1	16-in. Metcalf	1.15–2.50	....
8......	8.9–13.3	Whole and half ap.	11-in. Draper	0.75	....
9–12...	2.7–14.5	Prismatic comp.	8-in. Draper	5.00	211–25, 20
13......	2.7–14.3	Pleiades comp.	Various	5.00	211–25, 20
14......	2.7– 7.7	Circ. diaphragm	1-in. Cooke	1.50	8, 4
15......	2.7–11.9	Circ. diaphragm	1-in. Cooke	2.28	26.5, 9.3
16......	8.9–14.3	Circ. diaphragm	11-in. Draper	2.00	280, 112
17......	8.9–15.9	Circ. diaphragm	11-in. Draper	2.00	280, 112
18......	16.0–19.0	Circ. diaphragm	60-in. Refl.	4.81	1511–584, 152
19......	10.5–18.9	Circ. diaphragm	60-in. Refl.	2.97	1511–584, 356
20......	10.5–19.1	Circ. diaphragm	60-in. Refl.	2.97	1511–584, 356

The color-equations of the various instruments were found by comparing the Mount Wilson color-indices[1] with the differences on pp. 196–198 of Miss Leavitt's memoir.[2] Certain equations controlling the consistency of the results were also obtained by comparing the Mount Wilson magnitudes with the various groups of Sections A and B in L, 186–188, 195. To avoid the influence of differences of scale for the bright and the faint stars, only those stars between magnitudes ten and fifteen (Nos. 12 to 14*s*, L, 186–188) were used in the comparisons with Mount Wilson. Disregarding constant terms, the results are as shown in Table II.

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[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, 41, 206, 1915.

[2] *Op. cit.* For brevity this will be referred to by the letter L followed by the page number.

TABLE II

COLOR EQUATIONS

Equation	Source
MW−MC = +0.33 C	Section A Group 6, 7
MW−C = +0.08 C	" " 17
MW−I = +0.06 C	" " 9–12
MW−I = +0.06 C	" B " 23–30
I−AI = −0.13 C	" B compared with Section C, Gr. 31
I−ACe = +0.10 C	" " " " " " " 32, 34
I−L = +0.14 C	" " " " " " " 37–39
I−MC = +0.24 C	" " " " " " " 48–53
I−ACe red. = −0.07 C	" " " " " D " 62
I−L = +0.28 C	" " " " " " " 64
I−MC = +0.23 C	" " " " " " " 66
I−ACe = −0.14 C	" A and C revised with E " 68

The symbols in the left members of the equations refer to the instruments listed in L, 55. C in the right members is the Mount Wilson color-index. Beginning with the fifth equation, I includes the AC (later 1-inch Cooke) instrument as well as the 8-inch Draper and signifies the standard system chosen by Miss Leavitt. Owing to a lack of data she was unable to make a direct comparison of the color-systems of these two telescopes; but the differences used in deriving the foregoing equations for L (4-inch Cooke), which include AC as well as I results, show that they afford a really homogeneous system.

Although the coefficient for the second L comparison is twice that of the first, the evidence seems unmistakable; the difference shows clearly in both the AC and the I results. Again, the two ACe comparisons give discordant coefficients; but there seems here also to be a real difference, although the first comparison is not very reliable. On the other hand, the large MC (16-inch Metcalf) correction is constant and well determined. Results from the other instruments appearing in Miss Leavitt's discussion agree sensibly with the standard system.

The next operation was the revision of the mean magnitudes for each of the five sections. Beginning with Section A the results for each group were corrected with the aid of the foregoing equations. The mean magnitudes for individual stars were then formed exactly as described in L, 185, thus giving results which define the homogeneous scale, just as the original results for this section determined the HS. The remaining sections were then similarly treated, and finally each received a correction to reduce it to the revised scale of Section A, a procedure necessitated by the fact that the color-corrections modify the results for the different sections by different amounts. This reduction was made graphically, and the results for all sections were then collected in a table similar to that in L, 207–209. The weighted means were then found for each star in the manner followed by Miss Leavitt (L, 206) and corrected by +0.08 mag. for zero-point, thus giving what may be called the Harvard Homogeneous Scale (HH). In revising the results care was taken throughout to make allowance for the color-corrections already applied by Miss Leavitt (L, 204).

The agreement of the results from the different sections was very good before the revision, the average deviation of the mean magnitude for a single section being only 0.022 mag.; the reduction to a homogeneous system has decreased this to 0.019 mag., which is an indication of the remaining accidental errors. The main significance of the results, however, is to be found in the fact that, owing to the systematic dependence of color upon magnitude,[1] the scale itself has been appreciably modified. The amount of this change is indicated by the quantities in the fourth column of Table III. The application of these corrections to the values of HS gives the Harvard Homogeneous magnitudes in the HH column.

The comparisons of Mount Wilson with the original results for Sections A and B revealed an appreciable color-equation between the Mount Wilson and AC-I systems. This same relation should appear from the differences MW−HH in Table III; and, in fact, we find

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[1] *Mt. Wilson Contr.*, No. 81; *Astrophysical Journal*, **39**, 361, 1914.

$$MW-HH=+0.24+0.06\ C,$$

which is in agreement with the results in Table II. Star 11*s* (MW 15.31) was the faintest object used in deriving this relation, although all the brighter objects excepting 1*s* were included after applying a correction for scale-difference between HH and MW. This correction has the form

$$MW-HH=+0.061\ (HH-10),$$

which holds for objects brighter than 10.

For the final comparison of the Harvard and Mount Wilson results, we must reduce them to the same color system by means of the first of the foregoing equations. The necessary corrections for the reference of HH to the Mount Wilson system are in the column headed HH_w-HH, and their subtraction from MW−HH shows the outstanding differences of scale between Mount Wilson and Harvard. These appear under the heading $MW-HH_w$ in Table IX of *Mount Wilson Contr.*, No. 97; but for the purpose of the present paper it is convenient to have the two scales coincide from the tenth magnitude onward. On this account the constant term of 0.24 in the color equation was also taken into account in forming the differences $MW-HH_w$ in Table III.

By way of recapitulation, the latter differences indicate the relation between the Harvard (HS) and Mount Wilson scales after the former has been subjected to the following operations: (1) reduction to a homogeneous color system; (2) correction for color to reduce to the MW system; (3) correction for zero-point to produce coincidence with MW in the region of parallelism (mags. 10–15). The last column of Table III shows that in the intermediate region the agreement is very satisfactory. Between the ninth and sixteenth magnitudes there are but two differences exceeding 0.10 mag., namely, those from the red stars 4*r* and 6*r*.

Neither of the color-corrections is very large, but together they form an appreciable quantity, which is important because of its differential effect between bright and faint stars. Both enter in a direction such as to decrease the original divergence between HS

TABLE III

CORRECTIONS FOR COLOR AND COMPARISON OF SCALES

Star	MW	HS	HH *minus* HS	HH	CI	MW *minus* HH	HH$_W$ *minus* HH	MW *minus* HH$_W$
1s....	2.54	2.79	....	(2.79)	0.46	(−25)	....	(−49)
1.....	4.39	4.55	0	4.55	0.02	−16	0	−40
2.....	5.30	5.32	− 1	5.31	0.02	− 1	0	−25
3.....	5.83	5.82	0	5.82	0.27	+ 1	+ 2	−25
4.....	5.91	5.99	0	5.99	0.07	− 8	0	−32
2s....	6.45	6.46	0	6.46	0.12	− 1	+ 1	−26
5.....	6.45	6.47	0	6.47	−0.02	− 2	0	−26
3s....	6.64	6.62	+1	6.63	0.29	+ 1	+ 2	−25
1r....	6.61	6.77	+1	6.78	1.52	−17	+ 9	−50
6.....	7.11	7.06	+1	7.07	0.06	+ 4	0	−20
7.....	7.41	7.26	0	7.26	−0.11	+15	− 1	− 8
2r....	7.94	7.82	−1	7.81	1.59	+13	+10	−21
8.....	8.32	8.18	+2	8.20	0.19	+12	+ 1	−13
3r....	8.96	8.70	+1	8.71	1.40	+25	+ 8	− 7
9.....	8.88	8.78	+1	8.79	0.07	+ 9	0	−15
10.....	9.12	8.97	0	8.97	0.05	+15	0	− 9
4r....	9.22	9.05	0	9.05	0.96	+17	+ 6	−13
11.....	9.73	9.50	+2	9.52	0.20	+21	+ 1	− 4
12.....	10.09	9.81	+2	9.83	0.29	+26	+ 2	0
5r....	10.13	9.86	+2	9.88	1.48	+25	+ 9	− 8
4s....	10.25	10.07	−1	10.06	0.42	+19	+ 2	− 7
13.....	10.53	10.25	+1	10.26	0.16	+27	+ 1	+ 2
6r....	10.46	10.25	+2	10.27	1.25	+19	+ 8	−13
14.....	10.98	10.60	+2	10.62	0.44	+36	+ 3	+ 9
7r....	10.94	10.65	+1	10.66	1.04	+28	+ 6	− 2
5s....	11.09	10.76	+1	10.77	1.03	+32	+ 6	+ 2
15.....	11.22	11.02	+1	11.03	0.33	+19	+ 2	− 7
6s....	11.38	11.05	+3	11.08	0.65	+30	+ 4	+ 2
8r....	11.44	11.14	0	11.14	1.00	+30	+ 6	0
16.....	11.62	11.34	+1	11.35	0.39	+27	+ 2	+ 1
17.....	11.87	11.57	+4	11.61	0.59	+26	+ 4	− 2
9r....	....	11.64	+2	11.66	1.20	....	+ 7	....
18.....	12.27	12.00	+3	12.03	0.38	+24	+ 2	− 2
10r....	....	12.29	+3	12.32	0.72	....	+ 4	....
7s....	12.61	12.39	+3	12.42	0.51	+19	+ 3	− 8
19.....	12.69	12.36	+3	12.39	0.41	+30	+ 2	+ 4
20.....	13.02	12.67	+3	12.70	0.51	+32	+ 3	+ 5
21.....	13.33	12.90	+9	12.99	0.85	+34	+ 5	+ 5
11r....	13.22	12.91	−1	12.90	1.16	+32	+ 7	+ 1
22.....	13.44	13.13	+5	13.18	0.63	+26	+ 4	− 2
23.....	13.60	13.28	+4	13.32	0.50	+28	+ 3	+ 1
12r....	13.84	13.42	+3	13.45	1.38	+39	+ 8	+ 7
24.....	13.93	13.58	+3	13.61	0.60	+32	+ 4	+ 4
25.....	14.08	13.81	+2	13.83	0.48	+25	+ 3	− 2
8s....	14.49	14.14	+4	14.18	0.71	+31	+ 4	+ 3
26.....	14.64	14.27	+4	14.31	0.92	+33	+ 6	+ 3
9s....	14.75	14.41	+4	14.45	0.90	+30	+ 5	+ 1
27.....	14.91	14.63	+3	14.66	0.58	+25	+ 3	− 2
10s....	15.29	14.96	+3	14.99	0.80	+30	+ 5	+ 1
11s....	15.31	14.99	+4	15.03	0.96	+28	+ 6	− 2
28.....	15.27	15.05	+3	15.08	0.73	+19	+ 4	− 9

TABLE III—*Continued*

Star	MW	HS	HH *minus* HS	HH	CI	MW *minus* HH	$\mathrm{HH_W}$ *minus* HH	MW *minus* $\mathrm{HH_W}$
12*s*....	15.33	15.05	+3	15.08	0.64	+25	+ 4	− 3
13*s*....	15.54	15.17	+4	15.21	1.00	+33	+ 6	+ 3
29.....	15.82	15.60	+2	15.62	0.61	+20	+ 4	− 8
14*s*....	15.99	15.69	+3	15.72	0.92	+27	+ 6	− 3
30.....	16.18	15.98	+2	16.00	0.74	+18	+ 4	−10
31.....	16.41	16.16	+2	16.18	0.79	+23	+ 5	− 6
15*s*....	16.57	16.36	+1	16.37	0.86	+20	+ 5	− 9
32.....	16.76	16.56	+1	16.57	1.18	+19	+ 7	−12
16*s*....	16.86	16.66	+1	16.67	1.36	+19	+ 8	−13
33.....	17.06	16.90	+2	16.92	1.09	+14	+ 7	−17
17*s*....	17.19	17.05	+1	17.06	1.30	+13	+ 8	−19
34.....	17.24	17.19	0	17.19	0.95	+ 5	+ 6	−25
35.....	17.63	17.44	+2	17.46	0.69	+17	+ 4	−11
36.....	17.78	17.69	−1	17.68	0.98	+10	+ 6	−20
37.....	18.01	17.88	−1	17.87	1.20	+14	+ 7	−17
18*s*....	17.94	17.85	−2	17.83	1.03	+11	+ 6	−19
38.....	18.20	18.25	−2	18.23	1.15	− 3	+ 7	−34
19*s*....	18.16	18.44	−3	18.41	1.21	−25	+ 7	−56
39.....	18.58	18.61	−3	18.58	1.45	0	+ 9	−33
20*s*....	18.60	18.64	−3	18.61	1.41	− 1	+ 8	−33
21*s*....	18.66	18.82	−4	18.78	1.33	−12	+ 8	−44
22*s*....	18.75	18.87	−4	18.83	1.62	− 8	+10	−42
23*s*....	18.70	19.05	−5	19.00	1.29	−30	+ 8	−62
24*s*....	18.88	18.96	−8	18.88	1.54	0	+ 9	−33
40.....	18.87	18.97	−4	18.93	1.58	− 6	+ 9	−39
25*s*....	18.84	19.11	−5	19.06	1.46	−22	+ 9	−55
41.....	19.02	19.22	−5	19.17	1.55	−15	+ 9	−48

and MW for the bright stars. Thus the unexplained difference between the sixth and tenth magnitudes has been reduced from 0.40 mag.[1] to 0.24 mag.

We now turn to the question whether this remaining divergence, as well as that for the faint stars, is capable of further reduction. In accordance with the explanation on p. 2 we adopt for this investigation, as accurately known, the HH magnitudes between ten and fifteen, and proceed first to an extension of this scale in the direction of the brighter stars.

III. THE DIVERGENCE FOR THE BRIGHT STARS

An examination of Table I shows that the groups of observations there listed may be classified as follows: (*a*) Groups 1–4 and 14, including stars brighter than the ninth magnitude (approximately);

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[1] *Mt. Wilson Contr.*, No. 97, p. 16; *Astrophysical Journal*, **41**, 221, 1915.

(*b*) Groups 6–8, 16, and 17, including stars fainter than the ninth magnitude; (*c*) Groups 5, 9–13, and 15, which connect the results from (*b*) with those of (*a*).

The range in (*c*) is from about the third to the fourteenth or fifteenth magnitudes. The zero-point of the magnitudes fainter than 9 depends wholly upon the connecting groups in this division. Groups 18–20 include the fainter stars and are not considered here.

Since the Harvard scale, except for accidental errors, depends exclusively upon the observations of Section A, it is necessary to examine in detail the data for the above-mentioned groups. These we now consider in the order specified.

a) Groups 1–4 and 14

The reductions for this division have not been carefully examined as it is not possible to apply here independent methods of reduction; moreover, the data are not always accessible. The agreement for the different groups is good (see residuals L, 187), although individual plates are sometimes rather discordant (L, 153). One would anticipate that accordance of results by such widely different methods as varying foci, Iceland spar, and circular diaphragms would be strong evidence of a high degree of precision in the mean scale. But we also find close agreement among the Göttingen, Yerkes, and Mount Wilson results[1] for this region (4.0–7.5), and yet these three series of magnitudes show a mean divergence of about 6 per cent from the Harvard scale.

It is of interest to note that the same divergence between Harvard and Göttingen appears when the results for the Pleiades are compared. The Harvard magnitudes for the latter region were derived from Iceland spar and prismatic companion-plates (L, 53, 154, 176) and have been used for the reduction of Groups 3 and 13 of the Polar Sequence. Their deviations from the Göttingen results are shown in the fifth column of Table IV. Inasmuch as the Göttingen series[2] does not include fainter stars, I have extended the scale to the ninth magnitude with the aid of Schwarzschild's Vienna observations.[3]

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[1] *Mt. Wilson Contr.*, No. 97.

[2] *Aktinometrie*, B, p. 14, 1912.

[3] "Beiträge zur photographischen Photometrie der Gestirne," *Publ. der von Kuffner'schen Sternwarte*, **5**, 62, 1900.

For further comparison, Hertzsprung's magnitudes[1] of the brighter Pleiades stars (diminished by 0.26 mag.) are also included in Table IV. Although derived by different methods, the Göttingen and Hertzsprung series are in close agreement, and hence the latter also shows the characteristic deviation from Harvard.

TABLE IV

COMPARISON OF SCALES FOR THE PLEIADES

Harvard No.	Bessel No.	Gött.	Hertz.	Gött.−H.	Hertz.−H.
1.........	*η*	2.87	2.85	−14	−16
2.........	*f*	3.57	3.59	− 9	− 7
3.........	*b*	3.59	3.67	− 8	0
4.........	*c*	3.82	3.85	−12	− 9
5.........	*d*	4.21	4.13	− 7	−15
6.........	*e*	4.22	4.20	− 7	− 9
7.........	*h*	4.94	....	+ 4	....
8.........	*g*	5.37	5.38	+ 5	+ 6
9.........	*k*	5.69	5.70	− 3	− 2
10.........	*l*	6.35	6.34	+ 4	+ 3
11.........	24	6.84	6.87	+ 3	+ 6
12.........	29	7.02	7.01	+ 2	+ 1
13.........	4	7.25	7.28	− 1	+ 2
14.........	10	7.43	7.38	+ 6	+ 1
15.........	39	7.57	7.52	+16	+11
16.........	37	7.70	7.55	+14	− 1
17.........	33	8.15	8.14	+16	+15
18.........	....	....	8.67	....	+28
19.........	21	8.94	8.82	+25	+13
20.........	....	....	8.86	....	+ 6
21.........	2	9.00	8.95	+14	+ 9
23.........	36	9.26	9.10	+10	− 6

Some caution must be exercised, however, in judging of scale differences in cases like the present, for there is a progressive change in color with increasing magnitude, and hence a possibility of confusing a color-equation with differences in scale. An examination of the circumstances seems to exclude such an explanation, however.

b) Groups 6–8, 16, and 17

As the scales established by these groups usually begin at the ninth magnitude, they cover but one magnitude of the divergent region which includes the brighter stars. The calculation for the few stars falling within this interval is easily controlled with the aid of the adopted magnitudes in the region 10–15.

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[1] *Astronomische Nachrichten*, **186,** 181, 1910.

The plates show at least two exposures—full aperture (*a*) and reduced aperture (*b*). The Harvard scale readings for (*a*) and (*b*) were plotted against the adopted magnitudes as ordinates, beginning in each case with Star 13 (10.26). The two curves thus defined should be parallel, and the difference in corresponding ordinates should equal the reduction constant Δm. Any deviation from parallelism, or any lack of agreement with the adopted constant Δm, indicates an abnormality which must affect the scale derived from the plate in question. Non-parallelism of the curves signifies differences in gradation for the two exposures. Lack of equality between the ordinate-difference and Δm means that the effective reduction-constant differs from the adopted value. This may be caused by changes in transparency or in the sensitiveness of the plate during the exposures. With the method of reduction here used this is of no consequence, for as long as the curves are parallel, or nearly so, it is only necessary to displace the *a*-curve vertically until it coincides with the *b*-curve. This automatically extends the *b*-curve into the region of smaller scale-readings and gives a curve from which the magnitudes of the brighter stars of the *b*-exposure may be interpolated. The results are independent of changes in transparency, of the phenomenon affecting first and last exposures,[1] and of errors in the adopted reduction-constants.

By an extension of this process it is also possible to make use of scale-readings upon the images of the bright stars shown by the *a*-exposure, when such are present. This tends to diminish the accidental errors, but does not otherwise contribute toward a knowledge of the scale. A complete reduction of all the plates was made by both processes before the necessity of a detailed examination of the influence of color became apparent, although only the results by the simpler process first outlined have been used. A third reduction was subsequently made for Groups 6 and 7, which relate to the Metcalf 16-inch and are most likely to be influenced by color. But inasmuch as the first two reductions had been based upon white stars alone, the final reduction of Groups 6 and 7, in which the influence of color was taken into account, made no appreciable difference.

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[1] *Mt. Wilson Contr.*, No. 64, p. 7; *Astrophysical Journal*, **36**, 374, 1912.

Nearly all of the plates in the five groups of this division appear to be of very satisfactory quality. The magnitude-curves deviate from parallelism by only a few hundredths of a magnitude, and in all cases the calculated reduction-constant is nearly equal to the value adopted by Miss Leavitt.

The results for each plate are shown in Table V in the form of corrections to the HH magnitudes of Table III. The last two lines exhibit the agreement between the observed and the theoretical reduction-constants. The first series of values for Plate 229 depends upon the mean of two reduced-aperture exposures with screens C and D, respectively (L, 159), and has accordingly been given double weight. Plate 186 has also received double weight because of exceptional quality (L, 63) and duplicate measures. In addition to the four stars listed in Table V, corrections were derived from Plate 248 for four others as follows:

No. 6, −21; No. 2*r*, −29; No. 8, −24; No. 9, −24

The mean corrections are characterized by a rather marked persistence of the negative sign. The probable errors, which are expressed in thousandths of a magnitude, suggest that the HH magnitude for Star 10, and perhaps that for No. 12, actually require small corrections.

c) The Connecting Groups 5, 9–13, and 15

We now consider the important groups connecting the bright stars with those fainter than the tenth magnitude. With the exception of Group 13, the data have been independently reduced, partly by methods equivalent to those used by Miss Leavitt, and partly by other processes.

Group 5 (wire screen B2, $\Delta m = 2.50$, on 1-inch Cooke anastigmat, two exposures of 30^{m}).—Since the faintest star registered on the three plates of this group is 11.2, the magnitudes between 10 and 15 are of no assistance in deriving the scale or in testing the value of the reduction-constant. The usual method of reduction[1] has therefore been applied.

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[1] *Mt. Wilson Contr.*, No. 80, p. 24; *Astrophysical Journal*, **39**, 330, 1914.

TABLE V

CORRECTIONS TO HH FROM DIVISION (*b*)

Star	Group [illegible]			Group 7 [illegible]					Group 8				Group 16			Group 17		Mean	P.E.	Wt.
	231	232	233	229		240	243	248	166	167	170	171	182	184	185	186	194			
10........	−10	−22	+ 5	−26	−23	−13	+ 7	− 3	−24	+11	+24	− 9	− 6	−15	− 6	−10	−12	− 9	019	19
11........	− 2	+24	+ 3	−15	− 6	− 5	+14	+14	.....	.....	− 6	.....	.....	+ 9	− 5	− 2	+ 3	+ 1	018	15
12........	− 9	+17	0	−27	− 8	− 1	+ 2	+ 8	− 3	−18	− 5	+15	− 5	+ 2	− 6	−11	+ 9	− 4	017	19
4*s*.......	−19	+ 6	+11	+ 5	−14	−20	+ 6	0	.....	.....	−15	.....	.....	.....	.....	.....	+ 1	− 3	026	11
Obs. Δm..	1.60	1.50	1.43	1.50	1.12	1.52	2.40	2.52	0.73	0.72	0.77	0.80	2.00	1.95	1.90	2.10	1.86			
Theor. Δm.	1.55	1.55	1.55	1.56	1.15	1.55	2.50	2.50	0.75	0.75	0.75	0.75	2.00	2.00	2.00	2.00	1.90			

Although there are differences for individual plates, the mean scale for the group is substantially that found by Miss Leavitt, from which it may be inferred that our reduction-processes are equivalent. The zero-point was determined by making the mean magnitudes of the eight or ten stars fainter than magnitude 10 equal to the mean of the corresponding HH magnitudes. For stars brighter than magnitude 10 there is a preponderance of negative corrections (i.e., the HH magnitudes are relatively too faint) and a divergence in the direction of the Mount Wilson results (compare residuals, L, 187).

A point of fundamental importance is the reduction-constant, which was calculated from the measured dimensions of the mesh on the assumption that the absorption in magnitudes for point sources is twice that for luminous surfaces. The screen is of rather fine wire gauze (L, 159): wires 0.0940 mm, spaces 0.1206 mm), and for dimensions such as these the assumption that the constant, Δm, is twice the absorption for luminous surfaces is not entirely justifiable. Du Bois and Rubens have investigated the question[1] and their results indicate that for this case the double of the absorption for surfaces should be increased by 0.05 mag., in order to obtain the constant for point-sources. This correction makes the magnitudes of the bright stars relatively brighter, the change between 6 and 10 being 1.6 times the correction, or 0.08 mag. The final results, which are based upon the revised constant ($\Delta m = 2.55$), appear in the fourth column of Table X. As before, they are expressed as corrections which, applied to the HH magnitudes, will yield the values derived from the re-reduction of the group.

Groups 9–12 (prismatic companions, with the 8-inch Draper telescope).—For Groups 9–11, the exposures, with one exception, were 10^m. For Group 12, they were 60^m, excepting two plates which received 93^m and 120^m, respectively. The reduction-constant calculated from the clear aperture of the prism (cemented to the center of the objective) and the unobstructed area of the objective is 5.10 magnitudes. This requires correction for differential absorption owing to the different thicknesses of glass traversed by the two beams—that directly transmitted and that deflected by the prism. It was not found possible to determine this directly, and in consequence the following procedure was adopted (L, 172):

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[1] *Annalen der Physik*, **49**, 593, 1893.

> It was shown above that the center of the objective of the 11-inch Telescope transmits more light than the average for the entire lens by at least 0.1 magn. If we assume a similar effect in both the lenses of which the 8-inch Draper Doublet is composed, we shall have a correction of −0.2 magn. to the computed difference. On the other hand, we may allow a loss of light of 0.1 magn. arising from the reflection and absorption of the small prism. The adopted difference is $5.1-0.2+0.1=5.0$.

The fact that the absorption for the center of the 11-inch Draper objective is less than the average for the whole objective indicates that the main factor involved is not absorption in the ordinary sense, but residual aberrations in the optical system. These, however, are peculiar to each instrument; and the assumption that what applies to the 11-inch Draper also applies, and in the manner indicated, to the 8-inch instrument, seems open to question. One is therefore inclined to suppose that the adopted value of the constant is subject to some uncertainty. The circumstance is unfortunate, as the bulk of the material (23 plates) connecting the fainter stars with the international zero-point is contained in these four groups.

The matter is further complicated by the fact that the reduction of plates involving so large a constant presents special difficulties. These are pointed out by Miss Leavitt, and to meet them in so far as possible she has employed a special process (L, 147).

The uncertainty arising from this source can now be removed by a method similar to that described on p. 12. With the aid of the adopted scale between 10 and 15 we can derive for the brighter stars magnitudes which are free from systematic errors arising from the reduction-process; but in this instance we are obliged to assume the validity of the reduction-constant, for only a few of the brighter stars of known magnitude show prismatic companions.

We plot the scale-readings of the primary exposures against the HH magnitudes which are fainter than 10, and read from the curve thus defined the magnitudes corresponding to the scale-readings of the prismatic companions. The subtraction of five magnitudes from each value gives the required results. Prismatic companion readings which do not fall within the limits of the curve cannot at once be utilized. Nor can the readings upon bright stars of the primary exposure be used directly; but by plotting these against the corresponding magnitudes found by the process just described, a curve will be defined which should be an extension of that originally drawn. If, however, an erroneous value of Δm has been used, or if there are differences of gradation between the two exposures, a discontinuity will occur, and the curves must be adjusted so that a smooth junction is effected.

The use of the adjusted curves tends to eliminate gradation-differences and reduce the accidental errors of observation, but in the mean of several plates probably does not materially affect the result. They have not been used in the present case, because the majority of the bright stars are beyond the limit of distance from the center of the plate for which the primary images may be used, and, further, because it was desired to bring into view any peculiarities affecting the method of prismatic companions.

The corrections to the HH magnitudes for the individual plates of the four groups are in Tables VI and VII. The preponderance of negative signs is at once evident and indicates a systematic difference between the results of the present reduction and that of Miss Leavitt. The difference appears clearly in the mean residuals for Groups 9–11, although not in those of Group 12 (Table VIII). For individual plates, the difference is not constant, but shows a marked progression—see, for example, Plate 84 of Group 9 and Plate 116 of Group 12—which suggests either differences in gradation between the two series of images or systematic errors of measurement which easily arise when the secondary images are not closely comparable with those of the primary series. A difference in the slope of the HH scale for the bright stars as compared with that in region of magnitudes 10 to 15 would also produce a progressive change in the residuals; but any such progression would be the same for all the plates.

The systematic difference between the mean residuals for Groups 9–11 and Group 12 is rather remarkable. Two circumstances may be noted: first, the average exposure-time for the last group was about six times that for the others, and, second, four of the plates of

Group 12 either presented some abnormality or were difficult to measure (L, 102, remarks). On the other hand, Plates 98 and 103 were measured more than the usual number of times and by two different observers (L, 62, 63).

TABLE VI

RESIDUALS, GROUPS 9 AND 10

Star	Group 9					Group 10				
	84	85	94	95	104	105	106	107	113	118
2s	+14	−18	+ 8	+16	+ 8	......	......	......	......	......
5	+13	+ 1	+11	+ 6	−23	−34	−27	−27	−17	+ 6
6	+15	+ 1	−13	−14	+16	−29	+ 2	−23	−13	+14
7	+16	−24	−23	+32	− 4	......	−18	−27	−18	−13
2r	−18	−15	− 5	−14	......	−25	+ 9	−26	−13	−13
8	−24	−22	−20	−51	− 7	− 5	− 5	− 6	−30	+ 7
3r	−27	−11	− 2	−24	......	......	......	......	......	......
9	−14	− 9	−22	−10	− 7	+ 8	−12	−48	+ 5	− 3
10	−43	− 6	− 9	−41	−16	−18	−19	− 5	+ 9	−28
4r	......	......	− 4	−37	− 5	−19	−32	−47	− 7	− 5
11	−25	+15	+ 7	− 8	0	......	−26	−19	......	−33
12	......	− 5	− 3	−18	......	......	−26	−39	......	......
5r	......	......	......	−16	......	......	......	......	......	......
4s	......	......	......	−17	......	......	......	......	......	......

TABLE VII

RESIDUALS, GROUPS 11 AND 12

Star	Group 11					Group 12							
	97	98	99	103	109	108	114	121	148	150	115	116	117
2s	.....	−24	.....	+ 2	.....	.....	.....	+24	.....	.....	.....	.....	....
5	+10	−21	.....	−21	−42	+23	+ 1	+23	+37	+68	+ 5	−14	+ 3
1r	.....	.....	.....	.....	.....	.....	−33	.....	.....	.....	.....	.....	....
6	+ 8	−27	−41	−12	+ 4	+12	0	+25	−11	+20	+18	−21	+ 4
7	.....	−29	.....	−17	+ 9	−15	− 6	+40	.....	.....	.....	0	....
2r	+19	−10	−26	−33	+ 6	.....	−21	+17	.....	.....	− 3	− 3	+ 7
8	+14	−20	− 4	−54	− 5	−29	− 9	+24	+26	+ 9	−22	+11	− 4
3r	.....	+ 4	.....	−34	.....	−17	.....	+ 4	.....	.....	.....	+ 3	....
9	+ 2	.....	− 1	−39	+18	−15	−10	+ 5	0	−18	−17	+ 5	− 3
10	−27	−12	− 9	− 4	+21	−13	+ 2	−14	− 9	+ 1	− 8	− 2	+19
4r	.....	−34	.....	−25	+13	−32	− 6	− 2	.....	.....	− 6	+13	+32
11	.....	.....	−13	−35	−12	− 4	−21	+ 4	+25	− 6	− 8	+ 6	+20
12	.....	.....	.....	.....	.....	−44	+14	− 6	+13	+ 5	−29	− 1	+ 1
5r	.....	.....	.....	.....	.....	+16	.....	− 1	.....	.....	.....	+11	....
4s	.....	.....	.....	.....	.....	−39	−12	−22	− 2	−21	− 8	+31	−37

One is tempted to speculate as to the possibility of an influence depending upon exposure-time, for it is conceivable that the image of a prismatic companion might increase in diameter with increasing exposure at a rate different from that of the image produced by the remainder of the objective; but the data are too slender to permit any definite conclusion.

Group 13.—The results from this group were obtained by comparisons of the Pole with the Pleiades, for which a scale was established by the method of prismatic companions. As there are no Mount Wilson results for the Pleiades, a direct comparison cannot be made, but the divergence of the Harvard magnitudes for this region from those of Göttingen and Potsdam has already been referred to in connection with Table IV. As the method used for Groups 9–12 and several of the plates included in Group 11 (Nos. 97–99, 103) were also used for Group 13, it is to be inferred that the results found above also apply here.

TABLE VIII

MEAN RESIDUALS, GROUPS 9–12

Star	HH Mag.	Mean Residuals				No. Values	Wtd. Mean	P.E.	Wt.
		9	10	11	12				
2*s*..........	6.46	+ 6	.....	−11	+24	5, 0, 2, 1	+ 4	038	8
5..........	6.47	+ 2	−20	−18	+18	5, 5, 4, 8	− 1	035	22
1*r*..........	6.78	.....	.....	.....	−33	0, 0, 0, 1	−33	.....	1
6..........	7.07	+ 1	−10	−14	+ 6	5, 5, 5, 8	− 3	027	23
7..........	7.26	− 1	−19	−12	+ 5	5, 4, 3, 4	− 6	034	16
2*r*..........	7.81	−13	−14	− 9	− 1	4, 5, 5, 5	− 9	024	19
8..........	8.20	−25	− 8	−14	+ 1	5, 5, 5, 8	−10	028	23
3*r*..........	8.71	−16	.....	−15	− 3	4, 0, 2, 3	−12	035	9
9..........	8.79	−12	−10	− 5	− 7	5, 5, 4, 8	− 8	019	22
10..........	8.97	−23	−12	− 6	− 3	5, 5, 5, 8	−10	020	23
4*r*..........	9.05	−15	−22	−15	0	3, 5, 3, 6	−12	035	17
11..........	9.52	− 2	−26	−20	+ 2	5, 3, 3, 8	− 7	026	19
12..........	9.83	− 9	−32	.....	− 6	3, 2, 0, 8	−11	037	13
5*r*..........	9.88	−16	.....	.....	+ 9	1, 0, 0, 3	+ 2	046	4
4*s*..........	10.06	−17	.....	.....	−14	1, 0, 0, 8	−14	041	9
Mean.....	....	− 9	−16	−12	0		− 7		

Group 15.—This includes four plates taken with a circular diaphragm of 9.3 mm placed centrally over the 26.5 mm objective of a Cooke anastigmat. Two exposures of 45^{m} were made on each

plate. The calculated constant, 2.28 magnitudes, was used without correction for differential absorption within the objective. A re-reduction by the normal method, using the foregoing value of the constant, gives substantially the result found by Miss Leavitt. For two of the plates, however, the deviations are very large (see residuals, L, 178).

To test further the agreement with the HH scale, readings for both diaphragm and full aperture exposures were plotted against the HH magnitudes, using all the stars measured. From the two curves thus defined were read the differences in the ordinates for each half-interval of scale-reading. The deviations of these observed values of Δm from the adopted constant are shown in Table IX. The agreement for Plate 54 alone is really satisfactory, while Plates 55 and 58 are seriously discordant. Although the errors for these two balance each other, they reduce considerably the weight of the mean scale for the group. Changes in the observed reduction-constant of the character of those shown in Table IX are an indication of differences in gradation or of lack of comparability in the two series of images. Their appearance in the present instance is not surprising in view of the long exposures involved. In addition to these irregularities there is the uncertainty in the reduction-constant arising from differential absorption,

TABLE IX

GROUP 15, OBSERVED Δm—ADOPTED Δm

Scale Reading	Plate 54	Plate 55	Plate 56	Plate 58
3.0	....	−23	....	....
3.5	....	−18	....	....
4.0	+7	−21	+26	....
4.5	+4	−23	+20	....
5.0	+2	−26	+12	+49
5.5	+2	−26	+ 5	+46
6.0	+2	−28	+ 6	+42
6.5	+2	−28	0	+42
7.0	−3	−28	+ 3	+41
7.5	−5	−20	+ 3	+32
8.0	−3	−16	+ 7	+16
8.5	−4	−13	+17	− 8
9.0	−4	0	+22	−13
9.5	−6	+11	+12	− 8
10.0	0	+16	− 3	+ 2

so that Group 15 contributes little weight to the determination of the scale.

Summary for plates connecting bright stars with those fainter than the tenth magnitude.—The preceding paragraph completes the discussion of the data which connect the faint stars with those in the vicinity of the sixth or seventh magnitude. As the zero-point for all the fainter objects must be derived through this connection, its importance for the final scale is obvious. In the region covered there is considerable difference between the HH and MW scales, and the data and reductions have been examined for evidence which might afford an explanation of the divergence. The following summarizes the results:

Three methods have been employed—a wire gauze screen, prismatic companions, and a circular diaphragm. In none of these is it clear that the reduction-constant is certainly free from suspicion. That of the screen is derived from measures of the mesh and apparently requires a correction of +0.05 magnitude. In the case of the other two methods, differential absorption in the objective is not wholly accounted for; for the prismatic-companion plates a correction of doubtful validity is used, while for the diaphragm plates the effect is disregarded altogether. As for the prismatic companion constant, the error is probably small in spite of the assumptions involved, otherwise the portion of the scale below the tenth magnitude thus established would not agree so well with the HH scale, which in this region must be accepted as substantially correct.

With the revised screen-constant, we find that the scale thus established agrees closely with MW (compare MW−HH, Table III, with corrections in the fourth column of Table X). Re-reducing the prismatic companion plates with the aid of the adopted scale from 10 to 15, we find a systematic deviation from the Harvard Homogeneous Scale in the direction of the Mount Wilson results, but less, in the mean, than the differences between HH and MW. The examination of the diaphragm plates in Group 15 shows that the mean result agrees well with HH; but two of the plates show serious internal inconsistencies. On the whole, the re-reduction of the connecting plates points toward the necessity for some correction of the HH scale in the region of the brighter stars. The indicated change is in the same direction as that derived by comparison with MW, but somewhat less in amount.

d) *Results of the Re-reduction*

The corrections to the HH magnitudes found in sections *a*), *b*), and *c*) for all of the groups re-reduced excepting No. 15 are brought together in Table X, their relative weights (number of values) being indicated by the quantities in parentheses. Had Group 15 been included, several of the mean corrections in the sixth column would have been decreased one- or two-hundredths of a magnitude; but this change probably would have been offset by one in the opposite direction had it been possible to reduce Group 13 by the foregoing methods. In view of the uncertainties affecting Group 15 it is likely that the results are more reliable as they stand.

TABLE X

COLLECTED RESULTS OF RE-REDUCTION—CORRECTIONS TO HH MAGNITUDES

Star No.	HH Mag.	RE-REDUCTION *minus* HH				Wt.	MW *minus* HH_W	MW *minus* Re-reduction
		Gr. 6–8, 16, 17	Gr. 5	Gr. 9–12	Wtd. Mean			
1*s*...	(2.79)	....	−53 (3)	....	−53	3	(−49)	(+ 4)
1....	4.55	....	−33 (3)	....	−33	3	−40	− 7
2....	5.31	....	−30 (3)	....	−30	3	−25	+ 5
3....	5.82	....	−20 (3)	....	−20	3	−25	− 5
4....	5.99	....	−20 (3)	....	−20	3	−32	−12
2*s*...	6.46	....	−19 (3)	+ 4 (8)	− 2	11	−26	−24
5....	6.47	....	−22 (3)	− 1 (22)	− 4	25	−26	−22
3*s*...	6.63	....	−15 (3)	....	−15	3	−25	−10
1*r*...	6.78	....	−20 (3)	−33 (1)	−23	4	−50	−27
6....	7.07	−21 (1)	−14 (3)	− 3 (23)	− 5	27	−20	−15
7....	7.26	....	− 3 (3)	− 6 (16)	− 6	19	− 8	− 2
2*r*...	7.81	−29 (1)	−13 (3)	− 9 (19)	−10	23	−21	−11
8....	8.20	−24 (1)	+ 2 (3)	−10 (23)	− 9	27	−13	− 4
3*r*...	8.71	....	− 6 (3)	−12 (9)	−10	12	− 7	+ 3
9....	8.79	−24 (1)	−17 (3)	− 8 (22)	−10	26	−15	− 5
10....	8.97	− 9 (19)	−13 (3)	−10 (23)	−10	45	− 9	+ 1
4*r*...	9.05	....	−12 (3)	−12 (17)	−12	20	−13	− 1
11....	9.52	+ 1 (15)	−14 (3)	− 7 (19)	− 4	37	− 4	0
12....	9.83	− 4 (19)	+ 3 (3)	−11 (13)	− 6	35	0	+ 6
5*r*...	9.88	....	− 8 (3)	+ 2 (4)	− 2	7	− 8	− 6
4*s*...	10.06	− 3 (11)	− 8 (3)	−14 (9)	− 8	23	− 7	+ 1

The agreement from the seventh magnitude downward of the third, fourth, and fifth columns seems to show pretty clearly the necessity for corrections of the order indicated; but the relation of the fainter HH magnitudes to the international zero-point is unfortunately less definitely established. The results from the prismatic-companion plates (Groups 9–12), which are the most numerous of all the connecting plates, become relatively uncertain just at the critical point, for here the magnitudes are at the end of the scale derived from these groups. The only groups in Miss Leavitt's data which satisfactorily cover the questionable region are Nos. 3, 5, 13, and 15. The last appears to be unreliable for reasons given above. Nos. 3 and 13 involve comparisons with the Pleiades, and, although accordant with each other, do not agree with the results from Group 5.

The corrections in the sixth column, with the exception of that for Star 12, are all such as to decrease the outstanding differences between MW and HH. With the corrections included, the comparison with MW is as shown in the last column of Table X. The MW scale is therefore in agreement with the results of the re-reduction of the Harvard data from about the eighth magnitude downward. Above the seventh magnitude the differences are uncertain, either because of the small number of observations included in the re-reduction or because the calculated magnitudes are at the extremity of the scale which they define.

IV. THE DIVERGENCE FOR THE FAINT STARS

The large difference between the Harvard and Mount Wilson results for the faint stars (see Table III above, or, better, Table IX, *Mt. Wilson Contr.*, No. 97) is to be ascribed to two circumstances: (*a*) the neglect of the distance-correction in reducing the MW plates which enter into the Harvard discussion; (*b*) the application of a correction to the majority of these plates to allow for the order of exposure.

a) Effect of Distance-Correction

When the Mount Wilson plates were reduced by Miss Leavitt the values of the distance-correction were not yet available. In consequence, an attempt was made to avoid the difficulty by including only stars which were within a limited distance of the center of the plate (L, 140). Subsequently some of the plates were re-reduced by Miss Leavitt, using values of the correction which in the meantime had been determined at Mount Wilson, but without finding any important modification of the results (*loc. cit.*).

The amount of the correction is, however, not negligible. The matter is most easily illustrated in connection with Group 18, as in other particulars these plates were similarly treated in both the Harvard and Mount Wilson reductions. The values for this group given by Miss Leavitt on p. 189 are referred to the zero-point of the provisional magnitudes on p. 141 (L, 180); but it is convenient to reduce them to the zero-point defined by the Mount Wilson magnitudes between 10 and 15. When this has been done the difference between Group 18 and the corresponding portion of the Mount Wilson scale will be of the form

$$\mathrm{MW}-(18)_{\mathrm{H}}=\mathrm{MW}-(18)+\mathrm{D.C.}$$

in which $(18)_{\mathrm{H}}$ represents Miss Leavitt's values for Group 18 corrected for zero-point, while (18) is what would have been obtained had the distance-correction been applied. The subtraction of the term D.C. will therefore leave MW−(18), which represents the difference between MW and the Harvard reduction after revision for distance-error.

The zero-point correction is found to be 0.39 mag. and its application to the magnitudes in L, 189, gives the quantities in the second column of Table XI. Values in parentheses are relatively uncertain. The deviations from MW are in the third column; it is seen at a glance that these are very similar to the distance-corrections in the following column, which have been derived from the Mount Wilson reduction tables. The differences between MW and $(18)_{\mathrm{H}}$ corrected for D.C. (in the last column) have a mean value of −0.04 magnitude; but, it is found from Table XII, *Mt. Wilson Contr.*, No. 97, that the weighted systematic deviation of the Mount Wilson reduction of Group 18 (Plates 196, 200, 204) from the final Mount Wilson scale is −0.03 magnitude. The neglected distance-correction, therefore, completely accounts for the difference between the Harvard and Mount Wilson reductions. Similar differences also affect the plates comprising Groups 19 and 20, but it does not seem necessary to make a detailed comparison. The effect upon the scale begins to be appreciable at about the sixteenth magnitude, and gradually increases to a maximum of about 0.4 magnitude for the faintest and least favorably situated stars.

TABLE XI

EFFECT OF DISTANCE-CORRECTION

Star	Gr. 18+0.39	MW−$(18)_H$	D.C.	MW−(18)
31	16.35	+ 6	+ 7	− 1
15*s*	16.59	− 2	+ 2	− 4
32	16.89	−13	0	−13
16*s*	17.06	−20	+ 2	−22
33	17.26	−20	− 3	−17
17*s*	17.39	−20	−25	+ 5
34	17.43	−19	−12	− 7
35	17.64	− 1	−21	+20
36	17.86	− 8	−23	+15
37	18.21	−20	−17	− 3
18*s*	(18.21)	(−27)	−30	(+ 3)
38	18.44	−24	−26	+ 2
19*s*	18.62	−46	−46	0
39	18.67	− 9	−28	+19
20*s*	(19.06)	(−46)	−11	(−35)
21*s*	18.98	−32	−34	+ 2
22*s*	18.93	−18	−15	− 3
23*s*	(19.32)	(−62)	−45	(−17)
24*s*	(19.25)	(−37)	−25	(−12)
40	19.09	−22	−12	−10
25*s*	19.34	−50	−16	−34
41	19.32	−30	−12	−18
Means		−19	−15	− 4

b) Correction for Order of Exposure

From the data used by Miss Leavitt it appears that when several exposures are made upon the same plate, the first and last being short and equal, the images of the first exposure are generally brighter than those of the last. The mean difference she finds to be about a quarter of a magnitude.

The question thus raised is of great importance for the reduction of plates involving successive exposures, for it has a direct and important effect upon the scale. A symmetrical arrangement of the exposures, or, failing this, a correction depending upon the order of exposure, immediately suggests itself as a means of eliminating the disturbance.

This, however, assumes that the phenomenon develops proportionally to the time, which in the absence of more definite information is a perfectly natural supposition. Miss Leavitt has followed this procedure, and accordingly plates MW 196, 200, 204 (Group 18), for which the arrangement of exposures was symmetrical, received no correction. The plates of Groups 19 and 20 (MW 230, 232, 235, and MW 310, 312, 313), however, were not symmetrical in their arrangement, and have therefore been corrected on the basis of systematic differences derived from the short preliminary and supplementary exposures impressed on each plate.

The values of the measured differences between first and last exposures and the fractional part applied to the images of each of the principal exposures are shown on pp. 122 and 124 of her memoir. As the order of the principal exposures on all of the plates in question was the same, namely, full aperture followed by reduced aperture, the correction always modifies the results in the same direction; the difference in the images with full and reduced aperture is diminished, so that in calculating the scale the faint magnitudes become relatively fainter than they otherwise would have been.

No extensive calculation is required to show how the correction has modified the results from Groups 19 and 20, for a direct indication is afforded by the details in Miss Leavitt's discussion. It was found that the short supplementary exposure on one of the plates (MW 310, H 285) in Group 20 could not be measured, and in consequence no correction was applied (L, 182). The other plates (MW 312, 313; H 286, 287), however, were corrected as usual. Turning now to p. 184 of Miss Leavitt's memoir, it is seen at once that for the faint stars there is a large difference between the mean result for the last two plates and that for the first. For convenience, the deviations of the two scales thus defined from the mean for Group 20 are given in the third and fourth columns of Table XII. The difference between these two series of residuals, which appears in the fifth column, expresses very approximately the effect of the correction.

That the systematic difference thus exhibited is not inherent in the plates themselves appears at once upon referring to Table XII of *Mt. Wilson Contr.*, No. 97, which gives for the Mount Wilson reduction (in which none of the plates was corrected) the deviations of individual plates from the mean Mount Wilson scale. For the region covered by the table above we find as a weighted mean:

TABLE XII

CORRECTION FOR ORDER OF EXPOSURE

Star	MW Mag.	Pl. 310− Gr. 20	½(312+313) −Gr. 20	Pl. 310− ½(312+313)	Gr. 18− Gr. 20	Gr. 18− Pl. 310	Gr. 18− ½ (312+313)
31	16.41	−20	+ 8	−28	−37	−17	−45
15*s*	16.57	−21	+ 6	−27	−29	− 8	−35
32	16.76	−12	+ 5	−17	−33	−21	−38
16*s*	16.86	− 6	+ 2	− 8	−23	−17	−25
33	17.06	−12	+ 4	−16	−19	− 7	−23
17*s*	17.19	−20	+13	−33	−22	− 2	−35
34	17.24	−28	+14	−42	−36	− 8	−50
35	17.63	−16	+ 9	−25	−39	−23	−48
36	17.78	−26	+12	−38	−43	−17	−55
37	18.01	−20	+10	−30	−23	− 3	−33
18*s*	17.94	−20	+14	−34	(−10)	(+10)	(−24)
38	18.20	−50	+25	−75	−49	+ 1	−74
19*s*	18.16	−33	+ 8	−41	−57	−24	−65
39	18.58	−50	+33	−83	−62	−12	−95
20*s*	18.60	−40	+27	−67	(−28)	(+12)	(−55)
21*s*	18.66	−38	+19	−57	−67	−29	−86
22*s*	18.75	−11	+ 8	−19	−87	−76	−95
24*s*	18.88	+ 7	− 7	+14	(−36)	(−43)	(−29)
25*s*	18.84	−38	+38	−76	−50	−12	−88

$$\text{MW reduction: Pl. } 310-\tfrac{1}{2}\,(\text{Pl. } 312+313)=+0.02 \text{ mag.}$$

When similarly treated, therefore, the three plates give closely accordant results. As a matter of fact, the effect of the correction is about a tenth of a magnitude greater than the differences shown by the fifth column of Table XII, since the mean distance of the stars from the center of the plate is about 5 mm greater for MW 310 than for 312 and 313; and since none of the Harvard measures were corrected for distance, the magnitudes for MW 310 are relatively too faint by approximately the amount mentioned.

It is of interest further to compare Miss Leavitt's results for MW 310 with those for Group 18. This is at once accomplished by combining the third and sixth columns of Table XII, the results for the latter column having been derived from L, 189. The differences in the seventh column reveal a relatively small systematic effect. In other words, the uncorrected plate MW 310, which presumably is affected by the phenomenon under discussion since all of the other Mount Wilson plates of unsymmetrical arrangement seem to be so affected, gives results agreeing closely with those for Group 18 from which the systematic effect has supposedly been eliminated by a symmetrical arrangement of the exposures. On the other hand, plates MW 312 and 313, which have received correction, differ widely in their results from those of Group 18 (last column Table XII).

This immediately raises a suspicion as to the validity of the assumption upon which the above-described procedure was based. The matter can be tested further by means of the Mount Wilson results for individual plates, part of which appear in Table XII of *Mt. Wilson Contr.*, No. 97. Of the plates there listed Nos. 230, 232, 235, 310, 312, 313, and 314, on the basis of the procedure followed by Miss Leavitt, would require correction for the order of exposure. None of the others, and none of the fifteen plates of shorter exposure which were also used in deriving the MW scale, would need such correction, either because of a symmetrical arrangement of the data or because of the minuteness of the differences between the first and last exposures (in the case of Nos. 769 and 808). A comparison of the scales from the two groups of plates should therefore show at once the result of having neglected the correction in the case of the questionable series. Moreover, the difference in the two scales should be a reliable indication of the effect produced, for the amount of the material is such as to reduce to a minimum the errors arising from other sources.

From Table XII, *Mt. Wilson Contr.*, No. 97, we find the weighted deviations of the mean scale for Plates 230, 232, etc., from the final Mount Wilson scale to be as shown in Table XIII. Since the weight of the results from the plates of symmetrical arrangement is considerably in excess of that from Nos. 230, 232, etc., the scale difference for the two groups is somewhat less than twice the systematic difference in Table XIII; but the result thus attained is within the limit of error which may arise from other sources, and we are led again to the conclusion expressed above, namely, that the assumption upon which the whole procedure is based is unjustifiable.

TABLE XIII

MW—(Pl. 230, 232, etc.)

MW Mag.	Syst. Dev.	Wt.	MW Mag.	Syst. Dev.	Wt.
11.1	−0.005	30	16.1	−0.018	154
12.2	+ .053	39	16.6	− .029	186
13.2	+ .031	52	17.1	− .005	159
13.7	+ .015	68	17.6	+ .016	233
14.1	− .028	61	18.1	+ .016	241
14.6	− .001	134	18.6	+ .040	358
15.1	− .024	174	19.1	+0.056	77
15.6	−0.019	127			

Moreover, we may say with some definiteness that the questionable group of plates requires no appreciable correction whatever in order to arrive at a correct scale (excluding unsuspected disturbances which may arise from other sources). Both our own measures and those by Miss Leavitt and Miss Leland show that in the main there is no sensible difference between the first and last exposures upon plates MW 196, 200, and 204. Plates 769 and 808 which were used only for the Mount Wilson scale give a similar result. The mean scale from these five plates should therefore be free from any influence due to the phenomenon under discussion; but this scale we find from Table XII, *Mt. Wilson Contr.*, No. 97, to be practically identical with that derived without correction from the plates which do show differences between initial and final exposures. Apparently, therefore, the principal exposures on these plates, which are the ones used in deriving the scale, contain little or nothing of the systematic difference that is revealed when the short preliminary and final exposures are compared.

The phenomenon is obviously photographic in origin, and Mees has suggested,[1] as the most probable explanation, that it is due to a depression of the sensitiveness of the plate through the absorption of water-vapor upon exposure to the air. No data are available as to the time required for a condition of equilibrium to be established, but it is perhaps not inconceivable that the major part of the change should occur within a comparatively short interval after the beginning of the exposure. It might thus easily happen that only the first of a series of equal exposures would be markedly different from the others; and if the exposures were long the difference might be more or less completely obscured.

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[1] *Astrophysical Journal*, **40**, 236, 1914.

The question will be discussed further at another time; but one point requires mention before the subject is dismissed, and that is the apparent susceptibility of the measures used in this part of the discussion to systematic error. This is well illustrated by the results for different observers and different scales given in L, 122, and again, by the comparison of these results with our own for the same plates as measured by Miss High and Miss Richmond.

A striking result is the fact that measures made here agree in fixing a much smaller value for the mean difference between the initial and final exposures than that found by Miss Leavitt; but most extraordinary is the circumstance that we find the variation of the difference with size of image to be entirely different in character from that illustrated in Miss Leavitt's memoir (Plate 3, Fig. 1). For the Mount Wilson plates she finds in general that the difference is a maximum for the faintest images, while for these we usually find no difference at all.

Had we corrected the Mount Wilson plates, using our own measures, the scale for the six or seven plates thus modified would have been deflected by rather less than half the amount entering into the Harvard results. The matter would be disconcerting were it not pretty clear that it should not enter into the discussion at all, and were it not further clear that, when similarly treated, Miss Leavitt's measures of the more essential portion of the data and our own give practically identical results for the scale.

The preceding discussion does not extend the comparison with the Harvard results below the nineteenth magnitude; it is unnecessary, however, to carry the matter further. The Harvard magnitudes of the faintest stars depend upon an extrapolation of the scale for the region just considered, and it is evident that the divergence which has entered in the manner indicated must produce even greater differences for still fainter objects. These are quite sufficient to account for the difference in the limiting magnitudes reached in the two investigations—21 for Harvard and 20 for Mount Wilson.

SUMMARY

1. The Mount Wilson color-indices of the Polar Sequence stars afford the possibility of investigating the influence of color upon the results obtained with the various instruments used in deriving the Harvard photographic scale. It is found that corrections of a few hundredths of a magnitude (4th col., Table III) are required to reduce this scale (that of *H.C.*, No. 170+0.08 mag.) to a homogeneous color-system (HH, Table III).

2. The comparison of the Harvard Homogeneous Scale (HH) with MW reveals a color-equation with a coefficient of 0.06. Including the scale-divergence for the bright stars, we have to the sixteenth magnitude, approximately

$$\mathrm{MW}-\mathrm{HH}=+0.061\,(\mathrm{HH}-6.0)+0.06\,C$$

in which C is the color-index, and in which, further, the first term is constant and equal to +0.24 magnitude from the tenth magnitude onward. The allowance for color has therefore appreciably diminished the divergence between the sixth and the tenth magnitudes given by the earlier comparisons. For the faint stars there remains a large difference between the two scales, which at the twentieth magnitude (MW) amounts to about one magnitude.

3. The acceptance of the HH magnitudes between 10 and 15 as an accurately established scale makes it possible to re-reduce much of the Harvard data for the bright stars by methods which are free from various uncertainties that necessarily affect the original reduction. It is thus found that the HH magnitudes apparently require small corrections which improve the agreement with MW and extend the region of parallelism for the two scales upward from the tenth to the eighth magnitude (6th and 9th cols., Table X). The remaining discrepancy of a quarter of a magnitude is thus thrown upon the stars which are brighter than the eighth magnitude.

4. The divergence for the faint stars is due to the neglect of the distance-correction in reducing the Mount Wilson plates used for the Harvard scale, and to the application of a correction intended

to remove the influence of the systematic difference sometimes affecting initial and final exposures on photometric plates. With proper allowance for these differences in procedure, the Harvard scale becomes parallel with that of Mount Wilson in the region of the faint stars.

5. The photographic phenomenon which shows itself as a systematic difference between first and last exposures seems for the most part to occur within a short interval after the beginning of the first exposure, for the principal exposures, on the Mount Wilson plates at least, require no correction in order that they may give results accordant with those from plates which do not show the effect at all.

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MISCELLANEOUS NOTES ON VARIABLE STARS

By HARLOW SHAPLEY

The following collection of notes and data on subjects related almost exclusively to variable stars represents the results of short or partial investigations that have been undertaken during the last two years. A part of the observational work involved was done at the Princeton University Observatory, but the reduction and discussion of the observations and the statistical and other computations were made at Mount Wilson. The incomplete series of observations on eclipsing variables are published now because there will be no immediate opportunity to continue the work.

The contents of the several notes are as follows:

1. Comparison of the observed solar darkening with the assumed stellar darkening—an inquiry into the applicability of the formula $J=J_0\ (1-x+x\cos\gamma)$ to the representation of the distribution of light over the disk of the sun. The investigation is based on published and unpublished observations from the Astrophysical Observatory of the Smithsonian Institution.
2. Observations of a minimum of the F-type eclipsing star U Pegasi and the derivation of new light-elements.
3. Photometric measures of *o* Persei for the purpose of testing its light-variability.
4. Photometric observations of R Canis Majoris.
5. The visual range of AE Cygni, with a provisional solution for uniform orbital elements.
6. The relation between spectrum and length of period for close double stars, including eclipsing variables and spectroscopic binaries of the non-Cepheid type.
7. On the number of naked-eye variable stars of different classes.

The manuscript data concerning recent measures of the distribution of brightness over the disk of the sun were kindly furnished by Mr. Abbot. The sixth and seventh notes were prepared and written by Mrs. Shapley.

I. ON DARKENING TOWARD THE LIMB OF THE SUN

In the study of the light-curves and orbital elements of eclipsing binaries the nature of the distribution of brightness over the apparent stellar surfaces has been found to be of paramount importance. The observed darkening toward the limb of the sun was taken as a basis for the assumption as to the general character of stellar darkening. Since the adoption of an empirical cosine formula to represent the diminution of light from center to limb, the actual existence of darkening for stars of various spectral types has been definitely demonstrated;[1] but the suitability of this particular formula remains undetermined, and the dependence of the darkening coefficient on wave-length and spectral type is as yet unknown.

In the case of the sun the applicability of the adopted formula, and also the dependence of the coefficient on the mean wave-length of the light used in obtaining the light-curve, can be directly studied with the aid of the bolometric observations of the Astrophysical Observatory of the Smithsonian Institution. For stars of other spectral types, however, we can now only speculate as to how the law of darkening may differ and what the degree of darkening may be for the light of different wave-lengths.[2] There is perhaps an indication that it is greater for the bluer stars, and a refined study of appropriate light-curves will doubtless contribute something to the solution of the problem.

A comparison of the bolometric results with the formula has not been made heretofore, and in the present note a brief discussion will be given of the diagrams, Figs. 1 and 2, in which such a comparison is presented.

Letting J_0 be the value of the apparent surface brightness J at the center of the disk, the assumed relation between J and the apparent distance from the center is $J/J_0 = 1 - x + x \cos\gamma$, where the darkening coefficient is x and γ is the angle between the line

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[1] *Contributions from the Princeton University Observatory*, No. 3, pp. 106–110, 1915.

[2] Relative to the uncertainty in orbital computations arising from the present indefinite state of the darkening-at-the-limb problem see the above-cited work, chap. iii, and *Astrophysical Journal*, **40**, 219, 1914.

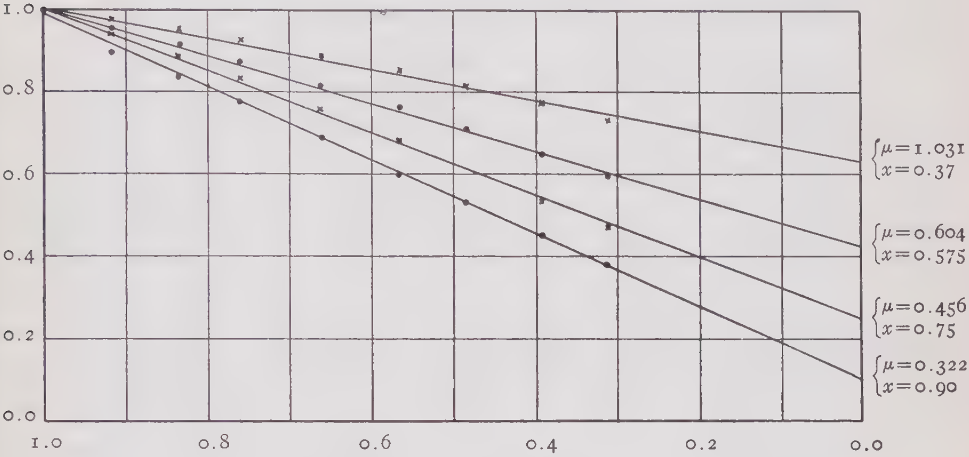

FIG. 1.—Solar darkening in 1907
Abscissae: $\cos\gamma$; Ordinates: J/J_0

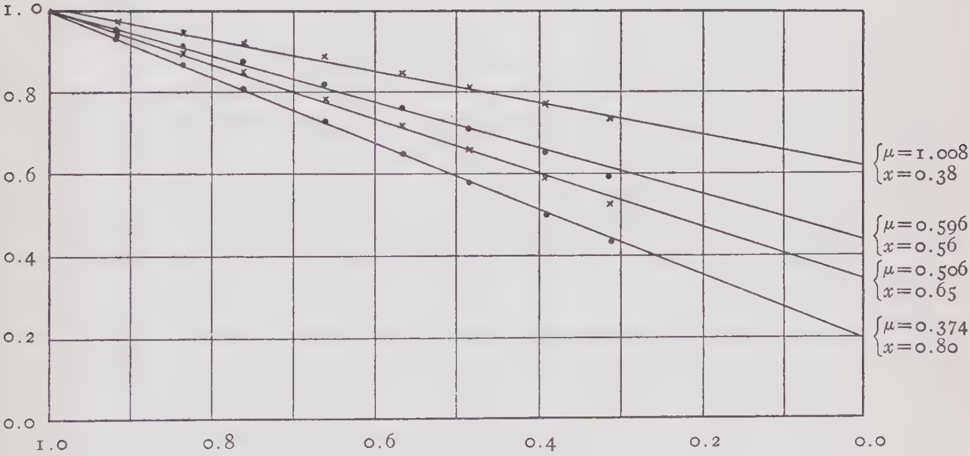

FIG. 2.—Solar darkening in 1913
Abscissae: $\cos\gamma$; Ordinates: J/J_0

of sight and the surface normal. The means of the bolometric measures made at Washington in the year 1907 (sun-spot maximum) are given in *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 157, 1913, and an accompanying diagram represents sin γ plotted against J/J_0 for several different wave-lengths. When we plot the measures against cos γ, as I have done for four representative wave-lengths in Fig. 1, the observed points should lie on a straight line if the foregoing formula is sufficient. The bolometric work at Mount Wilson during September, October, and November 1913 (sun-spot minimum) is represented for four wave-lengths in Fig. 2. In both diagrams the mean wave-length of the part of the spectrum used is written opposite the straight line that most nearly satisfies the observations, and the slope of the line, which is the darkening coefficient x, is also given. $1-x$ is the brightness at the limb in terms of the central brightness. The data for $\mu=0.322$ are from photographic results by Schwarzschild and Villiger.[1]

Three results of importance to this inquiry can be deduced from the diagrams:

1. The observed points, though lying on smooth curves, do not define straight lines for any of the wave-lengths of either series, and the deviations are evidently larger than the uncertainties of the observations. Therefore the formula adopted for the stellar investigation does not completely represent the observed solar darkening. The representation is quite satisfactory, however, since the error is generally less than 1 per cent and scarcely exceeds 3 per cent for any wave-length or for any distance from the center up to 95 per cent of the radius (cos $\gamma=0.31$). Beyond this point there is an indication that the brightness may fall off rapidly; but, to quote Abbot's *The Sun*, p. 107, footnote, "There is a tendency of all the data to show a less rapid fall of brightness from 95 to 97 per cent out on the radius than would be expected. This may be due to error." The effect on the light-curve of a variable star of the error in the assumed formula would be entirely negligible (even if we had an exact value of the coefficient) over at least 95 per cent of the curve at minimum. We conclude, therefore, that the adopted cosine law of darkening, though manifestly inexact, is a very good approximation and sufficiently accurate for the

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[1] *Astrophysical Journal*, **23**, 284, 1906.

stellar work of the present time. Knowledge of the amount of darkening for a given wave-length and for different spectral types is a more important and immediate need.

2. For the solar spectral type, at least, the value of the darkening coefficient is conspicuously greater for photographic light-curves than for visual—a well-known phenomenon. The average value of x is approximately 0.75 for the mean effective photographic light, and for the mean visual part of the spectrum, about 0.6. This of course does not necessarily mean that the coefficient would be greater for stars of bluer spectral types.

3. For a given wave-length the darkening coefficient decreases with the number of sun-spots, this change being more pronounced for the shorter wave-lengths. Or stated in another way,[1] "there was distinctly less contrast of brightness between the edge and center of the sun's disk in 1913 than in 1907," and the shorter wave-lengths change in contrast more than the longer. This can be shown easily by plotting μ against x for the two series. There are also changes in contrast accompanying the short-period irregular variations of the sun. Apparently, then, our problem may involve a variable darkening coefficient.

2. NEW LIGHT-ELEMENTS FOR U PEGASI

On the basis of the following series of observations made with the polarizing photometer during a single principal minimum of U Pegasi it is possible to make a small but definite correction to the period. The star B.D.$+14°5077$ (7.9), which is about three magnitudes brighter than the variable, was used for the comparisons.

The light-elements of U Pegasi generally adopted at present are those deduced by Roberts[2] from Wendell's observations at the Harvard Observatory:[3]

$$\text{Prin. Min.} = \text{J.D. } 2415021.2367 + 0^{d}.374784 \text{ E}$$

Apparently a small error exists in the yearly *Vierteljahrsschrift* catalogues since 1912 in giving the light-elements and ephemeris for this star.[1] The initial epoch there reads J.D. 2415021.2469; hence all times are predicted 15 minutes too late, which is a noticeable amount in this case of a nine-hour period.

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[1] *Report of the Secretary of the Smithsonian Institution for the Year Ending June 30, 1914*, p. 94.

[2] *Monthly Notices*, **66**, 135, 1906.

[3] *Harvard Annals*, **69**, 53, 1909.

TABLE I

PRIMARY ECLIPSE OF U PEGASI, OCTOBER 31, 1913

Gr. Helioc. Mean Time	Phase	Magnitude Difference	O−C
			mag.
$14^h\ 3^m$	$-0^h 13^m$	3.37	−0.04
14 13	−0 3	3.46	+ .01
14 25	+0 9	3.39	− .04
14 35	+0 19	3.35	− .01
14 46	+0 30	3.31	+ .06
14 59	+0 43	3.17	+ .03
15 19	+1 3	3.00	− .02
15 29	+1 13	2.98	.00
15 39	+1 23	2.87	− .07
15 49	+1 33	2.91	.00
16 1	+1 45	2.87	.00
16 12	+1 56	2.90	+0.04

The corrected heliocentric time of principal eclipse on October 31, 1913, predicted by means of Roberts' elements, is 13^h48^m, G.M.T. The observed time was 14^h16^m, G.M.T., with an uncertainty not exceeding two or three minutes. The difference of 27 minutes represents the accumulation of the error in the period during sixteen years. The accuracy of the elements determined by Roberts is sufficient to leave no doubt as to the number of intervening periods. There were 13,478 revolutions between the date of my observations and the published initial epoch, and about two thousand more between that arbitrary epoch and the middle of the interval over which Wendell's observations of the principal minimum extend. The correction to the period derived from these data is +0.10 seconds, so that the revised value is $0\overset{d}{.}3747852$, with a probable uncertainty of two or three units in the last place. The new heliocentric epoch of primary minimum is

J.D. 2420072.5754 = 1913, October 31, $13^h48\overset{m}{.}6$, G.M.T.

The phases given in the table above were computed by means of the new elements. The residuals in the last column show the agreement of the present short series of measures with the theoretical darkened curve based on Wendell's observations.[1] The representation is quite satisfactory, both as to depth and as to duration of the eclipse, and suggests that no marked change has occurred in the nature of the eclipse during the last fifteen thousand revolutions of the components. The data here available are obviously insufficient, however, to give this last conclusion much weight; but it may be pointed out that such extremely short-period systems as U Pegasi and W Ursae Majoris are deserving of careful and continuous attention from observers, with the object of establishing the presence or absence of sensible perturbations in the length of the period, the range of variation, or the general nature of the light-curve. Both systems are very close, their orbits are apparently circular, their spectral types are F and G, respectively, their components are the densest stellar bodies known, and both possess a high degree of ellipticity. In fact, one-third of the range of light-variation of U Pegasi at principal minimum is attributable to the rotation of the elongated components.

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[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **47**, 301, 1902.

3. PHOTOMETRIC OBSERVATIONS OF *o* PERSEI

The possibility that the spectroscopic binary *o* Persei is also an eclipsing variable has been recognized by several observers, and the constancy of its light has been put to test by Lau,[2] Guthnick,[3] and Hertzsprung.[4] Both Lau and Guthnick announce a variability of the light through a range of 0.35 mag., and the latter finds the time of minimum in agreement with the expectation of eclipse. Hertzsprung, on the other hand, finds no definite evidence of variability, but he points out that none of his observations were made within five hours of the predicted time of principal minimum. Before learning that the star was under investigation elsewhere the writer had commenced a series of observations with the Princeton polarizing photometer. The results of measures on three nights are given in Table II.

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[1] *Contributions from the Princeton University Observatory*, No. 3, p. 176, *et passim*, 1915.

[2] *Astronomische Nachrichten*, **196**, 427, 1914.

[3] *Ibid.*, **197**, 303, 1914. Since writing the above, Guthnick's discussion of his preliminary results for this star with the photo-electric photometer have been received (*Veröffentlichungen der Königlichen Sternwarte zu Berlin-Babelsberg*, **1**, No. 1, p. 57, 1914). The results, which are not as yet very definite, suggest a variation of a tenth of a magnitude.

[4] *Astronomische Nachrichten*, **199**, 140, 1914.

TABLE II

OBSERVED MAGNITUDES OF o PERSEI

Date	Gr. Geoc. M.T.	Phase from Super. Conj.	Magnitude Difference	Remarks
			mag.	
1913, Nov. 21 ...	13^h31^m	$+19^h27^m$	$a-o=+4.01$	
	13 44	+19 40	$a-o=+3.99$	
	13 55	+19 51	$a-o=+4.03$	
Dec. 12 ...	20 9	− 0 12	$b-o=+4.62$	
	20 24	+ 0 3	$b-o=+4.67$	
	20 40	+ 0 19	$a-o=+4.00$	
	20 55	+ 0 34	$a-o=+4.08$	Measure interrupted
	21 18	+ 0 57	$a-o=+3.99$	Star low
Dec. 15 ...	10 26	+62 5	$a-o=+3.90$	Moon
	10 36	+62 15	$a-o=+3.90$	
	10 50	+62 29	$b-a=+0.65$	

The phases were computed with the aid of the spectroscopic elements by Jordan.[1] The comparison stars are a=B.D.+31°644 (8.3), and b=B.D.+31°643 (8.2). Each observation is the mean of 16 comparisons and has a probable error of approximately ±0.03 mag. The fourth observation on December 12 should perhaps be given low weight.

The magnitudes in the table, so far as they go, give no indication of variability, although they were made apparently at the most favorable times. If, however, the orbit which was found by Jordan to be circular should be slightly eccentric, the principal minimum might be displaced sufficiently from the predicted time to render inconclusive the foregoing evidence against variability.

4. PHOTOMETRIC OBSERVATIONS OF R CANIS MAJORIS

The remarkable hump on the light-curve of R Canis Majoris at the end of the principal minimum was measured independently by Professors E. C. Pickering and O. C. Wendell at Harvard more than twenty years ago. The existence of the phenomenon at that time seems to be entirely beyond question; but so far as I know there has been no recent observation of the star, and no further investigation of the anomalous feature. With the hope of throwing some light on this peculiarity, a series of observations was undertaken with the Princeton polarizing photometer, but because of the southern declination of the star and the continued unfavorable weather during the short observing season, the work was given up before satisfactory results could be obtained. The observations given below may be of value to other observers, however, not only in showing that if the hump now exists it has perhaps shifted along the curve away from minimum, but also in showing that the light-elements by Chandler do not at present adequately represent the observed times of minima.

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[1] *Publications of the Allegheny Observatory*, **2**, 63, 1911. The period is 4.41916 days.

The predicted time of eclipse on March 22, 1913, was 15^h32^m, Gr. Hel. M.T. The observed time of mid-eclipse was about half an hour earlier. This difference may not indicate that a correction to the mean period is necessary. The orbit of R Canis Majoris has an exceptionally high eccentricity for an eclipsing binary;[1] furthermore, the line of apsides is evidently in fairly rapid motion, but with an unknown period.[2] The resulting oscillation in the

TABLE III

Observed Magnitudes of R Canis Majoris

Date	Gr. Geoc. M.T.	$a-v$	Date	Gr. Geoc. M.T.	$a-v$
1913		mag.	1913		mag.
Mar. 12...........	11^h48^m	2.11	Mar. 22..........	12^h59^m	2.02
	11 57	2.06		13 8	2.04
	12 14	2.06		13 26	1.90
	12 24	2.14		13 36	1.88
	12 40	2.10		13 47	1.88
	12 49	2.10		14 3	1.77
	13 2	2.14		14 13	1.66
	13 11	2.12		14 30	1.52
	13 23	2.09		14 44	1.49
	13 33	2.10		14 54	1.42
	13 43	2.19		15 4	1.37
	13 57	2.18		15 16	1.43
Mar. 22...........	12 37	2.12		15 30	1.43
	12 50	2.02		15 44	1.56

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[1] *Publications of the Allegheny Observatory*, **3**, 52, 1913.

[2] *Contributions from the Princeton University Observatory*, No. 3, p. 95, 1915.

time of the principal minimum may then easily be of the order of the discrepancy observed. Further investigation of the light-curve is much to be desired, not only to contribute to our knowledge of orbital changes in close binary systems of pronounced eccentricity, but also to elucidate the meaning and the present nature of the hump in the light-curve, which stands out as the most conspicuous unexplained irregularity known in the light-curve of any eclipsing binary.

The comparison star used in the measures above, $a=$ B.D. $-16^\circ 1886$ (8.0), is reddish. The last two observations of each night should receive low weight because of the low altitude and trouble with smoke and haze.

The observations on March 22, 1913, indicate that the range is at least a tenth of a magnitude greater than observed by Wendell fourteen years earlier. The results of computations on the photometric orbit and a discussion of them are given in *Contributions from the Princeton University Observatory*, No. 3.

5. NOTE ON THE PROVISIONAL ORBIT OF AE CYGNI

The following series of observations of the faint eclipsing variable AE Cygni are of value chiefly in showing the visual range of variation and in giving one epoch of minimum. They are not conclusive as to the existence of a secondary minimum nor of ellipticity. The comparisons were made with the star $a=$ B.D. $+29^\circ 4347$ (8.2). Each observation in Table IV contains 16 settings made with the polarizing photometer.

TABLE IV

OBSERVATIONS OF AE CYGNI

Date	Gr. Geoc. M.T.	$v-a$	Date	Gr. Geoc. M.T.	$v-a$
1912		mag.	1912		mag.
June 10	$16^h 53^m$	+3.04	Nov. 3	$15^h 15^m$	+2.40
	17 6	3.17		15 29	2.42
	17 23	3.16		15 48	2.40
	17 38	3.21		16 1	2.44
			1913		
Nov. 3	14 16	2.54	Jan. 1	11 11	2.39
	14 31	2.42		11 25	2.35
	14 48	2.46		11 47	2.46
	15 1	2.42		12 2	2.40

The light-elements given by Williams[1] represent the measures printed in Table IV with sufficient accuracy. His light-curve, however, which is based on 83 visual observations, presents unusual difficulties in the derivation of even approximate orbital elements. In order to investigate the system further my observations were undertaken, but the faintness of the star and the inconvenient length of the period ($23^h15^m.6$) have prevented decisive results from being obtained. The range of variation according to my measures is 0.77 mag. With this value the scale-readings given by Williams were converted into magnitudes and the resulting light-curve investigated. If the period is as given above, the provisional orbital elements are:

Ratio of radii =0.75
Inclination of orbit =85°
Radius of orbit =1.00
Radius of larger star =0.45
Light of larger star =0.50

But in this case a secondary minimum of three-tenths of a magnitude would be required, the existence of which is not verified by my observations; moreover, the components would be so close together that considerable elongation would be expected and the range as given above would then be illusory for orbital computations.

It seems more probable that the period is double the value given by Williams, that the stars are not markedly ellipsoidal, and that the alternate minima are probably of unequal depth. As a consequence, the components would be found to be of nearly equal dimensions, much smaller relative to the size of the orbit than given above, and the mean density of the system would be of the order of two-tenths that of the sun. If the shorter value of the period is correct, the mean density is one-half as large. The spectral type is unknown.

6. ON THE PERIODS AND SPECTRA OF CLOSE BINARY STARS

The following study has been made to supplement the recent work of Wicksell on the frequency of the periods of spectroscopic binaries,[1] and to determine whether the conclusions reached in his paper can be considered more than an apparent result depending on selection. He finds two distinct maxima in the frequency-curves for all spectral types and attempts to explain this on the assumption that in spectroscopic double stars the groups of long and short periods depend on two different principles or conditions of origin. He suggests, however, the possible insufficiency of his data.

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[1] *Astronomische Nachrichten*, **184**, 97, 1910.

Since there is essentially no difference between spectroscopic and eclipsing binaries, except that the latter are limited in the possible values of their orbital inclination, there is no reason why the data for the two classes of stars should not be combined in a study of this kind. Accordingly a card catalogue of eclipsing variables was made, which contains over 200 entries, and the 121 stars for which both period and spectral type are known form, together with Wicksell's data, the basis of this discussion. From Wicksell's list all those spectroscopic binaries which are known also to be eclipsing variables were removed; 14 spectroscopic binaries not in his list, but whose periods and spectral types are available, were added; and for one star, π_5 Orionis, a later value of the period was substituted. All Cepheid variables were excluded, since it is not certain that they are binary systems; hence four stars not called Cepheids by Wicksell were dropped from his list—β Cephei and α Ursae Minoris, two known Cepheids, and β Canis Majoris and λ Andromedae, whose orbits suggest that they also may belong to this class. A separate study was then made (1) of the eclipsing variables, (2) of the spectroscopic binaries not also known to be eclipsing binaries, and (3) of both the first and second groups combined.

Table V shows the total number of eclipsing and spectroscopic binaries of each spectral type in equal intervals of the logarithm of the period. Two Ma stars and one of type Oe5 were not considered.

Frequency-curves, similar to Wicksell's, were drawn for each spectral type in each of the three above-named groups, i.e., eclipsing binaries, spectroscopic binaries, and the total of both. Curves were also drawn for each group, using all the stars regardless of spectral type, and these are reproduced in the accompanying diagrams.

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[1] *Arkiv för Matematik, Astronomi och Fysik*, **10**, No. 6, 1914.

The supposed secondary maximum is found to occur definitely only in the case of the B-type stars in the spectroscopic group; and in that group it is due solely to six long-period stars. If, as shown by the dotted lines in the diagrams, all B-type stars are omitted, the phenomenon of the two maxima completely disappears. The same result is obtained by omitting only the six B stars of long period. These anomalous systems are: κ Velorum, $116^{\mathrm{d}}.65$; ϕ Persei, $126^{\mathrm{d}}.6$; ν Orionis, $131^{\mathrm{d}}.3$; ζ Tauri, 138^{d}; π Andromedae, $143^{\mathrm{d}}.67$; and μ Sagittarii, $180^{\mathrm{d}}.2$. ϕ Persei and μ Sagittarii are classified as having peculiar spectra. Evidently then the real problem lies in explaining the spectra or periods of these six stars. The next longest period among the B's is 31 days.

TABLE V

Logarithm of Period and Spectral Type

Spectrum	−0.8 to −0.4	−0.4 to 0.0	0.0 to 0.4	0.4 to 0.8	0.8 to 1.2	1.2 to 1.6	1.6 to 2.0	2.0 to 2.4	2.4 to 2.8	2.8 to 3.2	3.2 to 3.6	3.6 to 4.0	Total No. of Binaries
B..........	....	....	9	16	10	6	....	6	....	....	....	....	47
A..........	....	9	26	40	14	9	3	2	....	1	1	....	105
F..........	1	3	3	8	3	....	3	1	2	....	1	2	27
G..........	1	....	....	2	2	2	1	2	2	3	....	....	15
K..........	....	....	....	....	....	2	....	2	1	....	....	....	5
Total ..	2	12	38	66	29	19	7	13	5	4	2	2	199
F–G–K	2	3	3	10	5	4	4	5	5	3	1	2	

Thus, after more than doubling the data for close binary systems, it appears that the secondary maximum in length of period does not exist, although there are slight indications of it in some of the curves for the separate spectral types. These, however, disappear in the totals except for the six stars mentioned. It is found also that the maximum of all the frequency-curves falls between the values 0.4 and 0.8 for the logarithm of the period, that is, for the periods between approximately 2.5 and 6 days. The B-type stars seem to have a comparatively shorter range in length of period than any of the others (except the K stars, for which the data are too meager to justify any conclusions), namely, 1.45 to

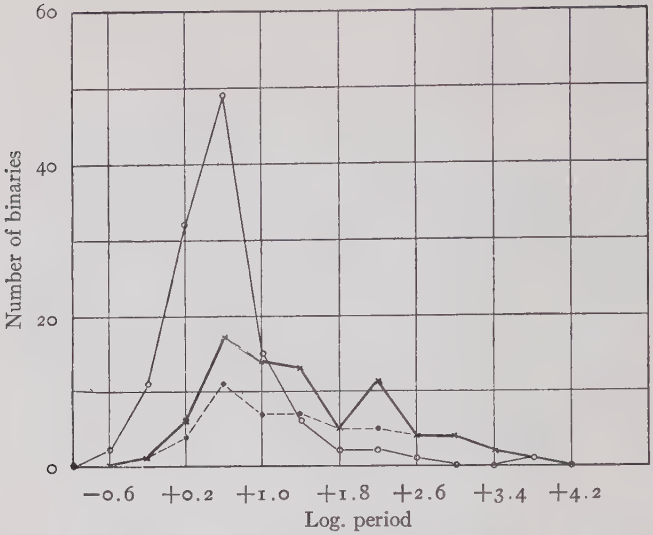

FIG. 3.—Frequency-curves: eclipsing variables, light line; spectroscopic binaries, heavy line; spectroscopic binaries excluding B-type spectra, broken line.

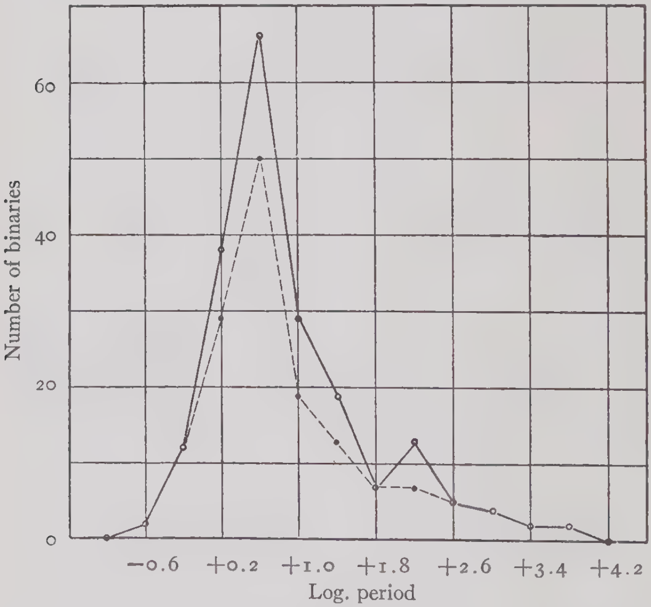

FIG. 4.—Frequency-curve for eclipsing and spectroscopic binaries combined (broken line excludes B-type spectra).

180.2 days; or, without the stars that may be anomolous, 1.45 to 31 days. For the systems with second-type spectra, in which the F's predominate, the range is the greatest, being 0.33 to 9905 days. It is of interest to note that one-third of the second-type systems have periods greater than 180 days and one-seventh shorter than 1.45 days, the outside limits given above for the binaries with B-type spectra.

7. ON THE NUMBER OF NAKED-EYE VARIABLE STARS

As a result of his search for variable stars in globular clusters, Bailey found that about 2.5 per cent of all the stars examined show distinct light-variation; this percentage was compared with the number of variables among the naked-eye stars, which he placed at 1 per cent.[1] Since that study was made the number of known variable stars has considerably increased. In order to get an idea of the percentage of naked-eye stars now known to be variable, and to see how they are distributed in the different variable star classes for each interval of magnitude, the data in Table VI were collected.

TABLE VI

Magnitude	Long Period	Short Period	Eclipsing	Miscellaneous	Total Variables	Total Stars	Percentage Variable
< 1.0 ..	0	0	0	5*	5	12	17 per cent*
1.0 to 1.9 ..	0	0	1	1	2	28	7
2.0 to 2.9 ..	1	1	5	1	8	105	8
3.0 to 3.9 ..	3	6	4	7	20	275	7
4.0 to 4.9 ..	5	4	4	9	22	750	3
5.0 to 6.0 ..	18	7	4	20	49	2160	2
Total ..	27	18	18	43	106	3330	3

* Three of the five variables of this magnitude are Novae, and, as they are now from the ninth to the twelfth magnitude in brightness, they are not included in the percentage.

The second, third, fourth, and fifth columns give the number of variables belonging to the different classes named, whose brightness at maximum light corresponds to the magnitude tabulated in the first column. In the class "Miscellaneous" are included variables with unknown or irregular periods and the Novae. Column six contains the total number of variables, and seven the total number of all stars, while the final column gives the percentage of variables in each magnitude division. It is seen that 3 per cent of all stars visible to the naked eye are known to vary in light.

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[1] *Harvard Annals*, **38**, 1, 1902.

In collecting this material the many bright stars recently suspected of small variation by Lau[1] and Guthnick[2] have not been considered, although many of them will probably be admitted soon to the lists of known variables. Had they been included the total percentage would be increased from three to more than five.

Mount Wilson Solar Observatory
February 1915

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[1] *Astronomische Nachrichten*, **196**, 427, 1914.

[2] *Veröffentlichungen der Königlichen Sternwarte zu Berlin-Babelsberg*, **1**, No. 1, 1914; *Astronomische Nachrichten*, **191**, 169, 1912.

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EFFECTIVE WAVE-LENGTHS OF 184 STARS IN THE CLUSTER N.G.C. 1647

By EJNAR HERTZSPRUNG

I. THE OBSERVATIONS AND THEIR REDUCTION

The following pages contain the details of my determination of effective wave-lengths in the cluster N.G.C. 1647 ($4^h40^m, +19^\circ$) briefly described in *Yearbook*, No. 12, p. 222, 1913, of the Carnegie Institution of Washington and referred to by Seares in his paper "The Color of the Faint Stars" in this *Journal* (**39**, 361, 1914). The instrument employed was the 150-cm (60-inch) Mount Wilson reflector diaphragmed down to 100 cm (40 inches) aperture in order to increase the diameter of the useful field.

Over the end of the tube of the reflector was placed a grating[2] consisting of overspun rubber cords 3 mm thick separated by free spaces of the same width. This grating forms short spectra on both sides of the central star image as shown in Fig. 1, which is enlarged 3.3 times from the original plate. The focal length of the reflector being 7606 mm (299.5 inches), the distance between the centers of the two spectra of first order is about 1 mm (0.04 inch). When the diameter of the wires of the grating is equal to the spaces separating them, all the spectra of uneven order are at their maximum intensity and all those of even order disappear. In this case the spectra of first order are π^1 times or 2.486 mag. fainter than the star image without grating. As the spectra with the dispersion here used are somewhat elongated, especially for the whiter stars, it will be safer to say that the limiting magnitude down to which, for a certain exposure time, effective wave-lengths can be determined is about 3 mag. less than that for which, under the same conditions without grating, measures of position can be made.[2]

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[1] Made by Toepfer of Potsdam.

[2] As the spectrum of a red star is sensibly shorter than that of a white one, the loss in light from elongation of the spectra is somewhat smaller for the red than for the white stars. It will therefore be possible to measure the effective wave-lengths of the former down to a magnitude a little fainter than in the case of the latter. It is easy to avoid an undesirable selection caused by this phenomenon by measuring only to a certain intensity of the central image.

Fig. 1 shows the appearance of two stars of extremely different color. The objects are B.D. +9°4367, which is a white star, and B.D. +9°4369, which has a spectrum of the fourth type. The plate received five exposures of 360, 114, 36, 12, and 4 seconds, respectively, so that one step in the exposure time corresponds to 0.5 in its logarithm or to about 1 mag. in intensity. The faintest images are not visible in the figure. A line is drawn between the images of longest exposure for the two stars. The distance is 12.58 mm on the original plate, or 341″.1 according to the *A.G. Catalogue.* Visually the two stars are of equal brightness—magnitudes 8.5 and 8.7, according to the *B.D.* and magnitudes 8.6 and 8.4, respectively, according to *A.G., Leipzig.* Photographically the difference is about 2 mag. in accordance with the difference in color.

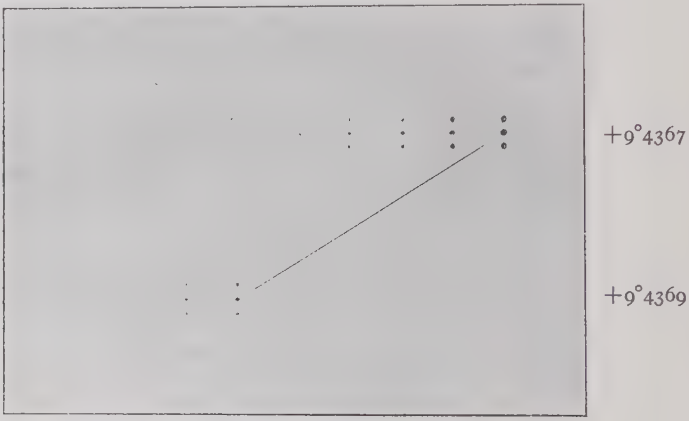

Fig. 1.—Central images and first-order spectra

It is seen at a glance, that the distance between the centers of the two spectra of first order is greater for the red (+9°4369) than for the white (+9°4367) star, owing to the difference in spectral distribution of energy. This distance between the two spectra determines the effective wave-length and may be taken as an equivalent of color. The mean error of one such distance for well-exposed images is about $\pm 6\mu$, corresponding to ± 26 A in the effective wave-length.[1] The effective wave-lengths of the two stars shown in Fig. 1 are found to be 4280 A for the white star B.D. +9°4367, and 4590 A for the red star +9°4369, the difference being 310 A.

The difference in effective wave-lengths for stars of spectra A and K is about 200 A, while the corresponding difference in color-index, $m_{pg} - m_{vis}$, is 1 mag. The mean error ±26 A in one effective wave-length therefore corresponds to a mean error of ±0.13 mag. in the color-index.

The effective wave-length varies not only with the color of the star but also sensibly with the strength of the spectrum image. To be considered as a color equivalent, therefore, a correction is needed to reduce the measured distance between the two spectra, or the effective wave-length, to a normal intensity of image. This correction will vary with the instrument (reflectors or refractors of different achromatization and focal length), the spectral sensitiveness of the plate, and the color of the star. Its accurate determination requires an undesirable increase of work. I therefore preferred to take several photographs of the same region with different exposures, and on each plate to measure only those spectra which are approximately of normal strength.[2] Actually the longest exposure given to the plates used in this investigation was 30 minutes, which, with 9.5 and 3 minutes, and 57, 18, and 6 seconds, forms a geometrical progression, the logarithm of whose constant ratio is 0.5.

The intensity of images just well exposed was taken as the normal. One effective wave-length from an image of normal intensity was given weight 6; and effective wave-lengths from images 1 and 2 mag. weaker or stronger than the normal intensity received weights 4 and 1, respectively, after the correction for deviation from normal intensity had been applied. No accurate knowledge of the correction to normal intensity is needed, therefore, unless the stars are very bright or very faint. Only for the very faintest stars, which do not reach normal brightness even on the longest-exposed plates, is the correction of any importance.

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[1] This accuracy, as is to be expected, is of the same order of magnitude as that attained in the photographic measurement of double stars. With the Copenhagen refractor $a = 20$ cm, $f = 480$ cm, I found for one exposure, on the average, a mean error of $\pm 4.2\,\mu$ for the distance between the two components of a double star. With the 50-cm Potsdam refractor $a = 50$ cm, $f = 1250$ cm, I found the corresponding mean error to be $\pm 4.5\,\mu$.

[2] From all this it appears that the zero-point of the effective wave-lengths is to a certain extent arbitrary. It would be practicable to define the color-indices and the effective wave-lengths in such a way that the indices 0 mag. and +1 mag. correspond to effective wave-lengths of 4200 and 4400 A, respectively. To this end 34 A would have to be subtracted from all the effective wave-lengths given in this and in the following note.

As an average, I found the effective wave-length to increase 19 A for each magnitude of increase in the intensity of the image measured. This rate has been adopted for all stars of all colors.[1]

A chart of the stars in the cluster N.G.C. 1647, used in this investigation, is given in Plate VI. The central star, No. 100, is A.G. Berlin 1290, $4^h40^m9^s.69$, $+18^\circ56'19''.1$ (1900). The diameter of the inner circle is 40′ and that of the outer 1°; the central portion being, of course, in much better field than the outer zone, the results derived for the central stars are therefore the most reliable.

TABLE I

LIST OF PLATES

Plate No.	Date	Sid. Time Middle of Exp.	Exp. Time	Kind of Plate
	1912			
249	Oct. 13	$5^h 23^m$	30^m	Lumière Σ
250	"	5 57	30	Seed 27
251	"	6 20	9.5	Seed 27
252	"	6 33	9.5	Lumière Σ
313	16	6 14	30	Lumière Σ
314	"	6 53	30	Seed 27
323	17	2 15	9.5	Lumière Σ
324	"	2 40	30	Seed 27
325	"	3 4	9.5	Seed 27
326	"	3 27	30	Lumière Σ
327	"	4 11	*	Seed 27
328	"	4 26	*	Lumière Σ

* 3^m, 3^m, 57^s, 57^s, 18^s, 18^s, 6^s, 6^s.

The plates used are listed in Table I.[2] As will be seen, there are two complete sets, one taken on Seed "27," the other on

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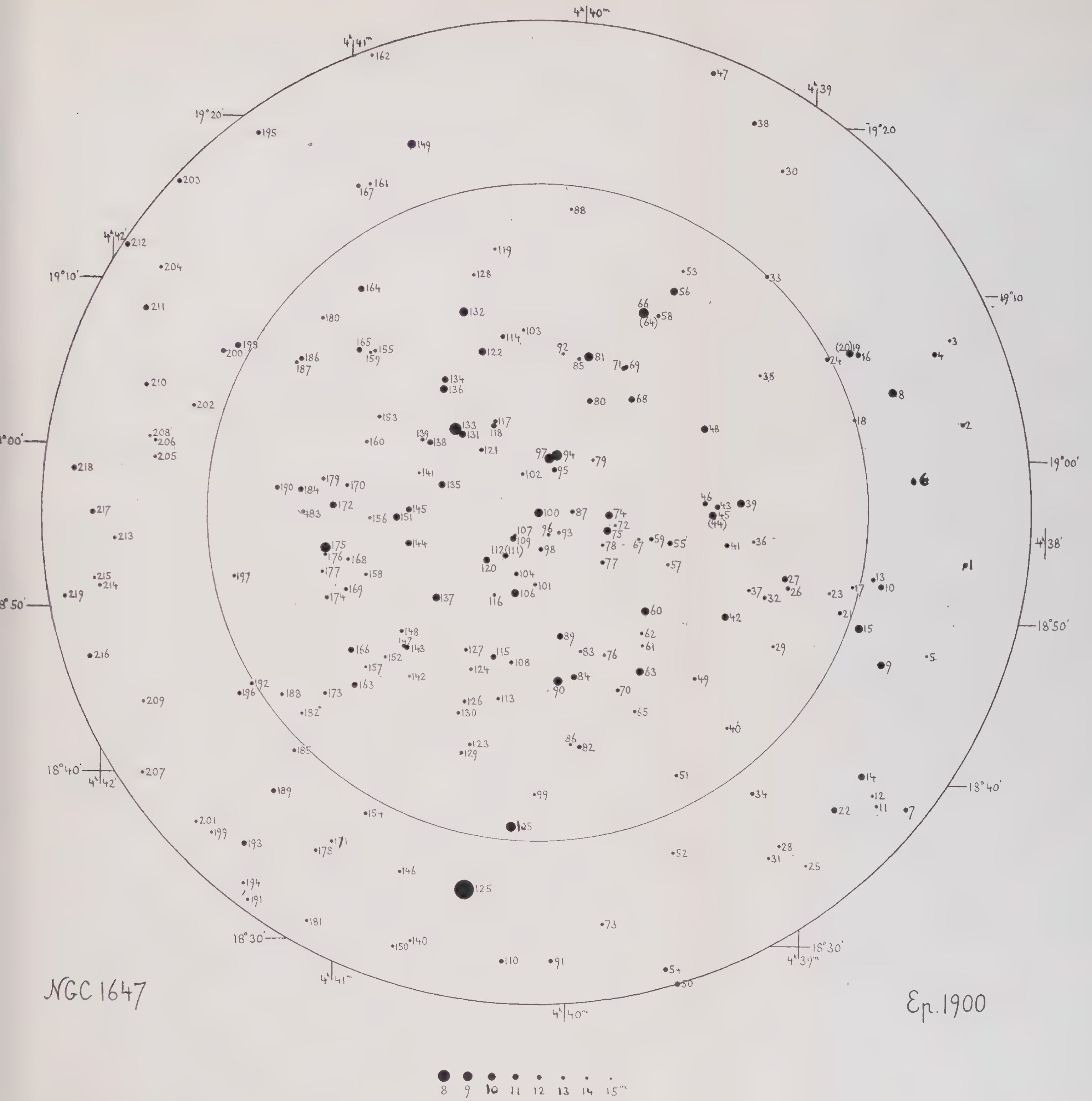
[1] For further details of the method of effective wave-lengths see *Potsdam Publ.*, No. 63, Part 1, 1911. A list of the earlier literature is found in *Astron. Nachr.*, **182**, 301, 1909.

[2] They were measured at the Astronomical Laboratory at Groningen on my return from Mount Wilson to Potsdam.

PLATE VI


4h 40m
4h 41m
4h 39
19°20'
19°20
19°10'
19°10
19°00'
19°00'
18°50'
18°50'
4h 38'
18°40'
4h 42'
18°40'
18°30'
18°30'
4h 39m
4h 41m
4h 40m
NGC 1647
Ep. 1900
8
9
10
11
12
13
14
15m

Lumière "Σ" plates. The two sets have been treated separately. As an average, the effective wave-lengths reduced to the normal strength of image proved to be 29.5 ± 1.5 A (mean error of median value) greater for the Seed "27" than for the Lumière "Σ" plates. Furthermore, there is a slight indication that this difference between the two sets increases about 2.9 A for each magnitude of decrease in brightness. The smallness of this magnitude equation is satisfactory.

To reduce the effective wave-lengths to the same system, the Seed "27" plates were first corrected for the constant difference of 29.5 A between the "Σ" and "27" plates. Secondly, the results from each kind of plate were corrected by half of the magnitude equation, or $\frac{1}{2} \times 2.9$ A per magnitude. Admitting that this magnitude equation arises only from errors of reduction in the two different sets of plates, the final effective wave-lengths are on the Lumière "Σ" system.

In this system I found from other plates an effective wave-length of 4234 A to correspond to a typical A star, the color-index of which $I_H = m_{pg} - m_{vis}$, Harvard, is zero according to the *Göttingen Aktinometrie*, B, 1912. It was furthermore found that a difference in the color-index I_H of 1 mag. corresponds to a difference in effective wave-length of 200 A. Hence we have the following formula for the relation between color-index, I_H, expressed in magnitudes and the effective wave-length λ_{eff} expressed in A:

$$200\ I_H = \lambda_{eff} - 4234 .$$

In Table II the reduced effective wave-lengths and their relative weights, as indicated above, are given for each set of plates separately. The weighted means contained in Table IV are the final values.[1]

It will be remembered that the weight 6 was assigned to an effective wave-length derived from a single image of normal

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[1] Star No. 218, which is 2′ inside the border of the field, has accidentally been omitted from this investigation. This mistake is of no importance for the general conclusions. The star is included in the catalogue for the sake of completeness. No. 219 was first called 218. For all other stars my original notation was kept in spite of there being a few objects which proved to be too faint for measurement of effective wave-lengths.

TABLE II

MEAN RESULTS FROM LUMIÈRE "Σ" AND SEED "27" PLATES

STAR NO.	LUMIÈRE Σ		SEED 27		STAR NO.	LUMIÈRE Σ		SEED 27	
	λ_{eff}	Rel. Wt.	λ_{eff}	Rel. Wt.		λ_{eff}	Rel. Wt.	λ_{eff}	Rel. Wt.
1	......	......	......	......	51	428	2	431	14
2	4337	10	4358	19	52	......	......	507	7
3	......	......	372	0	53	......	......	384	1
4	334	22	321	30	54	331	14	345	22
5	......	......	512	3	55	317	24	336	29
6	......	......	421	0	56	389	30	412	29
7	311	17	328	18	57	......	......	372	4
8	251	23	269	29	58	333	11	337	22
9	257	28	267	28	59	324	12	347	22
10	287	31	302	29	60	213	28	215	29
11	......	......	319	7	61	......	......	428	0
12	......	......	......	......	62	589	3	547	9
13	592	11	570	24	63	250	31	235	30
14	273	30	252	29	64	......	......	401	1
15	297	20	298	30	65	......	......	512	7
16	337	31	331	31	66	254	29	251	29
17	......	......	348	1	67	......	......	372	0
18	386	3	348	12	68	333	34	314	32
19	249	32	258	28	69	339	11	332	22
20	324	16	315	19	70	529	3	521	10
21	377	9	368	16	71	318	15	322	25
22	277	33	270	32	72	......	......	424	0
23	......	......	378	0	73	......	......	489	2
24	334	17	321	25	74	265	32	257	30
25	......	......	339	0	75	284	32	277	30
26	330	9	342	21	76	......	......	347	0
27	302	33	274	29	77	309	12	353	22
28	......	......	443	1	78	......	......	359	2
29	......	......	401	3	79	......	......	562	2
30	......	......	435	2	80	297	32	298	31
31	......	......	497	1	81	249	27	251	30
32	430	3	368	12	82	489	11	482	24
33	315	15	326	25	83	......	......	352	2
34	338	15	330	27	84	291	30	290	31
35	......	......	559	1	85	352	9	358	17
36	......	......	407	4	86	......	......	366	3
37	......	......	358	7	87	360	10	348	19
38	331	14	335	23	88	......	......	361	2
39	241	27	235	29	89	267	37	287	32
40	......	......	370	3	90	258	28	252	28
41	354	20	347	26	91	430	7	404	19
42	272	26	271	30	92	......	......	322	4
43	300	31	308	30	93	......	......	330	7
44	......	......	397	1	94	251	30	252	29
45	259	32	254	30	95	304	26	308	28
46	359	22	363	28	96	284	1	347	9
47	328	21	327	30	97	241	29	249	29
48	258	30	261	31	98	323	13	320	24
49	329	3	313	8	99	......	......	417	0
50	315	23	308	30	100	282	30	266	29

TABLE II—*Continued*

Star No.	Lumière Σ		Seed 27	
	λ_{eff}	Rel. Wt.	λ_{eff}	Rel. Wt.
101	288	0	314	7
102	323	0	368	10
103	......	......	379	3
104	331	16	330	24
105	510	21	515	27
106	263	29	268	30
107	......	......	375	1
108	406	3	463	12
109	287	32	306	32
110	331	10	343	19
111	352	4	385	18
112	387	21	391	27
113	......	......	388	0
114	332	15	327	24
115	293	28	292	30
116	......	......	408	0
117	388	10	393	19
118	311	25	306	27
119	......	......	484	0
120	371	31	388	31
121	359	10	327	17
122	266	30	270	31
123	......	......	353	6
124	......	......	333	0
125	498	16	512	21
126	......	......	395	8
127	......	......	338	7
128	......	......	367	3
129	......	......	510	0
130	......	......	429	0
131	......	......	326	4
132	280	30	282	30
133	242	31	245	29
134	......	......	225	0
135	355	2	347	10
136	344	11	327	19
137	266	30	256	30
138	295	29	299	31
139	439	5	462	12
140	......	......	......	......
141	......	......	......	......
142	......	......	......	......
143	320	25	301	30
144	283	32	297	32
145	315	29	312	30
146	......	......	483	1
147	298	3	340	9
148	346	6	313	11
149	491	23	543	30
150	......	......	398	1
151	286	32	258	32
152	......	......	482	0
153	524	4	501	12
154	391	4	372	14
155	516	1	483	9
156	......	......	463	1
157	......	......	......	......
158	......	......	338	3
159	......	......	392	3
160	......	......	321	0
161	......	......	432	0
162	......	......	370	4
163	278	34	271	32
164	338	32	330	33
165	307	31	287	31
166	311	33	297	29
167	334	12	375	22
168	......	......	360	0
169	357	8	334	16
170	433	0	357	6
171	316	0	342	7
172	287	30	287	32
173	351	0	378	6
174	323	6	332	10
175	254	25	249	29
176	......	......	......	......
177	......	......	449	1
178	305	1	318	9
179	421	2	422	9
180	......	......	355	4
181	243	22	235	30
182	......	......	392	0
183	325	5	309	8
184	308	26	303	30
185	307	1	304	8
186	326	19	329	26
187	360	5	350	11
188	......	......	581	0
189	314	16	317	21
190	228	17	207	25
191	......	......	339	4
192	365	3	357	6
193	311	31	305	30
194	......	......	368	3
195	341	15	342	24
196	325	8	352	10
197	......	......	395	3
198	301	32	278	32
199	......	......	352	0
200	461	21	468	27
201	......	......	464	1
202	399	2	418	7
203	551	17	548	25
204	382	2	403	7

TABLE II—*Continued*

Star No.	Lumière Σ		Seed 27		Star No.	Lumière Σ		Seed 27	
	λ_{eff}	Rel. Wt.	λ_{eff}	Rel. Wt.		λ_{eff}	Rel. Wt.	λ_{eff}	Rel. Wt.
205	......	......	......	......	213	......	......	388	4
206	......	......	405	4	214	299	0	306	7
207	......	......	374	6	215	......	......	382	0
208	......	......	408	0	216	306	21	323	28
209	......	......	......	......	217	315	22	331	28
210	336	15	343	24	218	......	......	......	......
211	558	25	570	31	219	339	10	334	12
212	306	22	325	28					

intensity, the mean error of such a result being about ±26 A. From the differences between the values derived from the Seed "27" and the Lumière "Σ" sets the mean error corresponding to weight 6 is found to be ±20 A. The agreement is satisfactory. There is some indication of a systematic error common to all effective wave-lengths derived from the same plate. So far as the material goes, this plate error seems to be of the order of ±10 A.

II. DISCUSSION OF RESULTS

Fig. 2 is a graphical representation of the results. The appearance of this diagram is very striking, the most curious fact being the lack of faint white stars. For a further discussion it must be remembered that the effective wave-lengths of the fainter stars, say below magnitude 13, are much less accurate than for the brighter ones. The ordinates of the dotted line in Fig. 2 indicate approximately the value of the mean error of the effective wave-lengths for different magnitudes. The faint stars consequently seem more scattered over the different colors than they really are. Everything taken into consideration, we may say:

The fainter the stars in the region examined, the greater the minimum effective wave-length. This minimum value, which increases with decreasing brightness, is rather sharply defined, and scarcely any stars are to be found with effective wave-lengths less than this limit.

The only faint star which is a pronounced exception to this rule is No. 190 of magnitude 12.40, showing an effective wave-length of only 4216 A; the next following of similar magnitude has an effective wave-length of 4316 A (No. 189), which is 100 A greater. The star No. 190 is found to be white on 6 plates, so that there can be no doubt about the reality of its exceptional color.

The question arises whether this relation between apparent magnitude and distribution of colors is to be explained by selective extinction of light in space or whether it may be accounted for in other ways. It must be considered that the region in question includes stars belonging physically to the cluster and also other stars belonging to the system of the Milky Way. The galactic latitude is $-15°$. All the physical members of the cluster may be considered as at the same distance, and hence for these stars we should expect the relation between color and *apparent* brightness to be about the same as that found for other clusters like the Hyades between color and *absolute* brightness. For the other stars, belonging to the general system of the Milky Way, we may, from all we know, adopt a distribution in space which, in its first approximation, is defined by a rather sharp outer limit beyond which

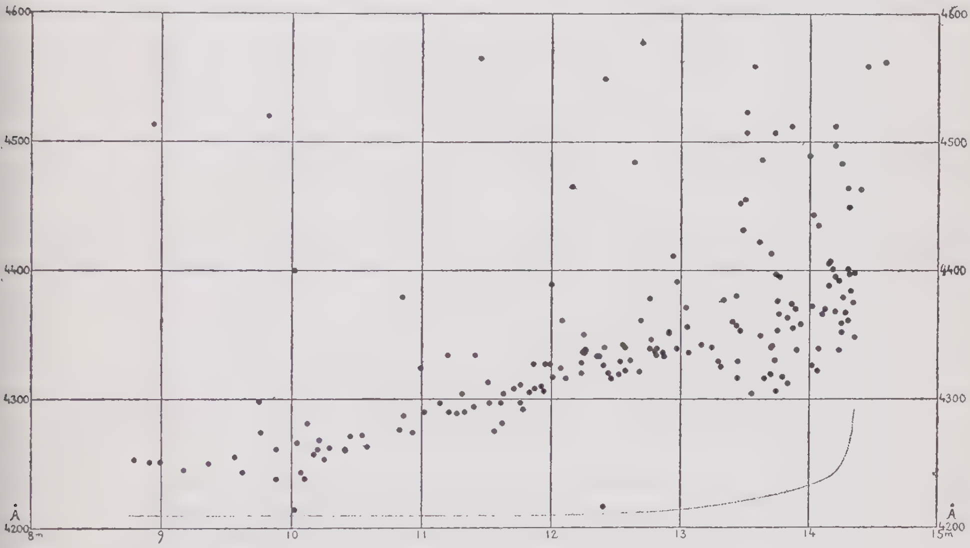

FIG. 2.—Effective wave-lengths and photographic magnitudes in N.G.C. 1647. Dotted curve indicates mean error of effective wave-length.

practically no stars occur. That is to say, stars of a certain apparent brightness cannot have an absolute brightness greater than that corresponding to the limiting distance. The maximum *absolute* brightness must therefore decrease with the *apparent* brightness.

Now for stars of known absolute brightness and color we find that the smaller the absolute brightness, the yellower is the minimum effective wave-length, and that only extraordinary stars occur which are whiter than the limit thus defined.[1] Dividing the stars of N.G.C. 1647 into groups of 29 or 30, I find the following relation between apparent magnitude and the median value of the effective wave-length:

Pg. magnitude	10.02	11.45	12.40	13.06	13.74	14.20	14.45
Eff. wave-length	4260	4300	4327	4347	4364	4389	4396A
$O-C$	+4	0	−2	−2	−6	+5	+4A

This relation as shown by the differences $O-C$ is well represented by the linear formula:

$$\lambda_{\mathrm{eff}}-4340=30.5\,(m-12.76).$$

The increase in median effective wave-length for each magnitude decrease in apparent photographic magnitude is 30.5 A. It is remarkable that this rate, all uncertainties taken into consideration, is practically the same as that, namely 26 A per magnitude, found for stars of known absolute brightness and color within the same interval of effective wave-lengths (4260–4396 A).[2] This is what we should expect for stars physically belonging to the cluster. But for the other faint stars we may also expect something similar, for it is a consequence of our ideas of the distribution of stars in space that the median distance of the fainter stars here in question will not vary much with the apparent magnitude.[3]

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[1] See the following article.

[2] See the following article. The fact that photographic magnitudes have been used here, and visual magnitudes in the following article, has been taken into consideration.

[3] The order of magnitude of this median distance of the fainter stars, say of magnitude 14, may be estimated in the following way. For stars of the photographic magnitude 14 we find the median effective wave-length to be 4378 A. It is obvious

We may therefore say that the results found in this note are in fair agreement with what we know about the distribution of stars in space and the relation between absolute brightness and color. They do not afford any evidence of selective extinction of light in space.

III. DETERMINATION OF THE PHOTOGRAPHIC MAGNITUDES

The photographic magnitudes used in this note have been determined in the following way: During the measurement of the effective wave-lengths, the diameters of the central stars were estimated. Assuming that a difference of 0.5 in the logarithm of the exposure time corresponds to a difference of one magnitude, the estimates were converted into provisional magnitudes. These were used in the manner described above to reduce all the effective wave-lengths measured to the same intensity of image. For the more accurate determination of magnitudes in the central portion of the cluster, the following Schleussner plates were taken on January 14, 1913, by the "Halbgitter" method, with the 80-cm Potsdam refractor:

	Plate No.			
	371	372	373	374
Halbgitter................	None	None	North	South
Exposure time............	2×10^{m}	30^{m}	30^{m}	30^{m}
Sidereal time.............	$3^{h}23^{m}$	$3^{h}52^{m}$	$4^{h}59^{m}$	$5^{h}52^{m}$

The plates were measured with the Hartmann microphotometer. The zero point of the magnitudes was fixed by

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that the "giant" yellow stars will form only a small minority among the yellow stars here considered. Now for "dwarf" stars of known absolute brightness and color the effective wave-length 4378 A corresponds to an absolute photographic brightness (referred to distance corresponding to $\pi = 1''$) of about $+0.5$ mag. Assuming the same absolute brightness for the stars of magnitude 14 here considered, we find their median parallax to be $0''.002$ and their median distance 1600 light-years. This is a plausible value. The parallax of N.G.C. 1647 itself may be estimated in the same way, supposing most of the tenth-magnitude A stars to belong physically to the cluster. Taking the absolute magnitude of these A stars to be -4, we find the parallax to be $0''.0016$. I imagine that the determination of effective wave-lengths of faint stars will be a valuable help in sounding the Milky Way in different directions. A considerable number of the plates I took at Mount Wilson are meant to serve this purpose.

TABLE III

Photographic Magnitudes

Star No.	Schleussner "Halb-gitter"	Lumière Σ Estimates	Seed 27 Estimates	Star No.	Schleussner "Halb-gitter"	Lumière Σ Estimates	Seed 27 Estimates
		mag.	mag.		mag.	mag.	mag.
1	...	...	14.45	52	...	13.77	13.70
2	...	12.86	12.95	53	14.32	...	14.30
3	...	...	14.40	54	...	12.40	12.42
4	...	12.04	11.86	55	11.82	11.95	11.92
5	...	...	14.20	56	10.07	9.92	9.94
6	...	...	14.45	57	14.01	...	14.07
7	...	12.04	12.42	58	12.89	12.76	12.85
8	...	9.81	9.94	59	12.76	12.67	12.85
9	...	10.31	10.28	60	10.04	10.03	9.94
10	...	11.50	11.31	61	14.51	...	14.45
11	...	...	13.70	62	13.55	13.60	13.62
12	...	...	14.51	63	10.06	...	10.11
13	...	12.76	12.64	64	...	...	14.30
14	...	10.67	10.50	65	13.90	13.77	13.79
15	...	9.68	9.82	66	8.83	8.71	8.68
16	...	11.40	11.42	67	14.38	...	14.45
17	...	...	14.35	68	11.00	10.95	11.01
18	...	13.41	13.44	69	12.81	12.76	12.85
19	...	10.22	10.28	70	13.48	13.60	13.53
20	mag.	12.50	12.53	71	12.43	12.50	12.42
21	13.04	12.95	13.15	72	14.57	...	14.48
22	...	10.95	10.91	73	...	...	14.00
23	14.47	...	14.51	74	10.41	10.40	10.39
24	12.39	12.50	12.42	75	10.09	10.22	10.17
25	...	...	14.51	76	14.49	...	14.50
26	12.80	12.67	12.75	77	12.82	12.76	12.75
27	11.32	11.13	11.21	78	14.25	...	14.26
28	...	13.86	14.20	79	14.65	...	14.35
29	14.17	...	14.20	80	11.50	11.50	11.62
30	...	...	14.07	81	9.37	9.27	9.41
31	...	...	14.20	82	12.64	12.67	12.64
32	13.43	13.41	13.44	83	14.23	...	14.35
33	12.62	12.50	12.42	84	11.31	11.31	11.42
34	12.36	12.58	12.20	85	13.10	12.86	13.05
35	14.47	...	14.35	86	14.07	...	14.20
36	14.18	...	14.07	87	12.93	12.76	12.95
37	14.01	13.77	13.78	88	...	...	14.30
38	...	12.40	12.31	89	10.79	10.95	10.90
39	10.16	10.03	9.94	90	9.56	9.55	9.55
40	14.10	...	14.20	91	...	13.04	12.85
41	12.25	12.22	12.31	92	14.15	13.77	14.00
42	10.48	10.40	10.39	93	13.70	13.77	13.79
43	11.66	11.50	11.62	94	8.91	8.85	8.95
44	14.31	...	14.30	95	11.90	11.95	12.09
45	10.18	10.13	10.17	96	13.70	13.77	13.70
46	12.09	12.13	11.97	97	9.13	9.27	9.26
47	...	12.13	11.86	98	12.69	12.67	12.64
48	10.46	10.31	10.28	99	14.40	...	14.50
49	13.80	13.86	13.70	100	9.77	9.81	9.69
50	...	11.95	11.58	101	13.88	13.77	13.70
51	13.46	13.69	13.35	102	13.79	13.77	13.62

TABLE III—*Continued*

Star No.	Schleussner "Halb-gitter"	Lumière Σ Estimates	Seed 27 Estimates	Star No.	Schleussner "Halb-gitter"	Lumière Σ Estimates	Seed 27 Estimates
	mag.	mag.	mag.		mag.	mag.	mag.
103	14.27	....	14.20	155	13.64	13.78	13.44
104	12.64	12.59	12.53	156	14.44	....	14.35
105	....	8.85	9.02	157	14.77	....	14.39
106	10.02	10.13	10.05	158	14.24	....	14.20
107	14.36	....	14.26	159	14.24	....	14.20
108	13.43	13.60	13.44	160	14.40	....	14.50
109	11.12	11.13	11.21	161	....	....	14.45
110	....	12.76	12.85	162	....	13.78	14.00
111	....	12.67	12.85	163	10.52	10.67	10.60
112	11.95	12.04	12.19	164	11.22	11.22	11.11
113	14.32	....	14.41	165	11.66	11.50	11.51
114	12.54	12.50	12.53	166	11.29	11.40	11.31
115	11.76	11.76	11.86	167	....	12.68	12.69
116	14.50	....	14.41	168	14.42	....	14.45
117	13.02	12.76	12.95	169	13.22	12.95	13.15
118	11.80	11.86	12.19	170	13.83	13.78	13.86
119	14.56	....	14.44	171	....	13.69	13.70
120	10.83	10.95	10.81	172	10.83	10.95	10.90
121	13.00	12.77	13.05	173	13.73	13.78	13.78
122	10.19	10.22	10.28	174	13.27	13.23	13.44
123	13.74	....	13.78	175	8.92	9.00	9.25
124	14.40	....	14.41	176	13.93	....	14.00
125	....	(6.6)	(6)	177	14.31	....	14.30
126	13.78	13.78	13.70	178	....	13.69	13.62
127	13.95	....	13.70	179	13.61	13.69	13.54
128	14.30	....	14.20	180	13.84	....	14.00
129	14.34	....	14.45	181	....	9.81	9.94
130	14.49	....	14.50	182	14.56	....	14.50
131	14.03	....	14.00	183	13.41	13.32	13.70
132	11.61	11.67	11.62	184	11.81	11.86	11.86
133	9.65	9.55	9.55	185	13.50	13.60	13.70
134	14.67	....	14.44	186	12.22	12.22	12.31
135	13.65	13.60	13.54	187	13.48	13.32	13.54
136	12.90	12.68	12.95	188	14.16	....	14.41
137	10.20	10.31	10.18	189	12.38	12.50	12.75
138	11.80	11.59	11.75	190	12.37	12.50	12.42
139	13.54	13.41	13.44	191	....	....	14.07
140	....	....	14.55	192	13.28	13.60	13.79
141	15.3	....	....	193	....	11.67	11.75
142	15.3	....	....	194	....	....	14.20
143	11.95	11.95	11.75	195	....	12.50	12.59
144	11.00	11.13	11.00	196	....	13.04	13.44
145	11.48	11.50	11.62	197	....	....	14.20
146	....	....	14.25	198	....	11.22	11.21
147	13.37	13.60	13.54	199	....	....	14.50
148	13.28	13.23	13.54	200	....	12.13	12.19
149	....	9.81	9.82	201	....	....	14.30
150	....	....	14.35	202	....	13.69	13.70
151	10.49	10.67	10.60	203	....	12.40	12.42
152	14.86	....	14.50	204	....	13.69	13.79
153	13.55	13.41	13.44	205	....	....	14.52
154	....	13.41	13.25	206	....	....	14.15

TABLE III—*Continued*

Star No.	Schleussner "Halbgitter"	Lumière Σ Estimates	Seed 27 Estimates	Star No.	Schleussner "Halbgitter"	Lumière Σ Estimates	Seed 27 Estimates
		mag.	mag.			mag.	mag.
207	....	13.86	13.86	214	....	13.78	13.70
208	....	....	14.45	215	....	....	14.48
209	....	....	14.55	216	....	12.13	12.09
210	....	12.50	12.64	217	....	12.04	12.09
211	....	10.40	10.50	218	....	....	....
212	....	12.04	11.98	219	....	12.77	13.35
213	....	....	14.15				

means of plate No. 371, which contained, besides the two exposures of N.G.C. 1647, one of the same duration (10^m) on the central part of the Pleiades.

The extinction constant of the Halbgitter used was not determined directly from photometric measures, but, as described in *Astron. Nachr.*, **199**, 247, from the ratio $O-C/C-S_1$, where O is the magnitude of the stellar image without halbgitter, C of the central image behind the halbgitter, and S_1 of the spectra of first order. The constant was thus found to be 2.14 mag. The results are contained in Table III together with the estimates from the diameters of the stars on the Lumière "Σ" and the Seed "27" plates, both reduced to the halbgitter scale. To these three series were assigned the relative weights 4, 1, and 1, respectively. The weighted means are contained in the catalogue, Table IV. The magnitudes of the stars outside the inner field of 40′ diameter are, of course, very uncertain owing to the neglect of the correction for distance from center of field.

IV. COMPARISON OF EFFECTIVE WAVE-LENGTHS WITH SPECTRA

It will be of special interest to compare the effective wave-lengths found here with the spectra of the same stars. With a mirror of 90 cm (35 inches) focal length, in connection with an objective prism 16×16 cm square, Professor Eberhard has kindly made a few exposures on the cluster, using Seed "30" plates. The best plate was taken on March 1, 1913, from 6^h0^m to 7^h32^m sidereal time, Potsdam. In order to reach stars as faint as possible, the dispersion was very small, the distance between Hβ and Hϵ being

TABLE IV

CATALOGUE

Star No.	α (1900)*	δ (1900)	Dist. from Center of Field	Ph. Mag.	Effective Wave-Length	Rel. Wt.†
1.........	$4^{h}38^{m}14^{s}.$	$+18^{\circ}53'.5$	27′	14.45	....	....
2.........	20.	19 2.2	26	12.91	4351	29
3.........	24.	19 7.3	27	14.40	4372	0
4.........	28	19 6.5	26	11.95	4327	52
5.........	28.	18 48.0	25	14.20	4512	3
6.........	31	18 58.6	23	14.45	4421	0
7.........	33.	18 38.6	29	12.23	4320	35
8.........	39	19 4.0	23	9.88	4261	52
9.........	40	18 47.4	23	10.29	4262	56
10.........	40.	18 52.2	21	11.40	4294	60
11.........	40.	18 38.8	27	13.70	4319	7
12.........	42	18 39.4	27	14.51	....	....
13.........	43	18 52.6	21	12.70	4577	35
14.........	44.	18 40.3	26	10.58	4263	59
15.........	46	18 49.6	21	9.75	4298	50
16.........	48	19 6.3	22	11.41	4334	62
17.........	48	18 52.1	20	14.35	4348	1
18.........	48.	19 2.3	20	13.43	4357	15
19.........	50	19 6.4	21	10.25	4253	60
20.........	50.	19 6.4	21	12.52	4319	35
21.........	51	18 50.5	19	13.04	4371	25
22.........	51.	18 38.5	26	10.93	4274	65
23.........	54	18 51.7	18	14.48	4378	0
24.........	56	19 6.0	20	12.40	4326	42
25.........	58.	18 35.0	27	14.51	4339	0
26.........	39 5	18 52.0	16	12.77	4346	30
27.........	5.	18 52.5	16	11.27	4289	62
28.........	5.	18 36.2	25	14.03	4443	1
29.........	8	18 48.4	17	14.18	4401	3
30.........	8.	19 17.5	26	14.07	4435	2
31.........	8.	18 35.4	25	14.20	4497	1
32.........	10.	18 51.4	15	13.43	4380	15
33.........	11.	19 11.1	20	12.57	4322	40
34.........	12.	18 39.4	22	12.37	4333	42
35.........	13	19 5.0	16	14.45	4559	1
36.........	14	18 54.8	13	14.16	4407	4
37.........	15	18 51.8	14	13.93	4358	7
38.........	16	19 20.4	27	12.35	4333	37
39.........	17.	18 57.1	12	10.10	4238	56
40.........	19.	18 43.3	18	14.12	4370	3
41.........	20.	18 54.5	12	12.25	4350	46
42.........	20.	18 50.1	13	10.45	4271	56
43.........	23.	18 56.9	11	11.63	4304	61
44.........	24	18 56.2	11	14.31	4397	1
45.........	24.	18 56.3	11	10.17	4257	62
46.........	26.	18 57.1	10	12.08	4361	50
47.........	26.	19 23.4	29	11.99	4327	51
48.........	27	19 1.7	11	10.41	4260	61

* The period after the seconds of right ascension indicates an additional $0^{s}.5$.

† The weight 6 corresponds to a mean error of ±20 A, and, consequently, weight 1 to ±50 A, 3 to ±28 A, 12 to ±14 A, 24 to ±10 A, 38 to ±8 A, 48 to ±7 A, 67 to ±6 A.

TABLE IV—*Continued*

Star No.	α (1900)	δ (1900)	Dist. from Center of Field	Ph. Mag.	Effective Wave-Length	Rel. Wt.
49	$4^h39^m29^s$	+18° 46′.4	14′	13.79	4317	11
50	31	18 27.7	30	11.76	4311	53
51	32.	18 40.4	18	13.48	4431	16
52	33	18 35.7	22	13.73	4507	7
53	33.	19 11.3	17	14.32	4384	1
54	34	18 28.6	29	12.41	4340	36
55	35.	18 54.6	8	11.86	4327	53
56	37	19 10.1	16	10.02	4400	59
57	37	18 53.3	8	14.02	4372	4
58	39.	19 8.5	14	12.86	4336	33
59	40.	18 54.9	7	12.76	4339	34
60	41.	18 50.4	9	10.02	4214	57
61	42	18 48.3	10	14.50	4428	0
62	42.	18 49.1	10	13.57	4559	12
63	42.	18 46.7	12	10.07	4243	61
64	43	19 8.6	14	14.30	4401	1
65	43.	18 44.2	14	13.86	4512	7
66	43.	19 8.7	14	8.79	4253	58
67	43.	18 54.8	6	14.39	4372	0
68	46	19 3.4	9	10.99	4324	66
69	48	19 5.4	10	12.81	4334	33
70	48	18 45.5	12	13.51	4523	13
71	48.	19 5.3	10	12.44	4320	40
72	50	18 55.6	5	14.55	4424	0
73	51	18 31.2	25	14.00	4489	2
74	51.	18 56.1	4	10.41	4261	62
75	52	18 55.3	4	10.12	4281	62
76	52	18 47.7	10	14.49	4347	0
77	52.	18 53.3	5	12.80	4337	34
78	53	18 54.4	4	14.25	4359	2
79	56	18 59.6	5	14.59	4562	2
80	57	19 3.2	8	11.52	4297	63
81	57.	19 5.9	10	9.36	4250	57
82	57.	18 42.0	15	12.64	4484	35
83	58	18 47.9	9	14.25	4352	2
84	59.	18 46.3	10	11.33	4290	61
85	40 0	19 5.8	10	13.05	4356	26
86	0	18 42.2	14	14.10	4366	3
87	1	18 56.4	2	12.91	4352	29
88	3	19 14.9	19	14.30	4361	2
89	3.	18 48.8	8	10.83	4276	69
90	3.	18 46.0	10	9.56	4255	56
91	4	18 29.0	27	12.94	4411	26
92	4	19 6.1	10	14.06	4322	4
93	4	18 55.1	2	13.73	4330	7
94	5	18 59.9	4	8.91	4251	59
95	5.	18 59.0	3	11.94	4306	54
96	7	18 55.0	1	13.71	4341	10
97	7	18 59.7	3	9.17	4245	58
98	8.	18 54.1	2	12.68	4321	37
99	9	18 39.1	17	14.42	4417	0
100	9.	18 56.3	0	9.76	4274	59
101	10	18 51.9	4	13.83	4312	7

TABLE IV—*Continued*

Star No.	α (1900)	δ (1900)	Dist. from Center of Field	Ph. Mag.	Effective Wave-Length	Rel. Wt.
102	$4^h40^m14^s$	$+18^\circ 58'.7$	3′	13.76	4366	10
103	14.	19 7.5	11	14.26	4379	3
104	15	18 52.6	4	12.61	4330	40
105	15	18 37.1	19	8.94	4513	48
106	15	18 51.4	5	10.04	4266	59
107	15.	18 54.9	2	14.34	4375	1
108	15.	18 47.1	9	13.46	4452	15
109	16	18 54.7	2	11.14	4297	64
110	16.	18 28.9	27	12.81	4339	29
111	17.	18 53.7	3	12.76	4378	22
112	18	18 53.7	3	12.00	4389	48
113	19	18 44.9	12	14.34	4388	0
114	20	19 7.1	11	12.53	4329	39
115	20	18 47.5	9	11.78	4292	58
116	20.	18 51.2	6	14.48	4408	0
117	21	19 1.9	6	12.97	4391	29
118	21.	19 1.6	6	11.87	4308	52
119	22	19 12.4	16	14.54	4484	0
120	22.	18 53.4	4	10.85	4379	62
121	24.	19 0.1	5	12.97	4339	27
122	25	19 6.1	10	10.21	4268	61
123	26	18 42.1	15	13.75	4353	6
124	26	18 46.7	10	14.40	4333	0
125	26.	18 33.2	23	6.	4506	37
126	27.	18 44.7	12	13.77	4395	8
127	27.	18 47.9	9	13.90	4338	7
128	27.	19 10.8	15	14.28	4367	3
129	28	18 41.6	15	14.36	4510	0
130	29	18 44.0	13	14.49	4429	0
131	29.	19 1.1	7	14.02	4326	4
132	30	19 8.5	13	11.62	4281	60
133	31.	19 1.4	7	9.62	4243	60
134	34.	19 4.4	10	14.62	4225	0
135	34.	18 57.9	6	13.62	4349	12
136	34.	19 3.7	9	12.87	4333	30
137	35.	18 51.0	8	10.20	4261	60
138	38	19 0.5	8	11.76	4297	60
139	40	19 0.7	8	13.50	4455	17
140	40	18 30.1	27	14.55	....	....
141	40.	18 58.6	8	15.3	....	....
142	41.	18 46.2	13	15.3	....	....
143	42.	18 47.9	11	11.92	4310	55
144	42.	18 54.3	8	11.02	4290	64
145	43	18 56.3	8	11.51	4313	59
146	43	18 34.3	23	14.25	4483	1
147	43.	18 48.0	11	13.44	4329	12
148	44	18 48.9	11	13.31	4325	17
149	44.	19 18.7	24	9.82	4520	53
150	44.	18 29.7	28	14.35	4398	1
151	46	18 55.9	9	10.54	4272	64
152	48	18 47.4	13	14.79	4482	0
153	51	19 2.0	11	13.51	4507	16
154	52	18 37.8	21	13.33	4377	18

TABLE IV—*Continued*

Star No.	α (1900)	δ (1900)	Dist. from Center of Field	Ph. Mag.	Effective Wave-Length	Rel. Wt.
155	$4^h40^m52^s$	+19° 6′.0	14′	13.63	4486	10
156	53	18 55.8	10	14.40	4463	1
157	53	18 46.7	14	14.69	......	....
158	53.	18 52.3	11	14.23	4338	3
159	54	19 5.9	14	14.23	4392	3
160	54	19 0.5	11	14.42	4321	0
161	54.	19 16.2	23	14.45	4432	0
162	55	19 24.1	30	13.89	4370	4
163	55.	18 45.6	15	11.56	4275	66
164	56.	19 9.8	17	11.20	4334	65
165	56.	19 6.1	15	11.61	4297	62
166	57	18 47.7	14	11.31	4304	62
167	58	19 16.1	23	12.69	4361	34
168	58	18 53.3	12	14.43	4360	0
169	58.	18 51.4	13	13.16	4342	24
170	59	18 57.8	12	13.83	4363	6
171	41 1	18 36.0	24	13.70	4340	7
172	2.	18 56.5	12	10.86	4287	62
173	3	18 45.1	17	13.75	4376	6
174	3.	18 50.9	14	13.29	4329	16
175	4	18 53.9	13	8.99	4251	54
176	4	18 53.5	13	13.94	....	....
177	4.	18 52.5	14	14.31	4449	1
178	5	18 35.4	25	13.65	4316	10
179	5	18 58.1	13	13.61	4422	11
180	6	19 8.0	18	13.87	4355	4
181	7	18 31.2	29	9.88	4238	52
182	9	18 43.6	19	14.55	4392	0
183	10	18 56.1	14	13.44	4316	13
184	11	18 57.4	14	11.83	4305	56
185	11	18 41.5	21	13.55	4304	9
186	11.	19 5.4	17	12.23	4328	45
187	12.	19 5.2	17	13.46	4353	16
188	14.	18 44.9	19	14.21	4581	0
189	16	18 39.0	23	12.46	4316	37
190	17	18 57.6	16	12.40	4216	42
191	22	18 32.4	29	14.07	4339	4
192	22	18 45.6	20	13.42	4360	9
193	23	18 35.8	27	11.71	4308	61
194	23	18 33.4	29	14.20	4368	3
195	23.	19 19.1	29	12.55	4342	39
196	25	18 44.9	21	13.24	4340	18
197	27.	18 53.5	18	14.20	4395	3
198	28	19 6.2	21	11.21	4290	64
199	31.	18 36.5	28	14.50	4352	0
200	31.	19 5.8	21	12.16	4465	48
201	35.	18 37.1	28	14.30	4464	1
202	38.	19 2.4	22	13.70	4413	9
203	43.	19 16.1	30	12.41	4549	42
204	48	19 10.8	27	13.74	4397	9
205	48.	18 59.3	23	14.52	....	....
206	48.	19 0.3	24	14.15	4405	4
207	49.	18 40.1	29	13.86	4374	6

TABLE IV—*Continued*

Star No.	α (1900)	δ (1900)	Dist. from Center of Field	Ph. Mag.	Effective Wave-Length	Rel. Wt.
208..........	$4^h41^m49^s$	$+19^\circ\ 0'.5$	24′	14.45	4408	0
209..........	49.	18 44.4	26	14.55	....	....
210..........	51	19 3.7	25	12.57	4340	39
211..........	51.	19 8.3	27	11.45	4565	56
212..........	56.	19 12.2	30	12.01	4317	50
213..........	58	18 54.3	26	14.15	4388	4
214..........	42 1.	18 51.4	27	13.74	4306	7
215..........	3	18 51.8	27	14.48	4382	0
216..........	3.	18 47.1	28	12.11	4316	49
217..........	4	18 55.9	27	12.07	4324	50
218..........	9	18 58.5	28	(11.9)	(4339)	....
219..........	10	18 50.7	29	13.06	4336	22

only 0.75 mm (0.03 inch). The classification of the spectra could therefore be made only roughly. The results I obtained are given in Table V, arranged according to effective wave-length. The median value of the effective wave-lengths of 23 A-type stars is 4266 A, which agrees well enough with the value given above for stars of zero color-index, viz., 4234 A.

TABLE V

EFFECTIVE WAVE-LENGTHS AND SPECTRA

Star No.	Sp.	λ_{eff}	Star No.	Sp.	λ_{eff}	Star No.	Sp.	λ_{eff}
60...	A?	4214	137....	A	4261	198....	A	4290
39...	A	4238	9....	A?	4262	109....	A	4297
181...	F?	4238	14....	A	4263	165....	A	4297
63...	F?	4243	106....	A	4266	15....	F?	4298
133...	A	4243	122....	A	4268	193....	A?	4308
97...	A	4245	42....	A	4271	143....	A?	4310
81...	A	4250	151....	A	4272	68....	A	4324
94...	A?	4251	22....	A	4274	55....	A?	4327
175...	F?	4251	100....	A	4274	164....	F?	4334
19...	A	4253	163....	A?	4275	56....	G, K	4400
66...	A	4253	89....	A?	4276	125....	G, K	4506
90...	A	4255	75....	A?	4281	105....	G, K	4513
45...	A	4257	172....	A	4287	149....	G, K	4520
48...	A	4260	27....	A	4289	211....	G, K, M	4565
8...	A?	4261	84....	A?	4290	....	....	....

POTSDAM
January 25, 1915

Contributions from the Mount Wilson Solar Observatory, No. 101
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EFFECTIVE WAVE-LENGTHS OF ABSOLUTELY FAINT STARS

By EJNAR HERTZSPRUNG

It is a well-known fact, of which the stars α and β Orionis may be taken as extreme examples, that absolutely bright stars may have all colors between red and white; but the fainter the absolute brightness, the more limited is the range of color within which the stars are distributed. This limitation is practically one-sided in the sense that only rarely are faint white stars to be found. The less the absolute brightness, the yellower is the color, with only a very few stars which are relatively white.

In order to test this relation between absolute brightness and color down to absolute magnitudes as faint as possible, I took with the 1.5-meter (60-inch) Mount Wilson reflector in the manner described in the preceding article[1] on N.G.C. 1647 a number of effective wave-length plates of faint stars having large well-known parallaxes. All exposures were made on Lumière "Σ" plates.[2] Care was taken to reduce the effective wave-lengths to the same zero-point as those for N.G.C. 1647. The values given here can therefore be directly compared with the former. The effective wave-lengths have been reduced to the same zenith distance.[3]

The results are given in Table I. This table contains among others the absolutely darkest stars so far known, which unlike the companion to Procyon are not too near a bright star for proper examination. The absolute parallaxes are those in *Groningen*

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 100; *Astrophysical Journal*, **42**, 92, 1915.

[2] The measures were made at Potsdam, a Toepfer machine with measurable movement of plate being used.

[3] Owing to selective extinction of light in our atmosphere, the effective wave-length of a star increases with its zenith distance. At sea-level this increase amounts to 35 A in passing from the zenith to a zenith distance of 60°. The greatest difference in corrections occurring here was 20 A for τ Ceti as compared with stars near the zenith.

TABLE I

Star	α (1900)	δ (1900)	Vis. Mag. Harv.	Abs. π	Vis. Mag. Red. to π=1″	λ eff	No. Images	No. Plates	Sp.¶
θ Persei A	$2^h37^m.4$	+48° 48′	$4^m.23$	0″.082	-1^m20	A 4299	2	1	G
η Cassiop. A	0 43.0	+57 17	3.67	.193	+0.10	4325	3	2	F8
o_2 Eridani A	4 10.7	− 7 49	4.48	.179	+0.74	4415	4	2	G5
τ Ceti	1 39.4	−16 28	3.65	.319	+1.17	4371	3	1	K
Mayer 20	0 43.1	+ 4 46	5.82	.169	+1.96	4462	4	1	K2
Σ 443 A	3 40.2	+41 9	8.35	.029	(+2.66)‖	4393	5	1	G6?
61 Cygni A	21 2.4	+38 15	5.57	.298	+2.94	4532	14	2	K8
Σ 443 B	3 40.2	+41 9	8.89	.029	(+3.20)‖	4527	4	1	G6?
Weisse 17^h322	17 20.8	+ 2 14	7.82	.126	+3.32	4575	3	1	K9
Σ 1280 A	8 46.0	+71 11	8.65	.099	+3.63	4543	2	1	Ma
Σ 1280 B	8 46.0	+71 11	8.65	.099	+3.63	4533	2	1	Ma
61 Cygni B	21 2.4	+38 15	6.28	.298	+3.65	4556	20	2	K8
B.D.+48°739	2 36.1	+48 34	9.62	.069	+3.81	4446	5	3	....
η Cassiop. B	0 43.0	+57 17	7.41	.193	+3.84	4512	7	2	....
Σ 1321 A	9 7.6	+53 7	8.0	.155	+3.95	4523	4	1	K8
Σ 1321 B	9 7.6	+53 7	8.0	.155	+3.95	4523	4	1	K8
OΣ 547 A	0 0.3	+45 16	9.31	.105	+4.42	4494	4	1	K5
OΣ 547 B	0 0.3	+45 16	9.31	.105	+4.42	4534	4	1	....
θ Persei B	2 37.4	+48 48	9.86	.082	+4.43	4504	2	1	....
Groom. 34 A	0 12.5	+43 27	7.97	.281	+5.21	4529	12	2	Ma
Lal. 46650	23 44.0	+ 1 52	9.09	.182	+5.39	4498	6	3	Ma
OΣ 547 C	0 0.3	+45 16	10.51	.105	+5.62	4498	4	2	....
Lal. 21185	10 57.9	+36 38	7.60	.410	+5.66	4512	9	1	Ma
o_2 Eridani BC	4 10.7	− 7 49	9.48	.179	+5.74	4179	6	1	Ao
AOe 17415−6	17 37.0	+68 26	9.1	.268	+6.24	4505	5	3	Mb
Krüger 60 A	22 24.4	+57 12	9.59	0.256	+6.63	4531	7	6	Mb

						A			
Σ 2398 A	$18^h41^m.8$	+59° 29′	$9^m.33$	$0''.293$	$+6^m.66$	4520	13	4	Ma
Σ 2398 B	18 41.8	+59 29	10.10	.293	+7.43	4530	9	5	....
Groom. 34 B	0 12.5	+43 27	10.99	.281	+8.23	4574	4	4	....
Krüger 60 B	22 24.4	+57 12	11.59	.256	+8.63	4544	5	4	....
2 yellow Hyades,* mean	....	....	3.98	.025	−4.04	4442	3	3	K
8 white Hyades,† mean	....	....	5.81	.025	−2.21	4301	18	5	F
5 satellites of Saturn,‡ mean	....	....	....	....	[−0.33]	4368	10	5	....
Planetoid (8) Flora	....	....	....	....	[−0.33]	4408	4	1	....
Color-indices transformed into effective wave-lengths:									
Helium stars, general mean	....	....	....	....	−8.58	[4188]	......	......	....
8 stars of the Ursa Major group §, mean	....	....	....	....	−4.38	[4237]			A
α Lyrae	18 33.6	+38 41	0.14	.097	−4.93	[4234]	......	......	A0
β Cassiop.	0 3.8	+58 36	2.42	.073	−3.26	[4318]	......	......	F5
α Aquilae	19 45.9	+ 8 6	0.89	.203	−2.57	[4280]	......	......	A5
γ Virg. A or B	12 36.6	− 0 54	3.66	.071	−2.08	[4282]	......	......	F
ζ Herculis	16 37.5	+31 47	3.00	.101	−1.98	[4337]	......	......	G0
α Canis Min.	7 34.1	+ 5 29	0.48	0.324	−1.97	[4308]	......	......	F5

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* B.D. +15°631 and +17°712.

† B.D +15°621, 625, 633, 636, 637, 639, 640, and 645.

‡ Tethys, Dione, Rhea, Titan, Japetus.

§ β, γ, δ, ε, and ζ Ursae Maj., α Coron. Bor., β Aurigae, α Canis Maj.

‖ The components of the double star Σ 443 have been omitted from Fig. 1 on account of the great relative uncertainty of their parallax.

¶ I am indebted to Mr. W. S. Adams for some unpublished data.

Publications, No. 24, due regard having been given to additional measures.

Judging from the common proper motion and radial velocity, the brighter stars belonging physically to the system of the Hyades are, as is well known,[1] divided into two distinct groups, one consisting of a few yellow stars and the other of white stars, which form the majority. The mean effective wave-lengths for two yellow and for eight white physical members of the Hyades have been entered in the table.

Concerning our sun, I have been able to determine the effective wave-length of its light only as reflected by five of the satellites of Saturn and by the planetoid (8) Flora.[2] The weighted mean (5 to 1 according to the number of objects), 4375 A, has been entered in the diagram (⊙) on the assumption that the quality of the sun's light is not seriously altered by the reflections in question.[3] Finally, for a few bright stars of well-known parallax, I have transformed the color-indices, I, given by E. S. King,[4] into effective wave-lengths by the formula

$$\lambda_{\mathrm{eff}}=4234+210\,I.$$

These results are to be found in brackets in Table I.

All the results are shown graphically in Fig. 1. The full line represents the radiation of a black body of the size of our sun. The points represent the stars, and the sign ⊙, the reflected sunlight, for which the effective wave-lengths have been determined. The circles show the color-indices by King, transformed into effective wave-lengths.

The appearance of this diagram is interesting. When we neglect the absolutely bright yellow stars, which are represented in the figure by the mean of only two bright yellow Hyades, we may state the following:

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[1] Cf. *Potsdam Publ.*, No. 63, p. 26, 1911.

[2] October 16, 1912, $23^{\mathrm{h}}.3$ sidereal time. Three underexposed plates of (433) Eros were obtained on September 11, 1912, between $20^{\mathrm{h}}.5$ and $21^{\mathrm{h}}.4$ sidereal time. The uncertain effective wave-length derived is 4506 A.

[3] It may be recalled that the satellite I of Jupiter (Io) shows a markedly greater effective wave-length than satellites II, III, and IV (*Potsdam Publ.*, No. 63, p. 40).

[4] *Harvard Annals*, **59**, 177, 1912.

On the supposition that stellar surfaces radiate approximately as a black body, the relation between absolute brightness and color from absolute magnitude −7 to +3 deviates from the same relation for a black body (full line) in the same way, and approximately to the same extent, as we should expect from the known increase in density[1] and decrease in mass[2] with decreasing absolute brightness. But below absolute brightness +3 mag., between +3 and +8 mag., the deviation from the radiation of a black body is too great to be explained by facts already known.

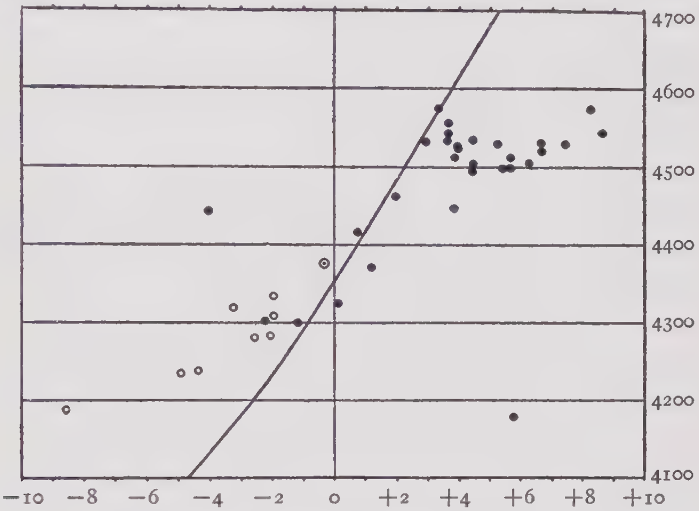

FIG. 1.—Abscissa: Absolute magnitude (sun = −0.33)
Ordinate: Effective wave-length

With the single exception of the white faint companion to o_2 Eridani[3] we may say that the stars between absolute magnitudes

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[1] Shapley, *Astrophysical Journal*, **38**, 158, 1913. The average density of a star of the first spectral type may be taken to be about 0.1 of that of our sun.

[2] The statistics of spectroscopic binaries have been specially investigated by H. Ludendorff, *Astron. Nachr.*, **189**, 145, 1911.

For 15 stars with known mass and parallax I find a decrease of 0.06±0.01 (mean error) in the logarithm of the mass for each magnitude of decrease in absolute brightness.

[3] This exception is, in fact, very strange. It is the well-known double star Σ 518, which has a proper motion of 4″.1 yearly in common with o_2 Eridani. The combined magnitude of the two components is 9.48 mag. and the difference about

+3 and +8 are approximately all of the same color.[1] Proceeding from the absolutely brightest stars to the fainter ones, evidently a new unknown element comes into action at about absolute brightness +3, which stops the further increase in effective wave-length with decreasing absolute magnitude.[2]

If the difference of 5 mag. in the absolute brightness between +3 and +8 mag., within which the color remains constant, were due only to differences in the size of the stars, the ratio of mass to density would be 1000 times greater at absolute magnitude +3 than at +8. We cannot imagine such a difference as due to change in density alone.

Concerning the mass, we are able to eliminate the effect of its variation on the size of a star when the object is one of the components of a double star the apparent orbit of which, together with the ratio of the masses of the two components, is known. Call the masses of the two components, in terms of the sun, M_1 and M_2; their apparent magnitudes, m_1 and m_2; the apparent major axis

1.7 mag. From the apparent orbit and the parallax, which are both approximately known, we find the sum of the masses to be $M_1+M_2=0.76\odot$. This is no exceptional value.

At the Solar Union, 1913, Professor H. N. Russell told me that this star had been found at Harvard to have a spectrum of Class A. This agrees with Adams, who found the spectrum to be A (*Publ. Astron. Soc. Pacific*, **26**, 198, 1914), and with my effective wave-length.

The exceptionally white color of the faint companion to o_2 Eridani recalls the white 12.4 mag. star No. 190 of the cluster N.G.C. 1647 (compare the preceding article). Perhaps there is also physical analogy between these two stars.

In this connection it may be noted that Adams has found the 7.3 mag. star Lal. 28607 ($15^h 37^m\!.7$, $-10°37'$; 1900), the proper motion of which is $1''\!.17$ yearly, to have an A spectrum and a radial velocity of −170 km. The absolute brightness of this star, when estimated from these data in the way described in *Astron. Nachr.*, **185**, 92, 1910, is found to be -1.3 ± 2.45 mag. (mean error). The absolute brightness may therefore very well be the normal value for an A star, namely, about −4 mag. The measured parallax, $+0''\!.03$, is very uncertain.

[1] From different sides it has been suggested to me that this phenomenon might be due to the rapid falling-off of the sensibility of the plate at the end of longer wave-lengths of spectrum. In objection to this I wish to mention that the linear relation between color-index and effective wave-length still holds for fourth-type stars with a color-index of about +2 mag. and an effective wave-length of about 4600 A.

[2] The earlier inferior results, *Potsdam Publ.*, No. 63, p. 40, 1911, showed only the general increase of effective wave-length with decreasing absolute brightness.

of the orbit in seconds of arc, α; and the period in years, P. The absolute magnitude (for a parallax $\pi=1''$) of a star having the same surface brightness and density as, for instance, the first component, and the same mass as the sun, will then be:

$$m_{1,\ \pi=1'',\ M=\odot}=m_1+\frac{5}{3}\log\frac{\alpha^3}{P^2\left(1-\frac{M_2}{M_1}\right)}\ .$$

The number of double stars of known mass ratio being small, I have adopted for other double stars equality of masses. Especially when the two components are nearly equal in brightness and color is this assumption plausible.

We may still increase the number of objects reasonably well suited to this investigation by including double stars when only the first trace of orbital motion has been observed. For such double stars we are able to calculate a minimum value[1] of α^3/P^2. Mr. C. Luplau-Janssen has kindly examined statistically all double stars with known orbits and found that the true value of α^3/P^2 is on the average $(1.65)^3$ times greater than the minimum value derived from a single element of the apparent orbit.[2]

By these more or less tentative methods the values given in Table II and shown in Fig. 2 were found. It will be admitted that the effect of differences in mass on the size of the stars has been eliminated to a great extent; nevertheless, Fig. 2 has the same general appearance as Fig. 1. We may therefore conclude that differences in mass are as little capable as differences in density of explaining the constancy of color between the absolute magnitudes +3 and +8.

It is an obvious suggestion for the explanation of the fact found above that the absolute magnitude +3, corresponding to the temperature 3400° Abs. of a black body of the size of the sun, represents the stage of a cooling star at which the formation of relatively dark solid matter on its surface begins, the remaining fluid part giving practically all the radiation. At any rate, these absolutely dark stars deserve further attention. They should, for one thing, be examined for variability in light.

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[1] *Astron. Nachr.*, **190**, 113, 1911.

[2] The proportion between the true value of $\alpha P^{-\frac{2}{3}}$ and its minimum value does not vary much for different double stars. The mean deviation from the logarithm of the mean ($\log 1.65=0.22$) is about ± 0.085, corresponding to ± 0.43 mag.

It must be kept in mind that the longer effective wave-lengths especially depend only upon the distribution of energy within a narrow part of the spectrum. It would be of special interest to have the results found here confirmed by measuring the color-indices.

TABLE II

Star	Vis. Mag. Harv.	Minimum Hypothetical π $M_1+M_2=\odot$	Hypothetical π $M_1+M_2=\odot$	M_2/M_1	Hypothetical Abs. Mag.	λ_{eff}
						A
ζ Orionis A	$2^{\mathrm{m}}.05$	$0''.008$	....	1	$-7^{\mathrm{m}}.85$	[4154]
γ Leonis A	2.61	.024	....	1	−4.90	[4442]
ζ Urs. Maj. A	2.40	.027	....	1	−4.86	[4240]
β Aurigae A	2.82	....	$0''.040$	1	−4.66	[4219]
α Urs. Maj. A	1.95	$.040^5$	....	1	−4.43	[4474]
α Gemin. A	1.99	....	.117	1	−3.17	[4257]
α Can. Maj. A	−1.58	....	.568	0.39	−3.05	[4272]
δ Gemin. A	3.53	.038	....	1	−2.99	[4314]
ι Urs. Maj. A	3.12	$.061^5$	....	1	−2.35	[4297]
α Can. Min. A	0.48	....	.352	0.33	−1.99	[4308]
ζ Herculis A	3.05	....	.128	0.43	−1.67	[4337]
γ Virginis A	3.65	....	.119	1	−1.47	[4282]
θ Persei A	4.23	.067	....	1	−1.05	4299
Sun	−26.9	....	....	....	−0.33	(4375)
η Cassiop. A	3.67	....	.200	0.76	−0.23	4325
Σ 443 A	8.35	.024	....	1	+0.84	4393
Σ 443 B	8.89	.024	....	1	+1.38	4527
Σ 1280 A	8.65	.048	....	1	+2.64	4543
Σ 1280 B	8.65	.048	....	1	+2.64	4533
61 Cygni A	5.57	.199	....	1	+2.65	4532
η Cassiop. B	7.41	....	.200	0.76	+3.31	4512
61 Cygni B	6.28	.199	....	1	+3.36	4556
OΣ 547 A	9.31	.059	....	1	+3.75	4494
OΣ 547 B	9.31	.059	....	1	+3.75	4534
Σ 1321 A	8.0	.120	....	1	+3.98	4523
Σ 1321 B	8.0	.120	....	1	+3.98	4523
Groom. 34 A	7.97	.123	....	1	+4.00	4529
θ Persei B	9.86	.067	....	1	+4.58	4504
o_2 Eridani B	9.69	....	0.163	1	+5.25	4179
Σ 2398 A	9.33	.123	....	1	+5.36	4520
Krüger 60 A	9.59	.134	....	1	+5.81	4531
Σ 2398 B	10.10	.123	....	1	+6.13	4530
Groom. 34 B	10.99	.123	....	1	+7.02	4574
Krüger 60 B	11.59	0.134	....	1	+7.81	4544

As all my plates were taken between July 18 and October 17, 1912, inclusive, a number of faint stars with well-known parallaxes could not be included in this note.

I am greatly indebted to the director of the Mount Wilson Solar Observatory, Professor George E. Hale, who not only immediately granted my request to use the 60-inch reflector for the determination of effective wave-lengths, but also offered his assistance in realizing my plans. I owe the main part of the necessary funds and five months' leave from Potsdam to the Prussian government and the Academy of Science of Berlin. I wish also to express here my best thanks for the kind and disinterested help given to me by all of the Mount Wilson Observatory staff, not the least to Mr. Hoge, the indefatigable night assistant.

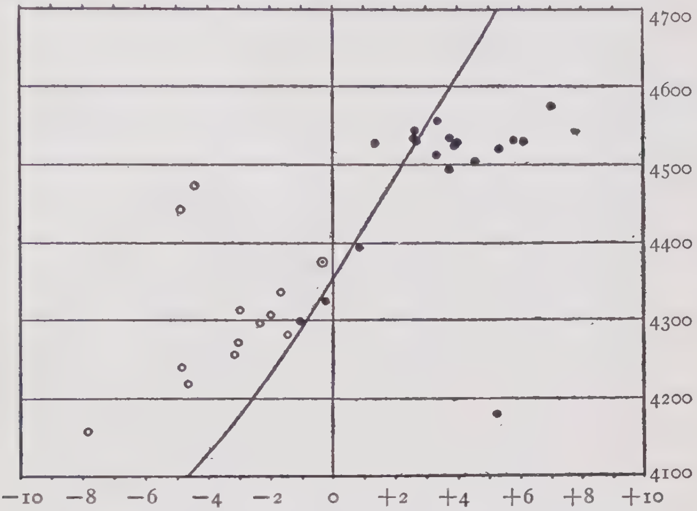

FIG. 2.—Abscissa: Hypothetical absolute magnitude (mass = ⊙)
Ordinate: Effective wave-length

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COLOR-INDICES IN THE CLUSTER N.G.C. 1647

By FREDERICK H. SEARES

I. INTRODUCTION

For the determination of the color of the stars two methods are available. The first, with the aid of a coarse objective grating, measures directly the mean effective wave-length of the light which impresses itself upon the photographic plate,[1] while the second compares the relative intensities of the radiation in two more or less widely separated regions of the spectrum. The latter method is most conveniently applied by choosing for the comparison the spectral regions whose intensities are expressed by the photographic and the visual, or photo-visual, magnitudes. The measure of the color is then

$$\text{Color-index} = C = Pg - Pv,$$

in which Pg and Pv are the photographic and photo-visual magnitudes of the star in question.

With suitable instrumental equipment the measurement of the effective wave-length is an expeditious and convenient method of procedure; but since precise magnitudes are required for the investigation of many questions, the determination of star colors is readily made incidental to the measurement of brightness, so that the second method also has its advantages.

The mutual control afforded by two processes so different in their observational details is invaluable, especially as both are liable to more or less troublesome systematic errors. The effective wave-length, for example, depends upon the intensity of the small spectral image from which it is derived, and the results must accordingly be reduced to a normal intensity. Hertzsprung avoids the greater part of the difficulty by making a number of exposures on the same plate with gradually increasing exposure time, and then selecting for measurement only those images which, as nearly as possible, are of a certain standard intensity. The color-index, on the other hand, as shown by the equation defining it, is seriously influenced by relative errors in either the slope or the zero-point of the magnitude-scales.

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[1] Hertzsprung, *Potsdam Publ.*, **22**, No. 63, 1911. The earlier literature is listed in *Astron. Nachr.*, **182**, 301, 1909.

As the precise determination of a scale of magnitudes is not an easy matter, the color-index is peculiarly liable to error, and the difficulty increases rapidly with decreasing brightness of the stars. To be useful, color results must be homogeneous; they must be directly comparable, whatever the relative brightness of the stars to which they refer. It is not sufficient, therefore, that the magnitudes should be relatively accurate for a limited range; in both scales and for all degrees of brightness, they must be correct relatively to the zero-point defined by the stars of the sixth magnitude. If either scale is in error, the equation

$$C=Pg-Pv$$

will give an incorrect result unless the other happens also to be in error by the same amount. The demand is therefore exacting, especially when it becomes a question of the color of the fainter stars, for with these the cumulative error in the scales has its greatest effect.

In fact, the difficulties hitherto experienced in establishing satisfactory scales of magnitude have been so considerable that it is advantageous to utilize in another direction the possibilities of the two methods of color determination by employing them to test the reliability of the scales. Given a series of carefully determined effective wave-lengths, we can, by means of stars of known color-index, turn the entire series into color-indices. A comparison with the corresponding results obtained directly from the magnitudes will then show at once the relative errors of the scales, although not their absolute values. Any systematic error in the effective wave-length is of course also involved; but this is small, and probably but little dependent upon the brightness of the objects observed, while the uncertainty affecting the color-indices derived from the magnitudes of faint stars is relatively large. It is not to be understood that the errors inherent in the colors derived from magnitudes are necessarily large, for when once we have been assured that reliable magnitudes are available, it is probable that the two methods will give results comparable in precision. The difficulty is that we have no such assurance—in fact, no independent control which is capable of specifying what the uncertainties affecting the magnitude-scales may be.

II. OBSERVATIONS

It is the purpose of this note to test in the manner indicated the photographic and photo-visual scales established at Mount Wilson for stars near the North Pole. Effective wave-lengths for these objects are not available, but Hertzsprung has derived such results for nearly 200 stars in another region, N.G.C. 1647, which are immediately applicable to the problem.[1] To effect a comparison it is only necessary to transfer the polar scales to the region of the cluster by photographs of multiple exposure.

To this end comparisons were made between the Pole and the cluster, using the same kinds of plates as in the investigation of the polar magnitudes—Seed "27" for photographic, and Cramer "Inst. Iso." with a yellow filter for photo-visual magnitudes. Five separate transfers of each scale were made, and to minimize atmospheric irregularities no two of the same kind were undertaken on the same night.[2] In all cases the exposures were symmetrically arranged, usually in the order: cluster, Pole, Pole, cluster; the exposure times were short, 2^m for Seed "27," and 5^m for "Iso." plates.

The photographs were measured and corrected for distance error in the usual way. The magnitudes of the stars in the cluster were read from curves derived from the scale-readings and the magnitudes of the Mount Wilson Polar Standards.[3] After correction for extinction the results were combined into the mean values which appear in the second and third columns of Table II. Owing to temperature fluctuations of unusual magnitude, the largest images on some of the plates were unsuitable for measurement. As these have been excluded, the objects listed begin at about magnitude 11.5; the faintest shown is 15.45. The plates were centered on star No. 100 of Hertzsprung's list and, within the limits of the useful field which is 23′ in diameter, there are nearly fifty stars available for comparison with his effective wave-lengths.

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[1] *Mt. Wilson Contr.*, No. 100; *Astrophysical Journal*, **42**, 92, 1915.

[2] I am indebted to Mr. P. J. Van Rhyn for one of the pairs of polar comparison plates here used.

[3] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

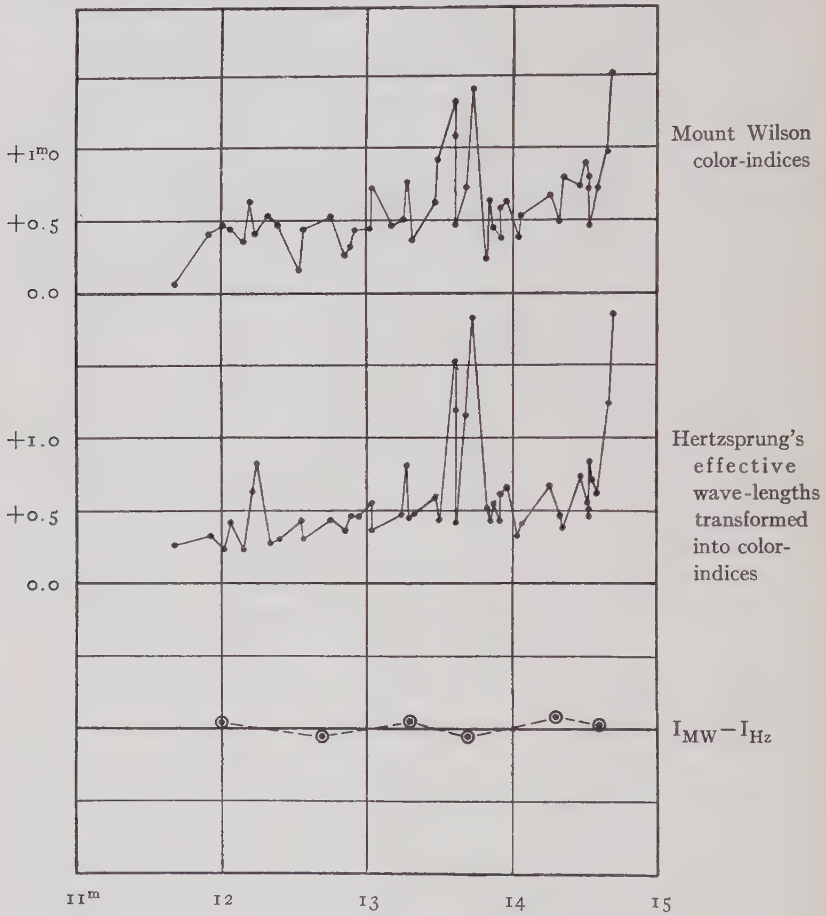

FIG. 1.—Color-indices and photographic magnitudes

As there is but little information bearing upon the precision with which polar comparisons can be effected, Table I may be of interest.

It shows the divergence of the results of individual polar comparison plates from the mean scales for the cluster. The unit is 0.01 mag. and the quantities in parentheses indicate the number of stars used in deriving each mean residual. The zero-point residuals relative to the mean zero-point established by all plates of the same kind are shown in the second line at the bottom of each section of the table. The average deviation of a single magnitude and the total number of stars also appear at the bottom of each section. The probable error of a single determination of the zero-point is ±0.06 mag., and of the mean from five plates ±0.025 mag. The probable value of the constant error in the color-indices arising from the transfer of the scales is therefore ±0.035 mag.

TABLE I

SCALE DIVERGENCE FOR MAGNITUDES IN N.G.C. 1647

MEAN MAG.	PHOTOGRAPHIC PLATES				
	1392	1405	1417	1428	VR1
12.1	+11 (8)	−6 (5)	+12 (8)	−12 (8)	− 8 (7)
12.7	+ 3 (5)	+4 (6)	+15 (5)	− 8 (6)	−11 (6)
13.3	+ 7 (7)	−9 (7)	+ 2 (7)	− 3 (7)	+ 2 (7)
13.7	+ 2 (9)	+5 (9)	− 4 (9)	+ 1 (9)	− 4 (9)
14.3	− 1 (6)	+4 (6)	− 5 (6)	+ 6 (6)	− 3 (6)
14.6	− 4 (8)	−1 (8)	0 (8)	+ 2 (8)	+ 4 (7)
14.9	− 7 (8)	−2 (8)	−10 (8)	+ 3 (8)	+16 (8)
A.D.	9.5	9.3	11.5	10.0	14.2
Z.P.	−6.2	+0.1	+14.3	−11.1	+ 2.8
No.	51	49	51	52	50
MEAN MAG.	PHOTO-VISUAL PLATES				
	1393	1406	1418	1429	VR2
12.1	− 3 (8)	−2 (8)	+ 4 (8)	+ 7 (8)	− 6 (7)
12.7	−10 (6)	−5 (6)	+ 4 (6)	+12 (6)	− 1 (6)
13.3	+12 (5)	−4 (7)	− 6 (7)	+10 (7)	− 8 (7)
13.7	+ 9 (3)	+5 (7)	− 5 (9)	− 7 (9)	+ 5 (9)
14.3	....	+5 (5)	+ 3 (5)	−13 (6)	+ 7 (6)
14.6	....	0 (2)	+16 (3)	− 9 (6)	+ 4 (6)
A.D.	10.5	7.3	8.2	11.3	11.0
Z.P.	−11.0	−1.5	+9.4	+ 7.2	− 3.6
No.	22	35	38	42	41

Table II contains the detailed results for individual stars. The numbers in the first column are those of Hertzsprung; the co-ordinates may be found by consulting his list.[1] The derivation of the Mount Wilson photographic and photo-visual magnitudes in the second and third columns has already been described. The number of values upon which each is based appears in the fifth column. In the fourth and sixth columns, respectively, are the Mount Wilson color-index and its weight. Then follow Hertzsprung's effective wave-length and its weight, the color-index obtained from λ_e, the difference between the two color-indices and its weight, and finally, as a check upon the scales, a comparison of the Mount Wilson photographic magnitudes with those of Hertzsprung.

III. TRANSFORMATION OF EFFECTIVE WAVE-LENGTHS INTO COLOR-INDICES

The data for the comparison of the effective wave-lengths with the color-indices were very kindly supplied in manuscript by Professor Hertzsprung some months before his paper was received for publication, and in consequence some of my results differ slightly from his. Allowance, however, is easily made for these differences.

The data bearing on the relation between λ_e and C here used are as follows: From 15 white stars, mean C (Harvard) $= +0.21$ mag., distance between first-order spectra is

$$D = 1.0780 \pm 0.0026 \text{ mm.}$$

From 11 red stars, mean C (Harvard) $= +1.22$ mag.,

$$D = 1.1338 \pm 0.0030 \text{ mm.}$$

$\Delta C = +1.01$ mag. is equivalent to

$$\Delta D = 0.0558 \pm 0.0040 \text{ mm or } \Delta\lambda = 222 \pm 16 \text{ A.}$$

From these and other data Hertzsprung adopts an increase of 200 A in λ_e as the equivalent of a change of 1 mag. in the Harvard color-index. We have, however, the following relations between the Harvard and Mount Wilson color-indices:

$$+0.21 \text{ Harvard} = +0.21 \text{ MW}$$
$$+1.22 \text{ Harvard} = +1.45 \text{ MW.}$$

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[1] *Mt. Wilson Contr.*, No. 100; *Astrophysical Journal*, **42**, 97, 1915.

TABLE II

MAGNITUDES, EFFECTIVE WAVE-LENGTHS, AND COLOR-INDICES

No.	MOUNT WILSON					HERTZSPRUNG			C_W *minus* C_{Hz}	WT. OF DIFF.	Pg_W *minus* Pg_{Hz}
	Pg	*Pv*	C_W	*n*	Wt.$_C$	λ_e	Wt.$_\lambda$	C_{Hz}			
43...	11.69	11.62	+0.07	3, 4	17	4304	61	+0.27	−0.20	13	+ 6
44...	14.53	13.80	+0.73	4, 2	12	397	1	+0.85	−0.12	1	+22
46...	12.21	11.57	+0.64	4, 5	22	361	50	+0.63	+0.01	15	+13
55...	12.08	11.64	+0.44	4, 5	22	327	53	+0.42	+0.02	16	+22
57...	14.27	13.58	+0.69	5, 4	22	372	4	+0.69	0.00	3	+25
59...	12.90	12.59	+0.31	5, 5	25	339	34	+0.48	−0.17	14	+14
61...	14.78	13.75	+1.03	5, 1	8	428	0	+1.04	−0.01	0	+28
62...	13.74	12.33	+1.41	5, 5	25	559	12	+1.84	−0.43	8	+17
67...	14.67	13.90	+0.77	5, 2	14	372	0	+0.69	+0.08	0	+28
69...	13.50	12.58	+0.92	5, 5	25	334	33	+0.45	+0.47	14	+69
71...	13.04	12.31	+0.73	5, 5	25	320	40	+0.38	+0.35	15	+60
72...	14.96	14.11	+0.85	5, 2	14	424	0	+1.01	−0.16	0	+41
76...	14.91	13.81	+1.10	5, 1	8	347	0	+0.54	+0.56	0	+42
77...	12.92	12.48	+0.44	5, 5	25	337	34	+0.48	−0.04	14	+12
78...	14.53	13.80	+0.73	5, 2	14	359	2	+0.61	+0.12	2	+28
79...	14.70	13.16	+1.54	5, 4	22	562	2	+1.86	−0.32	2	+11
80...	12.01	11.53	+0.48	5, 5	25	297	63	+0.23	+0.25	18	+49
83...	14.51	13.61	+0.90	5, 1	8	352	2	+0.57	+0.33	2	+26
85...	13.49	12.87	+0.62	5, 4	22	356	26	+0.60	+0.02	12	+44
87...	13.03	12.58	+0.45	5, 5	25	352	29	+0.57	−0.12	13	+12
92...	14.36	13.56	+0.80	5, 4	22	322	4	+0.39	+0.41	3	+30
93...	13.92	13.54	+0.38	5, 4	22	330	7	+0.44	−0.06	5	+19
95...	12.34	11.80	+0.54	5, 5	25	306	54	+0.29	+0.25	17	+40
96...	13.84	13.59	+0.25	5, 4	22	341	10	+0.51	−0.26	7	+13
98...	12.86	12.60	+0.26	5, 5	25	321	37	+0.39	−0.13	15	+18
101...	14.03	13.64	+0.39	5, 4	22	312	7	+0.33	+0.06	5	+20
102...	13.96	13.33	+0.63	5, 4	22	366	10	+0.66	−0.03	7	+20
103...	14.49	13.75	+0.74	1, 1	5	379	3	+0.74	0.00	2	+23
104...	12.55	12.40	+0.15	5, 5	25	330	40	+0.44	−0.29	15	− 6
107...	14.54	14.06	+0.48	5, 3	19	375	1	+0.72	−0.24	1	+20
108...	13.70	12.97	+0.73	5, 4	22	452	15	+1.19	−0.46	9	+24
114...	12.77	12.24	+0.53	1, 1	5	329	39	+0.43	+0.10	4	+24
115...	12.24	11.83	+0.41	5, 5	25	392	58	+0.82	−0.41	17	+46
116...	14.92	13.82	+1.10	5, 1	8	408	0	+0.91	+0.19	0	+44
117...	13.29	12.51	+0.78	5, 5	25	391	29	+0.81	−0.03	13	+32
118...	12.40	11.92	+0.48	5, 5	25	308	52	+0.30	+0.18	17	+53
121...	13.26	12.75	+0.51	5, 5	25	339	27	+0.49	+0.02	13	+29
124...	14.72	13.78	+0.94	5, 1	8	333	0	+0.45	+0.49	0	+32
127...	14.32	13.82	+0.50	5, 2	14	338	7	+0.48	+0.02	4	+42
131...	14.08	13.55	+0.53	5, 4	22	326	4	+0.41	+0.12	3	+ 6
134...	14.73	13.82	+0.91	5, 1	8	225	0	−0.22	+1.13	0	+11
135...	13.88	13.42	+0.46	5, 3	19	349	12	+0.55	−0.09	7	+26
136...	13.19	12.71	+0.48	5, 4	22	333	30	+0.45	+0.03	13	+32
138...	12.17	11.81	+0.36	5, 5	25	297	60	+0.24	+0.12	18	+41
139...	13.61	12.52	+1.09	5, 5	25	455	17	+1.20	−0.11	10	+11
141...	15.45	14.45	+1.00	5, 1	8	.....	....	....	....	....	(+15)
143...	12.58	12.14	+0.44	3, 5	19	310	55	+0.31	+0.13	14	+66
145...	11.92	11.51	+0.41	5, 5	25	313	59	+0.34	+0.07	18	+41
147...	13.86	13.23	+0.63	5, 3	19	329	12	+0.43	+0.20	7	+42
148...	13.61	13.12	+0.49	5, 4	22	4325	17	+0.41	+0.08	10	+30

TABLE II—*Continued*

No.	Mount Wilson					Hertzsprung			C_W *minus* C_{Hz}	Wt. of Diff.	Pg_W *minus* Pg_{Hz}
	Pg	Pv	C_W	n	Wt.$_C$	λ_e	Wt.$_\lambda$	C_{Hz}			
153...	13.62	12.30	+1.32	5, 5	25	4507	16	+1.53	−0.21	10	+11
156...	14.66	13.68	+0.98	5, 1	8	463	1	+1.25	−0.27	1	+26
158...	14.52	13.72	+0.80	5, 1	8	338	3	+0.48	+0.32	2	+29
160...	14.63	13.66	+0.97	5, 2	14	321	0	+0.39	+0.58	0	+21
168...	14.49	13.78	+0.71	1, 1	5	360	0	+0.62	+0.09	0	+ 6
169...	13.32	12.95	+0.37	1, 1	5	342	24	+0.50	−0.13	4	+16
170...	13.93	13.33	+0.60	1, 1	5	4363	6	+0.64	−0.04	3	+10

Hence $\Delta C(\mathrm{MW}) = +1.23$ mag. corresponds to $\Delta\lambda = 200$ A and, finally, for

$$\Delta C\ (\mathrm{MW}) = +1.00\ \text{mag.},\ \Delta\lambda = 163\ \text{A}.$$

The next step was the determination of λ_0, the value of λ_e which corresponds to $C = 0$. As the exact value of the grating constant was not available, λ_0 was obtained by combining the foregoing data as follows:

$$\lambda_0 = \frac{1.0780}{0.0558} \times 222\ \text{A} - 0.21 \times 163\ \text{A} = 4255\ \text{A}.$$

There is here a computational uncertainty of ±10 A owing to the fact that the factor 222 A is given only to the nearest angstrom, but probably this is within the uncertainty arising from other sources.

A rough control is afforded by Professor Hertzsprung's classification of the spectra of the brighter stars photographed by Eberhard, which was also kindly placed at my disposal. From the 26 A stars in this list the value $\lambda_0 = 4262$ was found on the assumption that the maximum frequency of occurrence is for stars of the type A0. From the same data Hertzsprung finds for the median $\lambda_e = 4266$. The close agreement of these two values with that found above must not be accepted as an indication of a corresponding precision in the result, for presumably the value $\lambda_0 = 4234$ adopted by Hertzsprung on the basis of all the material at his disposal is more reliable.

The difficulty in any comparison with spectra lies in the fact that the frequency distribution of the various types which applies to the stars in general does not necessarily hold for clusters. In fact, for this particular cluster it appears that the maximum frequency is for stars approximately of type F0, although there is a secondary, but much less conspicuous, maximum for the A stars. Then, too, there is a further difficulty with such a comparison in that we are not certain that the color-index of an A0 star belonging to the cluster is really zero.

In the absence of more definite information the value $\lambda_0=4260$ was adopted for the reduction, and the linear relation used for the transformation of the effective wave-lengths into color-indices was accordingly

$$163C_{\mathrm{Hz}}=\lambda_e-4260.$$

Had the value $\lambda_0=4234$ subsequently adopted by Hertzsprung been employed, the values of C_{Hz} would have been systematically larger by 0.16 mag. than those given in Table II.[1]

IV. COMPARISON OF RESULTS

A comparison of the color results found by the two methods is shown by the differences $C_{\mathrm{W}}-C_{\mathrm{Hz}}$ in Table II. Although there are several large values among them, the corresponding weights usually are low. For a final comparison reference may be made to Table III which gives mean results for groups of stars. Here the agreement is very good, both in the progressive increase of the

TABLE III

Mean Results for Groups of Stars

MW Pg. Mag.		MW C.I.	Rel. Wt.	C_{W} *minus* C_{Hz}	Rel. Wt.	No. Stars	Pg. Mag. MW *minus* Hz
Mean	Range						
12.1	11.69–12.34	+0.43	186	+0.02	132	8	+0.32
12.7	12.40–12.92	+0.35	149	−0.04	93	7	+0.26
13.3	13.03–13.49	+0.59	149	+0.04	83	7	+0.32
13.7	13.50–13.96	+0.78	253	−0.06	97	12	+0.24
14.3	14.03–14.49	+0.60	112	+0.10	20	7	+0.22
14.6	14.49–14.70	+0.91	119	+0.02	11	9	+0.23
14.9	14.72–15.45	+0.98	62	....	....	7	+0.30
Means	13.7	+0.66	....	0.00	....	....	+0.27

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[1] In a recent letter Professor Hertzsprung writes that $\lambda_0=4234$ corresponds to a color-index whose value is zero with a mean error of the order of 0.05 mag.

color-index and in the absolute values; but it should not be overlooked that the latter are subject to a small constant error because of the uncertainty in the value of λ_0.

The adopted system of weights requires a word of comment inasmuch as the two series $Wt._C$ and $Wt._\lambda$ which appear in Table II originated quite independently of each other and are not necessarily directly comparable. An examination of the mean errors corresponding to unit weight showed, however, that they are very nearly on the same system; in calculating the weights of the differences $C_W - C_{Hz}$ it was therefore assumed that $Wt._C$ and $Wt._\lambda$ are strictly homogeneous.

Of the 57 stars in Table II the value of $Wt._\lambda$ is zero for 9 of the fainter objects; for one there is no value of λ_e, so that the results in Table III are based on 47 stars. Had the value $\lambda_0 = 4234$ been used, the mean difference $C_W - C_{Hz}$ for all the stars would have been -0.16 mag. instead of zero as shown in Table III. These results, 0.00 and -0.16 mag., respectively, indicate the uncertainty affecting the comparison, and, at the same time, the probable limits of the relative errors in the Mount Wilson photographic and photovisual scales between the twelfth and the fifteenth magnitudes.

The only other scales extending into this region which have been connected with the international zero-point defined by the stars of the sixth magnitude are those of *H.C.*, No. 170. From the known relations which these bear to the Mount Wilson scales it is possible to compare at once the results from the effective wavelengths with those which would have been obtained had the Harvard magnitudes been used for the calculation of the color-indices.

For the region of the scale with which we are concerned we have

$$\begin{aligned} &\text{MW } Pg - \text{HH} = +0.28 \text{ mag.} \\ &\text{MW } Pv - \text{H } Vis = 0.00 \end{aligned}$$

in which HH represents what has been called the Harvard homogeneous scale,[1] namely, the photographic scale of *H.C.*, No. 170, reduced to a uniform color system and corrected by +0.08 mag. to refer it to the international zero-point; the second relation is approximate and, strictly speaking, holds only for the twelfth magnitude. We therefore find

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[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915. For the region in question MW $Pg - H.C.$, No. 170 = +0.40 mag. The color correction to *H.C.*, No. 170, which reduces the results to a uniform system is +0.04 mag. Applying this and the zero-point correction of +0.08 mag. we have the foregoing value for MW $Pg-$HH.

$$C_W - C_H = +0.28.$$

But from the preceding discussion

$$C_W - C_{Hz} = 0.00 \text{ or } -0.16$$

according to the value adopted for λ_0.

These two equations cannot, however, be combined as they stand, for the values of C_{Hz} in the second relation refer to the Mount Wilson color system which differs from that of Harvard. The necessary modification is that corresponding to the substitution of 200 for 163 in the equation on p. 9. Since the mean color-index for the 47 stars is 0.66 mag., the values of C_{Hz} on the Harvard system will be 0.11 mag. less, on the average, than those found above. We therefore find

$$C_H - C_{Hz} = -0.17 \text{ or } -0.33 \text{ mag.}$$

which represents the mean difference in the color-indices derived from the effective wave-lengths and from the photographic and visual magnitudes of *H.C.*, No. 170.

In addition to the color results, Table III also shows a comparison of the Mount Wilson photographic magnitudes with those found by Hertzsprung. The scale for the latter was established with the aid of a grating used in connection with the Potsdam 80-cm refractor ("Halbgitter" method). The agreement is good, although there is a constant difference of +0.27 mag.; but this is of no significance, as Hertzsprung has determined the zero-point of his magnitudes by a comparison with Pleiades stars instead of with the Pole.

V. PROBABLE ERRORS

It is of interest, finally, to compare the two methods of determining the color of the stars from the standpoint of precision. The average deviation of a single magnitude as found above is ±0.10 mag. The corresponding probable error for a color-index based upon one photographic and one photo-visual magnitude is therefore ±0.12 mag., while that for the mean of five such determinations is ±0.054 mag. This indicates sufficiently the precision with which relative values of the color have been determined. In estimating the uncertainty of the absolute values of the color-indices, allowance must also be made for the zero-point errors. In this case their effect upon the color-index is of the order of ±0.04 mag.

Hertzsprung gives as the mean error of a single effective wave-length 26 A, which corresponds to a probable error of about ±0.11 mag. For absolute determinations a zero-point error has also to be considered. This includes, first, the uncertainty in the adopted value of λ_0 which is used for the reduction of all the plates, and, second, a systematic plate error which seems to be of the order of 10 A.[1] The data relating to the error in λ_0 are not available; but since this is an instrumental constant whose value is determined once for all, its error can be made negligible by an appropriate investigation. In the matter of precision, therefore, there seems to be little choice between the two methods.

SUMMARY

Star colors may be determined by measuring the mean effective wave-length of the light of individual stars or by deriving their color-indices. Since the latter depend directly upon the photographic and the visual, or photo-visual, magnitudes, a comparison of color results found by the two methods affords a control upon the relative errors of the magnitude-scales; the fainter the stars the more important is the control.

Such a comparison has been made for 47 stars in N.G.C. 1647. The effective wave-lengths used were those by Hertzsprung. The color-indices were found by transferring the Mount Wilson photographic and photo-visual scales for stars near the Pole to the region of the cluster. The mean of the differences between the color-indices calculated from the effective wave-lengths and those derived from the magnitudes is

$$C_{\mathrm{W}}-C_{\mathrm{Hz}}=0.00 \text{ or } -0.16 \text{ mag.}$$

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[1] *Mt. Wilson Contr.*, No. 100; *Astrophysical Journal*, **42**, 92, 1915.

according as 4260 A or 4234 A is adopted as the effective wave-length of a star whose color-index is zero.

The comparison includes stars between photographic magnitudes 11.5 and 15. Within this interval the two series of color-indices show the same increase in the mean color with increasing magnitude. The relative errors of the magnitude-scales seem to be within the uncertainty affecting the reductions of the effective wave-lengths.

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THE TUBE-ARC SPECTRUM OF IRON AND A COMPARISON WITH DISSYMMETRIES IN SPARK SPECTRA

By ARTHUR S. KING

In previous papers,[1] the writer has described some of the phenomena of the "tube-arc." To produce this light-source, the tube used in the electric furnace is filed thin at the middle of its length, so that when the voltage is applied, the tube quickly burns apart, forming an arc carrying a very large current at low voltage. The spectrum shows a high intensity of the enhanced lines of metals, the spark spectrum of carbon, and the brighter hydrogen lines, together with an interesting difference in the intensity of the lines of various elements across the diameter of the tube when this ring-shaped arc is viewed axially.

Another feature mentioned in the former papers is a marked dissymmetry in the structure of certain tube-arc lines. This was not studied in detail, but it was shown that the dissymmetry is very marked for many lines which are symmetrical in the furnace spectrum and approximately so in the ordinary arc in air.

Recently a much larger transformer was installed in the Pasadena laboratory, giving improved facilities for work with the tube-arc. Its capacity is 100 kw and the voltage may be varied by 5-volt steps from 5 to 50 volts. With this apparatus, tube-arcs have been operated which consumed as high as 70 kw just after the rupture of the tube. An initial energy of about 40 kw, however, has given the best results, the exposure being made while the current fell from above 1000 to about 600 amperes.

The second order of a 4-inch plane grating was used in the vertical spectrograph, with an objective of 30 feet focal length, giving a linear scale of about 1 mm = 0.9 A. The interior of the tube, at the point where the arc was to form, was focused on the slit, so that the spectrum registered the radiation along the vertical diameter of the ring of arc when the tube burned apart. The arrangement of the tube and the inclosing jacket of graphite was similar to that in the earlier experiments, except that the point of break was located by simply filing a deep groove around the tube at its middle. The gap between the ends of the tube, after the arc had ceased, usually measured 10–15 mm above and 30–40 mm below, but was occasionally longer. The time of action of the arc was generally 30–40 seconds.

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[1] *Contributions from the Mount Wilson Solar Observatory*, Nos. 65, 73; *Astrophysical Journal*, **37**, 119; **38**, 315, 1913.

The present paper will consider the leading features of the tube-arc spectrum of iron, these being, first, the displacements resulting from the dissymmetry shown by many of the stronger lines, the degree of this dissymmetry being clearly connected with the classification of the lines in the furnace spectrum and in that of other sources, and, second, the enhancement of certain types of lines. The peculiarities in structure of tube-arc lines are found to be very similar to those exhibited by the spectrum of the condensed spark. A comparison of typical iron lines in tube-arc and spark will be given, together with a brief description of the variable dissymmetries found in other spark spectra, especially that of titanium.

DISSYMMETRIES OF IRON LINES IN THE TUBE-ARC

A study of the tendency of various light-sources to render lines unsymmetrical has the twofold object of showing what features in the light-source produce these dissymmetries and the degree to which various lines are subject to such disturbing influences. The tube-arc shows effects in a very pronounced degree which are present in a more or less incipient state in the ordinary arc in air, especially in the region near the negative pole. Such dissymmetries are naturally most conspicuous in the case of reversed lines, and a large number of iron lines are given in reversal by the tube-arc. Few of these show a fully symmetrical reversal when their intensities are such that they are satisfactory for examination. A few low-temperature lines to the violet of λ 4000 reverse so widely in the tube-arc that they are practically absorption lines, and little can be said regarding the relative strength of their edges. The iron lines showing distinct self-reversal usually have the red side stronger than the violet, and a large variability appears in the amount of the dissymmetry.

Lines of small or moderate dissymmetry toward the red.—In Table I, a list of lines is given whose dissymmetry of reversal does not exceed a moderate amount, with the red side not more than twice as strong as the violet. Table II contains lines which reverse very unsymmetrically. Some of the latter show but a trace of the violet side of the line on the plate. The second columns of Tables I and II give the estimated ratios of the two sides of the reversal.

TABLE I

IRON LINES SHOWING SMALL OR MODERATE DISSYMMETRY TOWARD THE RED IN THE TUBE-ARC

λ (Rowland)	Ratio of Violet and Red Sides	Furnace Class	Pressure Class	Pressure Group
3887.196	2:3	I	1	*b*
3888.671	2:3	II (I)	1	*b*
3906.628	2:3	I	1	*b*
3969.413	3:4	II	1	*b*
4005.408	3:4	II	1	*b*
4045.975	3:4	II	1	*b*
4063.759	3:4	II	1	*b*
4071.908	3:4	II	1	*b*
4132.235	2:3	II	1	*b*
4144.038	3:4	II (I)	1	
4202.198	3:4	II	1	*b*
4250.945	3:4	II	2	*b*
4271.934	3:4	II	1	*b*
4282.565	5:6	I (II)	1	*b*
4294.301	5:6	I	2	*b*
4308.081	2:3	II	1	*b*
4325.939	2:3	II	1	*b*
4376.107	3:4	I	3	*a*
4383.720	2:3	II	1	*b*
4404.927	2:3	II	1	*b*
4415.293	2:3	II	1	*b*
4427.482	2:3	I	3	*a*
4461.818	5:6	I	3	*a*
5227.362	9:10	II	4	*a*
5269.723	9:10	I	1	*a*
5270.558	9:10	II	4	*a*
5328.236	9:10	I	1	*a*
5371.734	9:10	I	1	*a*
5397.344	9:10	I	4	*a*
5405.989	9:10	I	4	*a*
5429.911	9:10	I	4	*a*
5434.740	9:10	I	4	*a*
5447.130	9:10	I	4	*a*
5455.834	9:10	I	4	*a*

TABLE II

IRON LINES SHOWING LARGE DISSYMMETRY TOWARD THE RED IN THE TUBE-ARC

λ (Rowland)	Ratio of Violet and Red Sides	Furnace Class	Pressure Class	Pressure Group
4187.204	1:6	II (III)	....	....
4187.943	1:6	II (III)	....	....
4198.494	1:6	III	....	....
4227.606	1:8	III	5	*d*
4233.772	1:6	III	5	*d*
4236.112	1:6	II (III)	5	*d*
4250.287	1:6	III	5	*c*
4260.640	1:6	III	2	*c*
4271.325	1:6	III	....	....
4299.410	1:6	II	5	*d*
4528.798	1:4	II	4	*c*
4859.928	1:8	III	5	*c*
4871.512	1:8	III	5	*c*
4872.332	1:8	III	5	*c*
4890.948	1:8	III	5	*c*
4891.683	1:8	III	5	*c*
4919.174	1:8	III	5	*c*
4920.685	1:8	III	5	*c*
4957.480	1:8	III	5	*c*
4957.785	1:8	III	5	*c*
5233.122	1:6	III	5	*d*
5324.373	1:6	IV	5	*d*
5586.991	1:6	IV	5	*d*
5615.877	1:6	IV	5	*d*

In some cases these are based on the curves of the Koch microphotometer, but eye-estimates are sufficiently accurate for classification, which is the main object of these tables. The third column gives the class in which the line is placed in the electric-furnace study[1] made by the writer. Later experience with the furnace spectra has slightly modified the method of classification, and the numbers given in parentheses for a few lines denote the class in which the line would probably fall if the classification were revised. The fourth and fifth columns contain the class and the group in which the line is placed by Gale and Adams[2] according to its appearance under pressure and to its pressure displacement, respectively.

The material presented in Tables I and II shows the following relations between the amount of dissymmetry imparted by the tube-arc and the behavior of the lines in the furnace and under pressure:

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[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

[2] *Mt. Wilson Contr.*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

1. Lines of small and moderate dissymmetry of reversal belong in the furnace Classes I and II, those most nearly symmetrical being as a rule among the "flame" lines of Class I. These are all lines occurring at low temperature, the rate of increase in strength with rise of temperature being more rapid for the lines of Class II.

2. Lines which are strongly unsymmetrical in their reversal in the tube-arc are with few exceptions in Classes III and IV. These require a higher temperature for their initial appearance and increase rapidly in strength as the temperature rises.

3. The connection with the behavior of the lines under pressure is very definite. With two exceptions, all lines in Table I are given as symmetrical in appearance under pressure (Classes 1, 3, and 4), and have small or moderate displacements (Groups *a* and *b*). Those in Table II are made unsymmetrical by pressure (Classes 2 and 5), and show large displacements (Groups *c* and *d*).

Lines displaced toward the violet in the tube-arc.—In the green-yellow an important group of lines is found, the strongest of which are λλ 5365.069, 5367.669, 5370.166, 5383.578, 5404.357, 5411.124, 5415.416, 5424.290. The tube-arc gives these lines very strong, diffuse, and nearly symmetrical, a slight shading appearing on the violet side. By measurements to be described later in the paper, these lines were found to give decided displacements toward the violet. In the furnace these lines are to a certain extent in a class by themselves, since no other lines as strong in the iron arc are so difficult to obtain in the furnace. They appear very faintly at high temperature and are an extreme type of Class V lines.

No reversed iron lines with the violet side strongest have been found, but two important cases of this dissymmetry occur in the H and K lines of calcium, which are found on several tube-arc plates of the iron spectrum. In the former paper[1] on the tube-arc, these lines were mentioned as symmetrical, but the greater power available in the later experiments gives them in every case with the violet side stronger, about in the ratio 6:5. The available pressure measurements for H and K indicate that they are displaced toward the red by pressure, so that they appear to present an exception to the general correspondence between the two effects.

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[1] *Mt. Wilson Contr.*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

Relation to pole-and-center differences in the iron arc.—When a plane-grating spectrograph is used for the observation of the ordinary arc spectrum, the image of the arc from pole to pole being projected on the slit, some lines of iron are found to remain nearly symmetrical from center to pole of the arc and others to become decidedly unsymmetrical near the pole. These differences are being studied in detail by St. John and Babcock. Without giving an extended comparison, it may be stated here that the dissymmetries shown by the tube-arc lines are of the same nature as for corresponding lines in the ordinary arc, but are greatly magnified in amount. This is true also for the lines considered in the last paragraph, which widen toward the violet near the pole of the arc. A comparison by Royds[1] of the pole and center of the arc gave considerable displacements toward the violet for these lines. Another group of lines in this region, found by Royds to give negligible displacements, is included among the tube-arc lines in Table I, while still others showing polar displacements toward the red are in Table II.

Micrometer measurements on unsymmetrical lines.—The group of strong green lines near λ 4900 affords an interesting example of the sort of wave-length measurements that are given by lines of this kind, and also of the relation of the measured displacement to the intensity of the line in question. These lines are in the same furnace class, also in the same pressure class and group. In the tube-arc they are all very unsymmetrical, the violet side of the reversal usually being so faint as to show merely as a shading off on that side, but in the case of two of the group, λλ 4891.683 and 4920.685, the strength of the lines was sufficient to show both sides of the reversal distinctly, so that the measuring microscope could be set on the reversal itself. For the other lines, the maximum was well enough defined to give closely concordant sets of measurements, but these settings were in reality chiefly on the red side of the partially reversed line. An arc spectrum on the same plate given by the center of a 6-ampere arc was measured for comparison. The reference line in each case was λ 4891.683, this being clearly reversed in the tube-arc spectrum. These measurements are given in Table III. The small negative displacement of λ 4920.685 is within the error of measurement for lines of this quality. The measured displacements of the others show a rough but distinct relation to the strength of the line. The effect clearly increases with the absolute width of the line, while the example shows the extent to which measured wave-lengths of such sensitive lines may be disturbed in the spectrum of a source of this kind.

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[1] *Kodaikanal Observatory Bulletin*, No. 40, 1914.

TABLE III

λ	Intensity	Displacement Tube-Arc − Arc
4859.928	8	−0.041
4871.512	25	+0.076
4872.332	12	+0.062
4878.407	5	+0.035
4890.948	15	+0.071
4891.683 (reversal)	30	0.000
4903.502	3	+0.028
4919.174	12	+0.074
4920.685 (reversal)	35	−0.004

Use of the registering micro-photometer.—This brings us to the generally recognized difficulty of making micrometer settings on unsymmetrical lines. One must either set on the reversal, if there is such, in which case no account is taken of the dissymmetry of the sides, or on the position of maximum density, which may well be only one side of a partially reversed line. The Koch microphotometer is of much assistance in this respect. Its use in registering the density-curves for various types of emission lines was illustrated in a previous paper.[1] The large apparatus recently constructed here serves to register the structure of individual lines and also to measure the wave-length interval between lines. If one or both of the lines are unsymmetrical, the position of the ordinate which best represents the maximum of each line must be decided upon. For most of the reversed tube-arc lines, the dissymmetry is chiefly at the top of the curve. Examples are shown in

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[1] *Mt. Wilson Contr.*, No. 77; *Astrophysical Journal*, **39**, 213, 1914.

Fig. 1 of two iron lines in different stages of reversal. The wave-length of the corresponding arc line is given closely by the minimum of the reversal, while the maximum of the unsymmetrical line is obtained by producing the slopes of the curves as shown, the ordinate of this maximum being, to a close approximation, midway between the two sides of the curve. If the curve as a whole is very one-sided, owing to shading of the spectrum line, it is more difficult to determine the ordinate for use in a wave-length measurement.

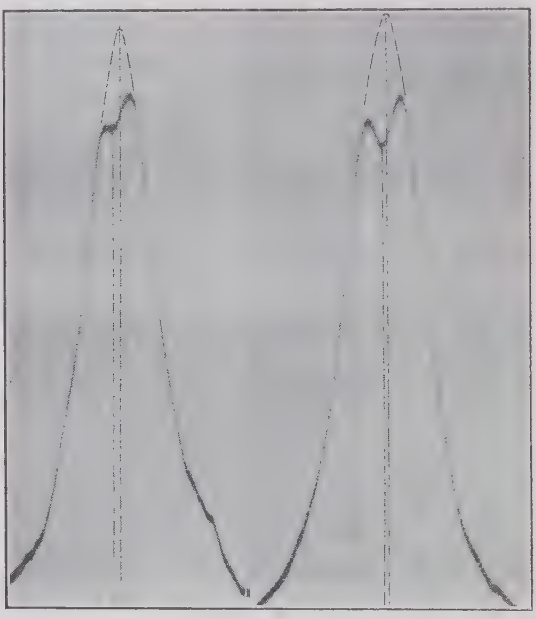


Fig. 1.—Curves of reversed iron lines showing different degrees of dissymmetry in the tube-arc.

Following the plan just outlined, an attempt has been made to measure the displacing action of the tube-arc by obtaining the distance between the centers of pairs of density-curves, one curve being for one of the highly unsymmetrical lines given in Table II, the other for a line, usually from Table I, which could be considered as giving an ordinate practically undisplaced, this ordinate being either that of the reversal of the slightly unsymmetrical line or of its maximum if narrow and unreversed. The curves were registered on direct and reversed runs of the screw to eliminate possible errors due to inertia in the instrument, which proved, however, to be extremely small. Curves were similarly made for arc lines on the same plate, the portion of the line given by the center of the arc being registered.

A further purpose in measuring these lines was to observe the effect of large differences in the quantity of iron vapor present in the tube-arc. An inspection of photographs made for large and small vapor densities showed the magnitude of the dissymmetry to vary greatly with the strength of the line. With a very small quantity of iron vapor, strong lines such as λ 4383.720 are reversed with the usual degree of dissymmetry; while lines less intense, including many of those which are very unsymmetrical with much vapor present, appear fairly sharp. The question is, Are these weaker lines moved out of their position by the tube-arc to the same degree that they are when strong, or is the effect dependent on the strength of the line? Evidently only a comparison of the distances between the density-curves can decide this question.

TABLE IV

VARIATION IN WAVE-LENGTH OF TUBE-ARC LINES WITH VAPOR-DENSITY

Unsymmetrical Line	Standard Line	Displacement, Tube-Arc—Arc	
		Low Vapor Density	High Vapor Density
4187.204.	4185.058	+0.009	+0.019
4187.943.	4185.058	+0.009	+0.015
4250.287.	4250.945	−0.002	+0.029
4260.640.	4271.934	+0.013	+0.040
4271.325.	4271.934	+0.008	+0.035
4299.410.	4294.301	+0.001	+0.036

Displacements toward the red.—The values in Table IV give the displacement of the center of density of the unsymmetrical line with reference to a neighboring line taken as standard. Thus a positive displacement indicates that the given line is toward the red with respect to its position in the arc measured from the same standard. λ 4185.058, used as standard for the first two lines, is a Class III line in the furnace and, though narrow, probably suffers some displacement in the tube-arc. The curves of the last three lines in the table were the most satisfactory for measurement.

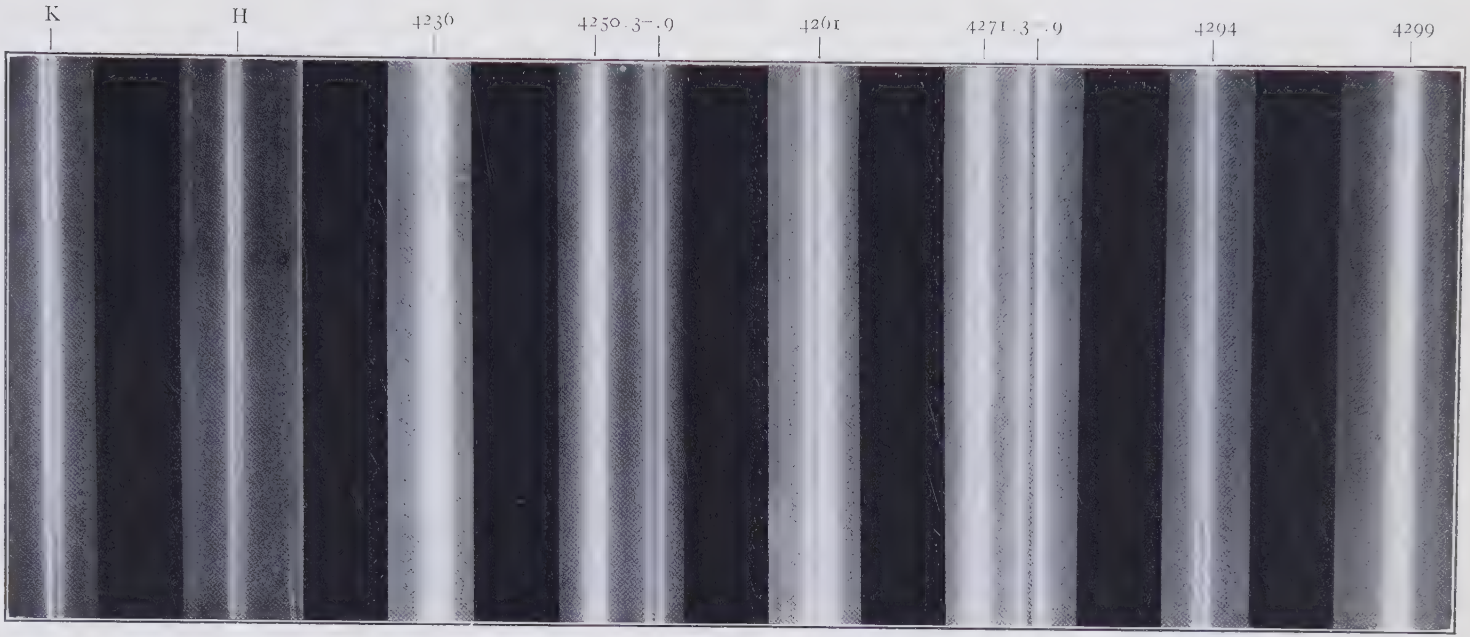
The last column of Table IV gives the magnitude of the displacement measured by the micro-photometer when the tube-arc line shows large dissymmetry; while the third column shows that this displacement becomes small when the width of the line is decreased by low vapor-density. As was previously noted, the tube-arc under the latter condition is still able to produce decided dissymmetry in strong lines such as λλ 4308, 4325, 4384, 4415. How far this effect may be due to differences in vapor-density, strictly speaking, will be discussed later; but evidently the dissymmetry of tube-arc lines does not depend entirely on the strength of the discharge conditions. If it did, the amount of measured displacement to be expected for a line in any source could be calculated from the strength of the enhanced lines in that source; but the effect is complicated by conditions attendant on the amount of vapor present.

The dissymmetries exhibited by some tube-arc lines are shown in Plate VII. First are the calcium lines K and H, which are reversed with the violet side stronger. They appeared thus on several plates which showed the stronger iron lines maintaining their usual dissymmetry toward the red. The iron lines shown are λ 4236.112, highly unsymmetrical, λλ 4250.287 and 4250.945, of large and small dissymmetry respectively, λ 4260.640, showing large dissymmetry, with two satellites on the violet side, λλ 4271.325 and 4271.934, similar to the pair at λ 4250, and λλ 4294.301 and 4299.410, showing a marked contrast in structure.

Plate VII is supplemented by the density-curves in Fig. 2, these being for the same lines as are shown in the plate, with the addition of the iron lines λ 3969.413 (from the same plate as the adjacent H line) and λ 4308.081, made from a plate taken with low vapor-density in the tube-arc.

Displacements toward the violet.—Four of the strong lines in the green-yellow described on p. 5 have been studied with the micro-photometer, the distances being measured in both tube-arc and arc between the curves of these lines and those of standard "flame" lines which reverse almost symmetrically in the tube-arc and may be regarded as probably undisplaced by this source. The arc-curves were made in each case for the part of the line

PLATE VII



Unsymmetrical Tube-Arc Lines from the Spectra of Calcium and Iron

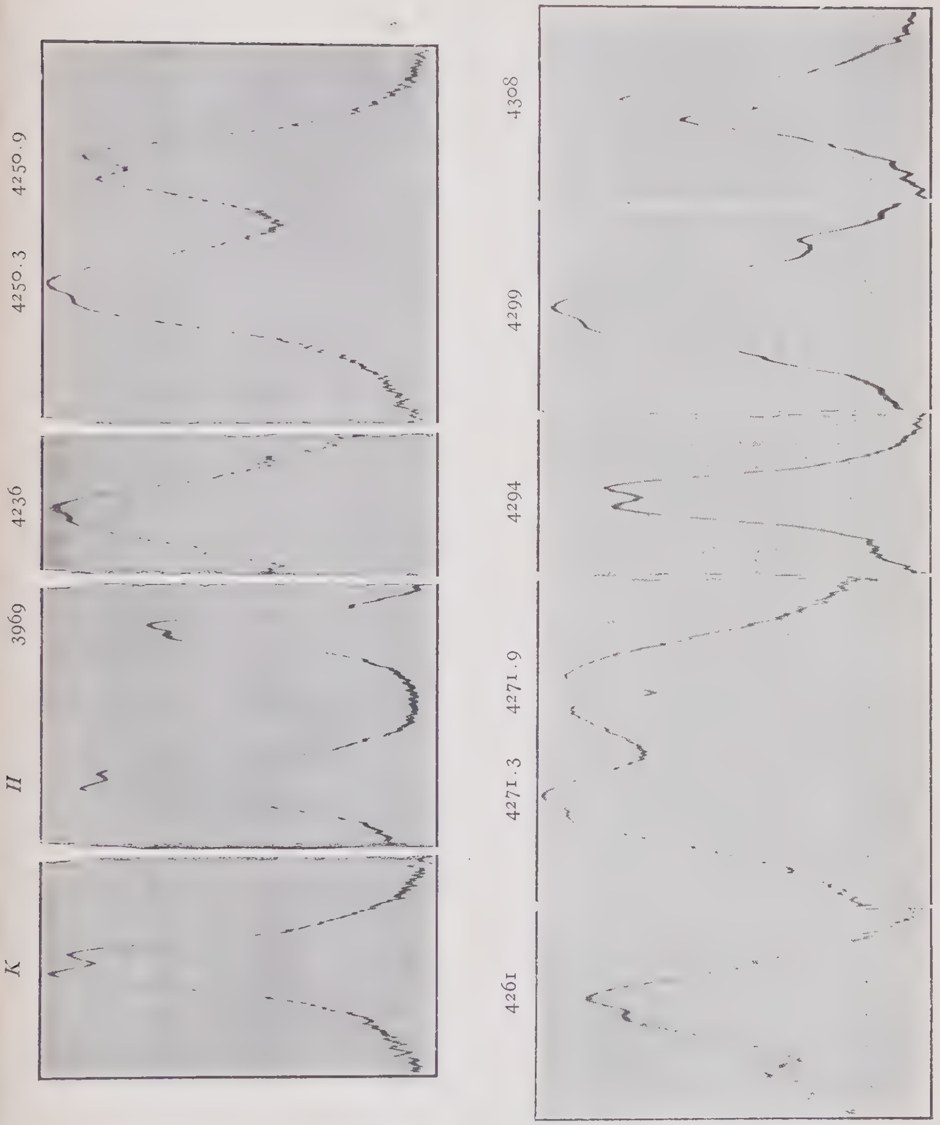


Fig. 2.—Density-curves of tube-arc lines, showing varying degrees of dissymmetry

given by the central region of the arc. The results are given in Table V.

TABLE V

Tube-Arc Lines Displaced toward the Violet

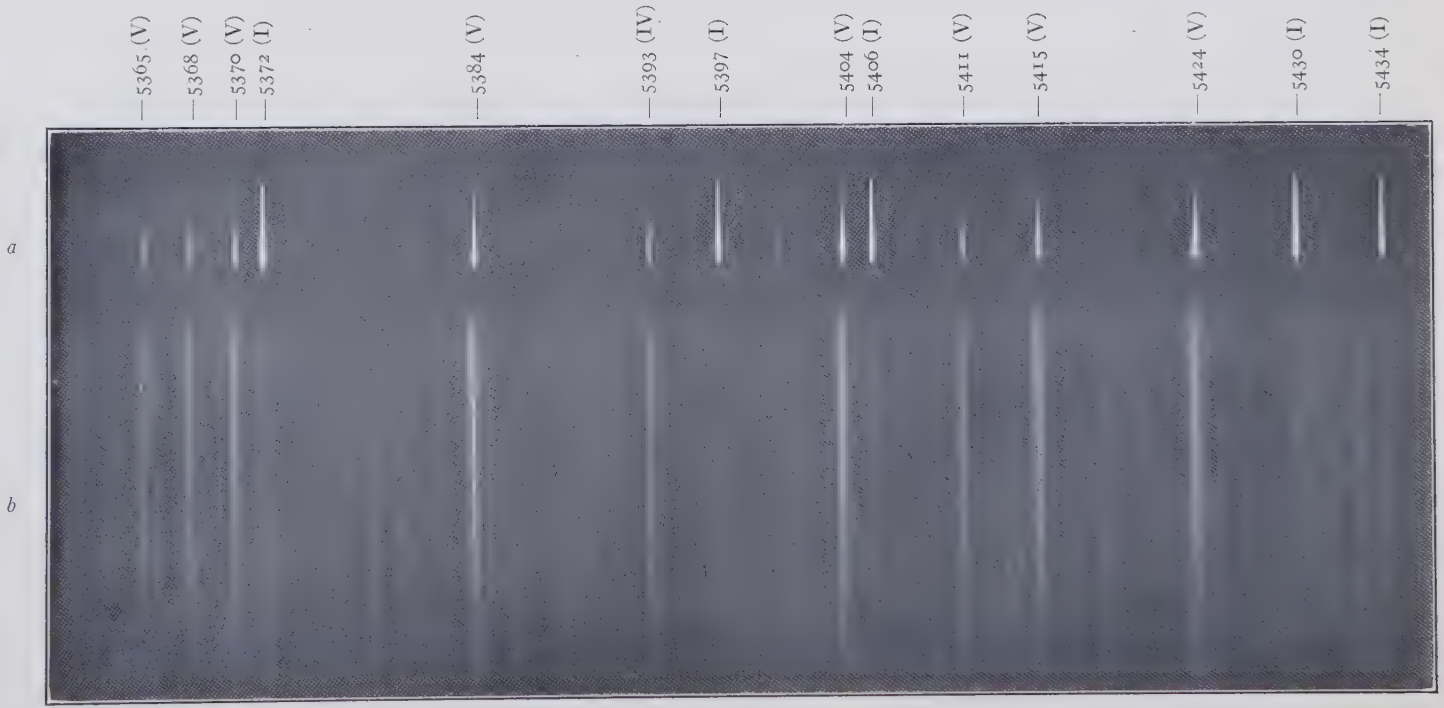
λ (Rowland)	Standard Line	Displacement
5367.669	5371.734	−0.036
5370.166	5371.734	−0.027
5411.124	5405.989	−0.056
5415.416	5405.989	−0.094

These displaced lines are unreversed and the measurements were taken well up on the curve, where it is nearly symmetrical. The bottom of the curve shows a shading toward the violet which would increase the displacement if it were taken into account; but the method employed measures as nearly as possible the position of the strongest part of the line. The displacement of λ 5415.416, the strongest of the group, is seen to be very large, an effect similar to that noted for the lines near λ 4900 (see p. 6), where the measured displacement, in a group of lines showing the same behavior, was found to increase with the strength of the line. The displacements given in Table V are the means of two sets of curves having different scales, made by changing the speed-ratio between the negative and the registering plate. The two sets gave almost identical results, and while no high degree of accuracy can be claimed for the absolute values of the displacements, there can be no question that any method of measurement would give displacements toward the violet of considerable magnitude for the centers of density of these lines.

LINES INTENSIFIED IN THE TUBE-ARC SPECTRUM

It is in harmony with the general resemblance of the tube-arc spectrum to that near the pole of the ordinary arc that the enhanced lines of iron should be brought out strongly and a decided intensification be given to those usually classed as "nebulous" in wavelength tables and thus denoted by the writer in the paper on the furnace spectrum. With few exceptions, they belong in Class V, being absent or extremely faint in the furnace spectrum. In the

PLATE VIII



The Spectrum of Iron

(*a*) In ordinary arc, (*b*) in tube-arc, showing the high intensity of lines in the higher furnace classes (indicated after the wave-length), as compared with the "flame" lines of Class I ordinary iron arc, these lines are much stronger in the highly luminous part near the negative pole. In the tube-arc, they are very strong, the well-known group in the green-yellow from λ 5365.069 to λ 5424.290 dominating the spectrum in this region. This condition is illustrated by Plate VIII, which shows the tube-arc spectrum with that of the ordinary iron arc for comparison. The strong lines of Class I are scarcely visible, but this is partly due to their being reversed for the greater part of their width, the edges of the reversal being almost symmetrical.

THE TUBE-ARC SPECTRUM IN THE ULTRA-VIOLET AND RED

The investigation of iron in the tube-arc has covered the spectrum from λ 3600 to λ 6600. At the violet end of this range, dissymmetries are numerous, but no features of special interest were observed. The stronger lines reverse with the red side stronger, the dissymmetry being moderate in amount, similar to the lines listed in Table I. The region from λ 5700 to λ 6600 required the first order of a very bright grating, and few deviations from the usual arc spectrum appeared. The stronger lines remained sharp, only a few showing shading toward the red. The diffuse lines were intensified, and the enhanced lines appeared as in other portions of the spectrum.

DISSYMMETRIES IN THE SPARK SPECTRUM

The writer has at intervals during several years photographed the spectrum of the spark under conditions giving a highly disruptive discharge. A powerful transformer spark with large condenser was used, the spark terminals being bluntly pointed rods of iron held in massive brass cylinders for cooling. After the discharge started, a series spark-gap was lengthened until the transformer was just able to maintain an exceedingly noisy spark. Comparison arc spectra were made on each plate by placing an arc in the position formerly occupied by the spark, leaving the lens system undisturbed. Sometimes the position of the spark line with reference to that of the arc was tested by means of the occulting screen, exposures with the arc being made before and after the spark exposure. For a large number of photographs a long slit was used, on which the image was projected with the axis of the spark at right angles to the slit. The middle of the spectrum line then registered the radiation from the interior of the spark, together with that from the outer vapor in the line of sight; while toward its ends the spectrum line was given by light from the outer vapor alone. Lines sensitive to reversal were frequently reversed almost to the point where the tip faded from the plate. The more striking features observed were as follows:

1. Dissymmetries were found to be very frequent in the spark spectrum and to vary greatly in magnitude for different lines.

2. The relative sensitiveness to dissymmetry among the lines of the spark appears to be quite the same as that observed for corresponding lines in the tube-arc spectrum.

3. In the spark spectrum of iron, two classes of lines, both unreversed in the spark, show no dissymmetry. These are the flame lines (furnace Class I) and the enhanced lines. As will be noted later, exceptions to this rule are found in other spectra when such lines are reversed.

4. The stronger arc lines in the blue and violet, usually of Class II, when photographed with a long slit, show large dissymmetry of the reversal at the middle of the line, the red side being the stronger. Receding from the central position reduces this dissymmetry as the interior vapor of the spark becomes less effective, until at the ends of the long line approximate symmetry is attained.

5. When adjacent spectra of the arc and of the core of the spark are made by means of the occulting screen, the position of the reversal is found to coincide closely in the two spectra, thus indicating that the outer vapors of the spark which cause the reversal give a line in the position of the arc line. This conclusion was arrived at by Kent,[1] who compared the lines given by the arc and spark in a concave-grating spectrum.

The features described in Nos. 4 and 5 are illustrated in Plate IX. In the upper section are enlargements of three reversed iron lines, each showing half of the length of the line photographed with the long slit across the spark-image. The unsymmetrical line given by the core is seen to change to a symmetrical line given by the

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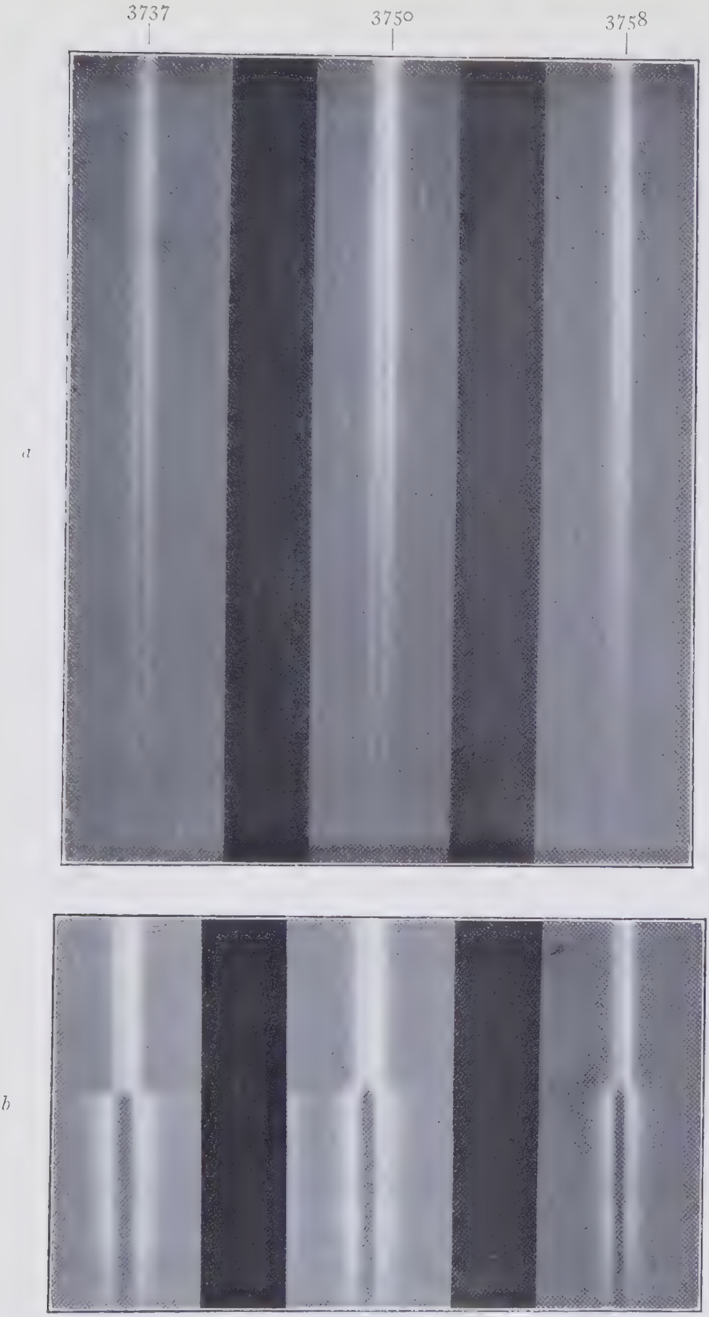
[1] *Astrophysical Journal*, **22**, 182, 1905.

PLATE IX



Iron Lines Showing Dissymmetry in the Disruptive Spark

a. Half of long line photographed with the slit at right angles to axis of spark.

b. Comparison of spark line (above) with arc line.

outer vapor. In the lower section of the plate, the same lines are shown as photographed by means of the occulting screen, the line given by the core of the spark having below it two exposures of the arc line made before and after the spark exposure respectively. The contrast in symmetry between the arc and spark lines is seen, and also the continuity of the reversal for the two sources.

The effect is further illustrated by the micro-photometer curves of Fig. 3. These show the structure of λ 3737.281 at the middle and near the end of the long spark line and also in the arc. Completing the curve for the unsymmetrical spark line shows the difference between the ordinate given by this and the ordinate of the reversal, which marks the position of the line in the arc.

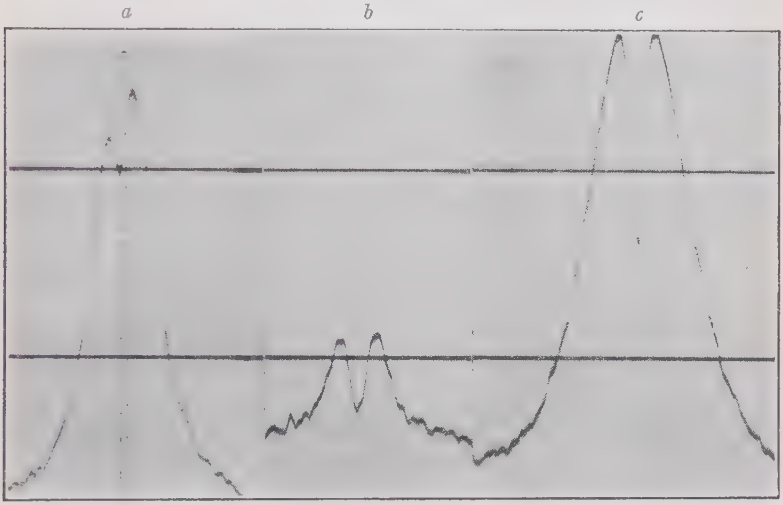


Fig. 3.—Curves of λ 3737 of iron; *a* and *b*, from middle and end of long spark line; *c*, from arc line.

Both in the appearance of the reversed lines, usually moderately unsymmetrical, and in the density-curves obtained from them, there is a close correspondence between the effects for the tube-arc and the condensed spark in the case of the iron lines listed in Table I. The lines of Table II, which are very unsymmetrically reversed in the tube-arc, do not reverse in the spark on any of my plates, but are winged strongly on the red side. Typical density-curves obtained from lines of this class are shown in Fig. 4.

In contrast with these lines, the strong diffuse lines of the green-yellow, which show dissymmetry toward the violet in the iron arc and are displaced toward the violet in the tube-arc, are shaded strongly on the violet side in the spark. These lines are given relatively strong by the core of the spark and by the most intense regions of the arc.

The evidence indicates that the core of the spark radiates lines which are in general displaced toward the red, probably to different degrees according to the class of the line. Such a displaced line might be symmetrical if a homogeneous mass of vapor from the core of the spark could be isolated. The line usually photographed is a composite given by the superposition on this core line of less-displaced radiations from the vapor enveloping the core. The density-curves of the strong reversed iron lines of moderate dissymmetry, usually belonging to furnace Class II, indicate the presence of a symmetrical line from the core of the spark differing in wave-length from the absorption line given by the outer vapor. These lines are quite similar in the tube-arc and spark. The iron

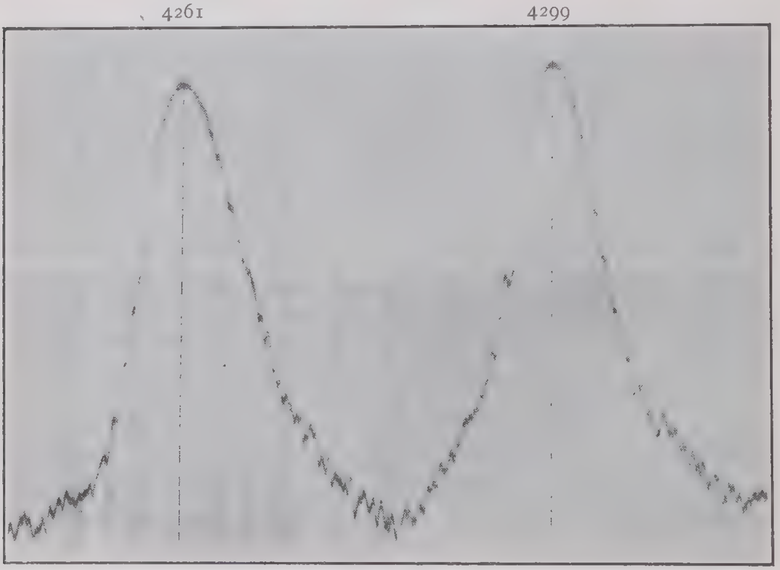


FIG. 4.—Curves of iron lines showing one-sided structure in the disruptive spark

lines at the two extremes of the furnace scale, the flame lines (Class I) and the enhanced lines (Class V E), are very nearly symmetrical in the condensed spark. It is shown, however, with the plane grating and long slit, that the flame lines are given chiefly by the outer vapor and are therefore out of the displacing region while the enhanced lines are confined to the core with no superposed radiation to render them unsymmetrical. The former should, therefore, remain symmetrical and undisplaced, while an enhanced line in the spark should be symmetrical but displaced with reference to the same line in the furnace or center of the arc. Only a few enhanced lines appear in the latter sources, however, and an accurate comparison presents many difficulties on account of the great difference in the strength of these lines in the spark and in the arc or furnace.

TUBE-ARC AND SPARK EFFECTS FOR OTHER ELEMENTS

A study in some detail has been made of the dissymmetries shown by titanium in tube-arc and spark, together with a few photographs of interesting regions in the vanadium and chromium spectra. Alloys of titanium and vanadium with iron were used to secure the highly disruptive spark. The main features observed may be briefly given.

Titanium shows a very close correspondence between tube-arc and spark in the degree to which various lines are made unsymmetrical. Among the strong reversed lines in the blue, certain groups show a decided difference in dissymmetry, this difference being alike in the two sources. λ 4298.89 is an example of symmetrical reversal, while λλ 4527.47 and 4457.61 have their sides unsymmetrical about in the degree 1:2 and 1:4 respectively. Lines of similar structure show a tendency to group in the neighborhood of these lines. While these are for the most part in furnace Class II, the more unsymmetrical lines approach the behavior of Class III lines in the furnace. The strong lines in the green near λ 5000 were examined in the condensed spark and show distinct differences in structure. λλ 4981.93 and 4991.24 show 1:2 dissymmetry while λλ 5036.08 and 5036.65 are more one-sided, about 1:4. Two

Class I lines in this region, λλ 5040.14 and 5064.79, partially reversed in the spark, are nearly symmetrical.

The enhanced titanium lines show considerable variation in the condensed spark. Those in the green are unreversed and very symmetrical in structure. Fig. 5 gives the density-curves for one of these and for a typical titanium arc line in this region. The enhanced lines from λ 3600 to λ 4600, however, are so strong in the spark that many of them are reversed, and these reversals, while usually unsymmetrical toward the red, are unsymmetrical toward the violet for λλ 4395.19 and 4444.00. Enhanced lines showing symmetrical reversal are λλ 3685.37, 3759.46, and 3761.47.

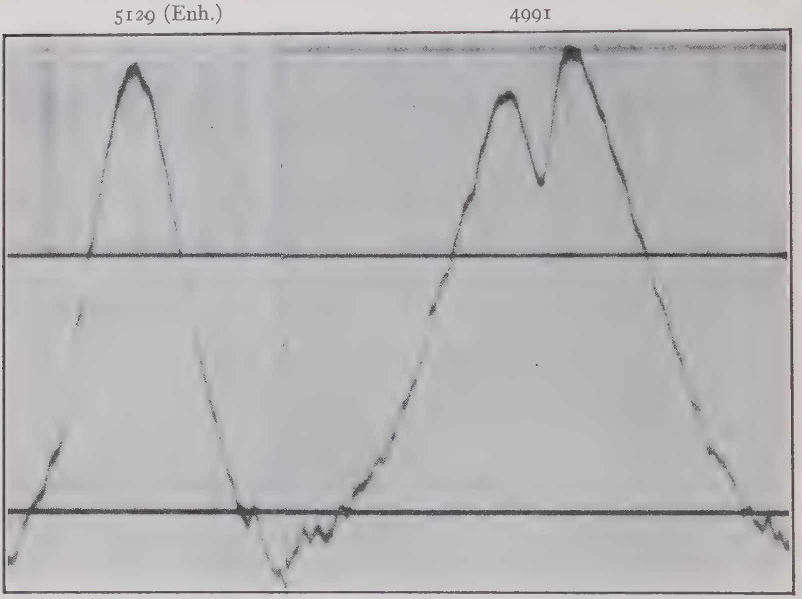


Fig. 5.—Curves of titanium lines in the disruptive spark

The spark spectra of vanadium and chromium were photographed for parts of the violet, blue, and green where distinctive strong lines occur. The plates show dissymmetries of varying amounts to occur very generally, but no decided difference has been noted from the condition prevailing in the iron and titanium spectra. Class I lines of vanadium and chromium show rather larger dissymmetries than those of this class in the other spectra, but these are very strong lines and the distribution of vapor between the core of the spark and the reversing envelope may be slightly different from that of iron and titanium lines in the same class.

COMPARISON WITH THE PRESSURE-EFFECT

It has been shown that the degree of dissymmetry exhibited by various classes of lines in the tube-arc and condensed spark corresponds closely with their susceptibility to pressure displacement. The question arises whether pressure may be active in the tube-arc and in the spark. Two lines of evidence are against this. First, the dissymmetries show no tendency to become larger with increasing wave-length, as do the pressure displacements, and, second, the tube-arc is operated in a partial vacuum, its action becoming more favorable to strong dissymmetries as the pressure is decreased. In addition to the difficulty of considering a high pressure in the ordinary sense as present in the spark in air, the resemblance of the spark effects to those of the tube-arc is so close that their causes would seem to be very similar. It will be shown later how the action of pressure may be different from that producing the tube-arc and spark displacements and yet produces equivalent effects.

THE INFLUENCE OF VAPOR-DENSITY

Examination of the tube-arc photographs and measurement of such lines as permit of it show clearly that, with the given electrical conditions, the dissymmetry of lines becomes much greater when their width is increased by the use of a large quantity of vapor. The measured displacement becomes small if the line is narrowed by the presence of only a small amount of vapor. However, the spectra given by the regular furnace offer evidence that high vapor-density alone will not render lines unsymmetrical. The writer has never yet observed a line in the furnace spectrum which was clearly unsymmetrical. To be sure, many of the lines most sensitive to this effect are in Class III or Class IV and are therefore narrow and unreversed in the furnace, but a number of strong titanium lines, very unsymmetrical in the tube-arc and spark, are reversed in the furnace at high temperature and remain quite symmetrical. The vapor-density in the furnace must be considerable, in spite of the large volume of the tube. This is attested by the width of the stronger lines and by the fact that, during the short run required for an exposure, two grams or more of iron may be vaporized so that the tube is afterward found incrusted with it from end to end. In the arc also, lines may be very wide when much vapor is present, and still remain symmetrical. When the poles of the arc are approached, however, the occurrence of enhanced lines indicates a condition resembling the spark, and in this region the lines most sensitive to displacement become one-sided. These dissymmetries are greatly magnified in the tube-arc and in the disruptive spark.

A considerable degree of harmony among these effects is obtained if it is permissible to consider as a necessary condition the presence of electrified particles moving at high velocities. These are supplied in the spark and at the poles of the arc by the high potential gradients. The cause for the very similar spectrum effects in the tube-arc, where the potential is low, is not so clear, and at present the writer can only refer to the suggestion offered in a previous paper[1] that the heated carbon ejects electrons which are given high velocities by the explosive conditions at the point of rupture. We know that at this point there is a great concentration of electrical energy, and the effects would seem to indicate that the action of the tube-arc produces a superheating of the carbon in virtue of which the electrons are given velocities comparable with those imparted in other sources by a high potential. If this possibility is admitted, the effect of high electronic speeds in causing a disturbance of period should be increased by the crowding together due to high vapor-density, and on this account the relation to the pressure-effect may be a close one. Spectrum lines from the furnace or from the center of the arc, given by particles having low velocity, are displaced by the crowding due to pressure. If the source does not have high pressure, but is such that the particles have high velocities, the tendency of these velocities to disturb the period would be expected to increase with the density of the vapor.

A consequence of this point of view would be that high pressure should give large displacements for lines which are strengthened by strong electrical conditions. This is borne out by the large pressure displacements found for lines in Classes III and IV of the furnace classification. These lines are much stronger in the arc and spark than in the furnace. Further, Gale and Adams[1] observed that the enhanced lines as a class are displaced by pressure more than the arc lines, and that for a given pressure the enhanced lines are displaced more in the spark than in the arc. They note also that when the arc and spark were observed at the same pressure, the lines in general show "a marked tendency toward higher values for the spark displacements." All of these effects would follow as a direct result of a combination of high electrical action with high pressure.

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[1] *Mt. Wilson Contr.*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

Royds,[2] in reviewing the displacement effects observed as a result of arc dissymmetries, arrives at the conclusion that vapor-density plays an important part, while recognizing the possible influence of ionization in the arc and spark, and suggests "density of ions" as a substitute for "vapor-density" as the displacing cause. I believe this idea is in the right direction and would make a further modification by suggesting *density of high-speed electrons* as a condition which appears to harmonize the results for the displacements of the arc, spark, and tube-arc with those for the pressure-effect and to account for the evident connection of all of these with the furnace classification.

SUMMARY

1. The results for the tube-arc and the disruptive spark show that these sources impart an unsymmetrical structure to many of the stronger spectrum lines, the micro-photometer indicating that this is caused by the region of strongest excitation giving a displaced line, usually to the red.

2. Much variability appears in the amount of this dissymmetry for different lines, its magnitude clearly being related to the furnace classification of the lines and to their displacement under pressure.

3. Iron lines of small and moderate dissymmetry in the tube-arc are low-temperature lines and show only moderate pressure-effect.

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[1] *Mt. Wilson Contr.*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

[2] *Kodaikanal Observatory Bulletin*, No. 40, 1914.

4. Lines of large dissymmetry are faint in the furnace relatively to the arc and are greatly displaced by pressure.

5. The degree of dissymmetry in the tube-arc, for each of the foregoing classes, has a distinct relation to the width of the line, so that, other things being equal, strong lines are more displaced by this effect than faint ones and an increase of vapor-density increases the dissymmetry.

6. The most symmetrical lines of iron in the tube-arc are the "flame" lines. The H and K lines of calcium reverse with the violet side strongest.

7. Lines unsymmetrical toward the violet in the spark and near the pole of the arc are displaced toward the violet in the tube-arc.

8. The nebulous lines of iron, requiring strong arc conditions, are much intensified in the tube-arc.

9. A powerful and disruptive spark produces line dissymmetries which in general are very similar to those of the tube-arc. The most symmetrical iron lines in this spark are the "flame" lines and the enhanced lines.

10. Analysis by the plane grating shows the radiation from the core of the spark to be displaced toward the red, in the case of most of the stronger lines, the outer vapor giving a symmetrical line in the position of the arc line.

11. The results indicate that a condition of strong electrical excitation, combined with high vapor-density, produces a condition the displacing action of which shows many resemblances to the effect of pressure.

The writer wishes to acknowledge the valuable assistance of Mr. Colby and of Mr. Monk, who operated the micro-photometer in registering the curves for this investigation.

Mount Wilson Solar Observatory
April 20, 1915

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A STUDY OF THE LIGHT-CURVE OF XX CYGNI

By HARLOW SHAPLEY and MARTHA BETZ SHAPLEY

XX Cygni, discovered in 1904 by Madame Ceraski,[1] is the shortest-period variable star known. It is classed as a Cepheid by Kron,[2] and the light-variation seems to have all the general characistics peculiar to that class of variables; but unlike other Cepheids for which we have the necessary data, it is said to have a greater range visually than photographically.[3] That such an abnormality of color should be attributed to a variable already exceptional on account of its unusual period, suggests that the peculiarity may not be due to observational uncertainties. Its explanation would be that at maximum the star is redder than at minimum, the greatest intensity of the spectrum shifting toward the red with increasing light, instead of toward the violet as with all other Cepheids.[4] This would indicate that, if there is a change of spectral type with change of light, the redder spectrum occurs at maximum and not at minimum, as has been observed for RS Boötis,[5] and as is presumably true for all other Cepheids whose photographic ranges are known to exceed the visual. To determine whether the visual range of XX Cygni is really greater than the photographic, and to investigate other peculiarities of the star,[6] a study of its light-curve has been undertaken at Mount Wilson, the results of which are contained in the present communication.

[1] *Astronomische Nachrichten*, **165**, 61, 1904.

[2] *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, **22**, Part III, 52, 1912.

[3] Parkhurst and Jordan find a photographic range of 0.63 mag. Kron's mean visual range is 0.76. But cf. Kron, *op. cit.*, p. 56.

[4] Campbell, *Stellar Motions*, p. 309, 1913.

[5] *Publications of the Astronomical Society of the Pacific*, **26**, 256, 1914.

[6] Such as irregularities in time and shape of maxima, or other features that might throw light on the cause of Cepheid variation; *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

Because of the favorable field and the abundance of close and suitable comparison stars, XX Cygni is well adapted for accurate eye-estimates, as well as for photometric and photographic observation; and since its discovery the light-variations have been followed regularly. In 1912 E. Kron[1] published a monograph on the star, containing 2705 observations by eight different observers,[2] together with a critical discussion of the data. The period derived is $3^h14^m12^s.3547$, with the addition of a quadratic secular term. Although Kron concludes that the existence of this term is clearly demonstrated, the Mount Wilson observations show that, quantitatively at least, it is not sufficient.

On the nights of August 21 and 23, and September 18, 1914, and March 17, 1915, series of observations were made with the 60-inch reflector. On each of the first two nights the star was followed throughout an entire period. In all the observations Seed "27" and Cramer "Instantaneous Isochromatic" plates (used with a yellow color-filter) were alternated, thus giving in quick succession photographic and photo-visual magnitudes. The importance of this procedure lies in securing complete photographic and photo-visual determinations of the same epoch of maximum with the same instrument. As we shall find later, neither the shape of the curve nor the range is constant for successive periods. Therefore it is important that every photographic-photo-visual comparison should relate to the same interval of variation.

The plates obtained were as follows: August 21 and 23, 13 of each kind on each night; September 18, 5 Seed 27 and 4 isochromatic; March 17, 8 Seed 27 and 7 isochromatic, but owing to haze five of the latter were not sufficiently exposed to insure reliable measurement. In addition there are nine polar comparison plates, one of which was rejected because the cycle of comparisons with the

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[1] *Op. cit.*

[2] Blažko, *Astronomische Nachrichten*, **172**, 58, 1906; Schwab, *ibid.*, **170**, 370, 1906; Parkhurst and Jordan, *Astrophysical Journal*, **23**, 84, 1906; Graff, *Astronomische Nachrichten*, **171**, 55, 1906; Luizet, *Bulletin Astronomique*, **25**, 251, 1908; Nijland, *Astronomische Nachrichten*, **188**, 149, 1911; Kron, *op. cit.*, p. 35; Hartwig, *Vierteljahrsschrift der Astronomischen Gesellschaft*, **41**, 309, 1906. The observations by Guthnick, as well as those by most of the observers named, are not published outside of Kron's monograph.

Pole is not closed and the altitude of the variable was so great that the extinction correction is large and uncertain. With the exception of the polar comparisons, each Seed 27 plate received four one-minute exposures,[1] while each isochromatic received three two-minute exposures.

The method of measurement and reduction closely follows that described by Seares in *Mt. Wilson Contr.*, No. 80.[2] Of each separate image of the variable and the comparison stars two independent measures were made, and all of the measures on each plate, corrected for distance from the center and for irregularities of the measuring scale, yielded a single mean value of the brightness of the variable.

Diaphragms were used to determine the scale of the comparison stars referred to an arbitrary zero-point; the magnitudes so obtained were reduced to the international zero by comparison with the Mount Wilson Polar Standards. Corrections for differential extinction between the variable and the Pole were made with the aid of the Potsdam tables,[3] interpolating for the altitude of Mount Wilson, and doubling the tabulated visual values to obtain the photographic corrections.

TABLE I

POLAR COMPARISON PLATES

PLATE		DATE	G.M.T.	NO. EXP. ON VAR.	NO. EXP. ON POLE	LENGTH OF EACH EXP.	EXTINCTION CORRECTION	REDUCTION TO INT. ZERO
No.	Kind							
1792..	Iso.	1914 Aug. 22	$23^{h}\ 8^{m}$	2	2	2^{m}	−0.00 mag.	10.83 mag.
1828..	Iso.	23	23 16	2	1	2	−0.02	10.74
2325..	Iso.	1915 Mar. 15	23 43	2	2	3	+0.03	10.75
2327..	Iso.	16	0 28	2	3	2	+0.06	10.86
1791..	S. 27	1914 Aug. 22	22 52	2	2	1	+0.02	10.19
1829..	S. 27	23	23 25	2	2	1	−0.04	10.08
1890*.	S. 27	Sept. 18	15 23	2	1	1	+0.22	.....
2324..	S. 27	1915 Mar. 15	23 51	2	2	1	+0.07	10.07
2326..	S. 27	16	0 25	2	3	1	+0.11	10.13

* Rejected as a polar comparison.

Table I gives a list of the polar comparisons. The last column, giving the value of the reduction constant derived from each plate, shows the satisfactory accordance of the results and indicates that that part of the probable error in the color-index which depends on the uncertainty of the zero-points is considerably less than a tenth of a magnitude.

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[1] Plate 1892 has five exposures; 1890 has two.

[2] *Astrophysical Journal*, **39**, 307, 1914.

[3] Müller, *Photometrie der Gestirne*, p. 515, 1897.

Table II gives the adopted magnitudes of the five comparison stars as determined from seven diaphragm-plates of each kind.

TABLE II

MAGNITUDES OF COMPARISON STARS

Star	Photographic Magnitude	Av. Dev.	Photo-visual Magnitude	Av. Dev.
1........	11.10	±0.09	10.76	±0.06
2........	12.56	±0.10	11.33	±0.09
3........	13.35	±0.04	....	....
4........	....	....	11.77	±0.02
5........	....	....	12.23	±0.07

On the Seed 27 plates, Nos. 1, 2, and 3 (see Fig. 3 and Table VI) were used; on the isochromatic plates, Nos. 4 and 5 were substituted for No. 3 which was too faint photo-visually to be suitable. The photographic magnitudes of Nos. 4 and 5 and the photo-visual magnitude of No. 3, which are missing from this table, were later determined, but less accurately, by a direct comparison with the Polar Standards (see Table VI).

In Table III we have the photographic and photo-visual material upon which the light-curves are based. The first column contains the number of the plate, the second the Greenwich heliocentric mean time of the middle of the exposure, the third the phase computed from Kron's elements II,[1] and the fourth the magnitude of the variable. Diaphragms were used in making the plates marked with an asterisk, the order of apertures being 60–32–60 for the isochromatic and 60–32–32–60 for the Seed 27 plates (with the exception of Plate 1892 on which the order was 60–40–32–40–60).

The data bearing on the accuracy of the light-elements are collected in Table IV. The times of maximum predicted from Kron's adopted elements (Kron II; see *op. cit.*, p. 47)[2] are given in the fourth column. They do not agree within allowable errors with the observed times, the greatest difference being 17 minutes. Using the

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[1] *Op. cit.*, p. 47.

[2] Max.=J.D. 2416563.41065, G.H.M.T. $+0\overset{d}{.}13486522E \; -0\overset{d}{.}1578\times 10^{-10}E^2$.

TABLE III

MOUNT WILSON OBSERVATIONS OF XX CYGNI

Photographic				Photo-visual			
Plate	G.H.M.T.	Phase	Mag.	Plate	G.H.M.T.	Phase	Mag.
	1914 Aug. 21				1914 Aug. 21		
1752	$20^{\mathrm{h}}\ 9^{\mathrm{m}}.2$	$+2^{\mathrm{h}}34^{\mathrm{m}}.9$	12.47	1753	$20^{\mathrm{h}}17^{\mathrm{m}}.2$	$+2^{\mathrm{h}}42^{\mathrm{m}}.9$	12.21
1754	27.2	52.9	12.48	1755	34.5	3 0.2	11.92
1756*	42.3	3 8.0	11.90	1757*	49.7	0 1.2	11.47
1758	57.2	0 8.7	11.54	1759	21 4.4	15.9	11.54
1760	21 11.9	23.4	11.68	1761	20.0	31.5	11.70
1762*	33.1	44.6	12.00	1763*	40.7	52.2	11.98
1764	48.3	59.8	12.33	1765	55.7	1 7.2	12.03
1766	22 3.4	1 14.9	12.23	1767	22 10.8	22.3	12.11
1768	18.4	29.9	12.29	1769	26.5	38.0	12.06
1770	34.8	46.3	12.52	1771	42.3	53.8	12.15
1772	49.8	2 1.3	12.45	1773	57.9	2 9.4	12.15
1774*	23 5.3	16.8	12.52	1775*	23 13.1	24.6	12.17
1776	21.2	32.7	12.48	1777	28.6	40.1	12.11
	Aug. 23				Aug. 23		
1794	15 37.7	0 44.5	11.75	1795	15 46.6	0 53.4	11.99
1796	55.0	1 1.8	12.08	1797	16 2.2	1 9.0	11.98
1798*	16 9.9	16.7	12.25	1799*	18.7	25.5	11.93
1800	27.6	34.4	12.37	1801	35.4	42.2	12.05
1802	46.5	53.3	12.50	1803	54.3	2 1.1	12.18
1804	17 1.5	2 8.3	12.41	1805	17 8.6	15.4	12.17
1806	15.8	22.6	12.46	1807	23.0	29.8	12.13
1808	30.4	37.2	12.50	1809	37.3	44.1	12.05
1810	45.1	51.9	12.40	1811	52.4	59.2	11.67
1812*	59.5	3 6.3	12.08	1813*	18 8.0	0 0.6	11.40
1814	18 19.7	0 12.3	11.82	1815	26.9	19.5	11.32
1816	34.1	26.7	11.74	1817	41.0	33.6	11.40
1818	48.4	41.0	11.81	1819	55.6	48.2	11.62
1829	23 27.2	2 5.6	12.56	1828	23 18.2	1 56.6	12.25
	Sept. 18				Sept. 18		
1890	15 26.7	3 6.3	12.31	1891*	15 34.0	3 13.6	11.48
1892*	44.5	0 9.9	11.61	1893	54.1	0 19.5	11.59
1894	16 1.7	27.1	11.76	1895	16 9.3	34.7	11.83
1896	17.8	43.2	12.16	1897*	26.9	52.3	11.86
1898*	35.9	1 1.3	12.29				
	1915 Mar. 17				1915 Mar. 17		
2338*a*†	22 38.9	2 46.9	12.64	2339*a*	22 47.0	2 55.0	11.86:
b	55.0	3 3.0	12.04	*b*	23 1.4	3 9.4	11.76
2340*a*	23 9.4	0 3.2	11.79	2341*a*	15.9	0 9.7	11.35
b	22.0	15.8	11.65	*b*	28.5	22.3	11.56:
2342*a*	34.9	28.7	11.70	2343*a*	41.3	35.1	11.54:
b	47.7	41.5	12.00	*b*	53.8	47.6	11.67:
2344*a*	24 0.1	53.9	12.18				
b	16.5	1 10.3	12.20				

† The plates marked *a* and *b* each have two complete sets of exposures.

elements Kron designates as II*a*, which involve a least-squares correction to Blažko's values of the initial epoch and period but do not contain a secular term, the residuals are somewhat reduced. Although the observational data here given are not sufficient to justify a correction to Kron's elements, as far as they go, they argue against the existence of the secular term.

TABLE IV

Date	Epoch	Julian Day	Predicted Hel. Max.	Observed Hel. Max.	Obs. − Kron II	Obs. − Kron II*a*
1914 Aug. 21	28202	2420366	$20^h48^m.5$	20^h57^m	+8	+3
Aug. 23	28216	2420368	18 7.4	18 24	+17	+11
Sept. 18	28408	2420394	15 34.6	15 49:	+14:	+8:
1915 Mar. 17	29745	2420574	23 6.2	23 21	+15	+8

Two of the light-curves tabulated in Table III are shown in Figs. 1 and 2. In each case we have two simultaneous records, one based on blue light, the other on light of longer wave-lengths; but both are determinations of the same maximum, so that the differences in shape, range, time of maximum, etc., which exceed the errors of observation, must be interpreted as differences of color. In shape there appears to be no great difference between the photographic and photo-visual curves on the same night, although there is a marked difference between the curves of August 21 and those of August 23. This, however, will be discussed later.

Within the errors of observation, the time of maximum is practically the same photographically and photo-visually. There is some indication, nevertheless, that the photographic maximum may come slightly later, as would be expected if there is an appreciable absorption of light in space.[1] It is in the amplitude, however, that we find the most marked difference, especially on August 21. On that date the photo-visual range is 0.72 mag. as against a photographic range of 0.97 mag. This difference in color is normal for Cepheids. The second night, on the other hand, gives a photo-visual range of 0.10 mag. greater than the photographic. The mean photographic range for the two nights is 0.86 mag., the mean

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[1] Kron finds the difference vanishingly small but, if anything, in the opposite sense; *op. cit.*, p. 56.

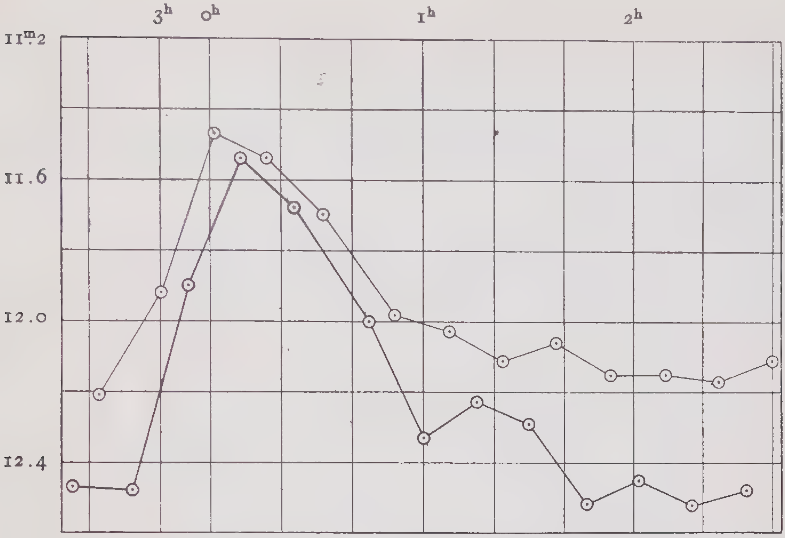

FIG. 1.—Photo-visual (above) and photographic light-curves of XX Cygni for August 21, 1914.

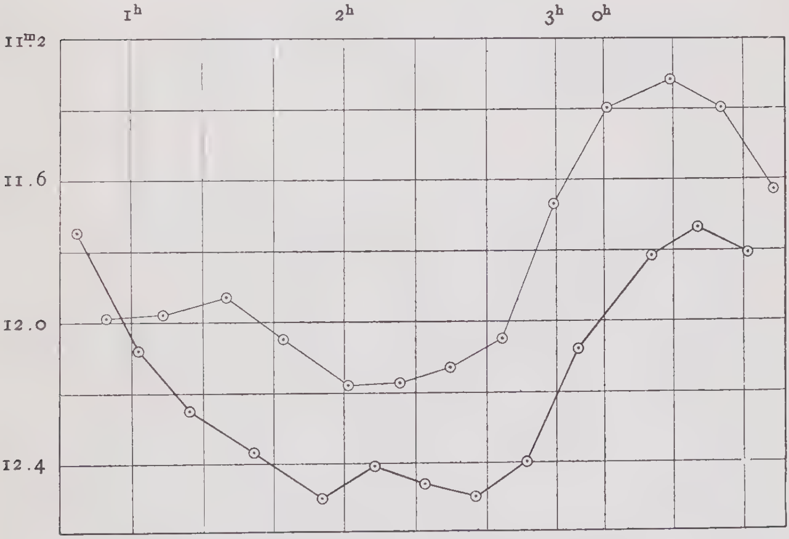

FIG. 2.—Photo-visual (above) and photographic light-curves of XX Cygni for August 23, 1914.

photo-visual is 0.78 mag.; the mean visual range found by Kron from all observers is 0.76 mag. On March 17 the photographic range is 0.99 mag., but minimum light depends on only one normal point. Thus it appears from the data now available that the photovisual range is not greater than the photographic, and is apparently somewhat smaller.

The color and magnitude at minimum (as shown in Table V) are sensibly constant, suggesting that the irregularities occur at maximum and that the minimum is the normal condition of the star. This is verified by the measures on the polar comparison Plates 1828 and 1829 which show a color-index at minimum of +0.31. The color-index at maximum, however, varies more than 0.3 mag.

TABLE V

COLOR DATA FOR XX CYGNI

	AUGUST 21, 1914			AUGUST 23, 1914		
	Photog. Mag.	Photo-vis. Mag.	Color-Index	Photog. Mag.	Photo-vis. Mag.	Color-Index
Minimum.....	12.50	12.17	+0.33 mag.	12.49	12.17	+0.32 mag.
Maximum.....	11.53	11.45	+0.08	11.74	11.32	+0.42
Range........	0.97	0.72	0.25	0.75	0.85	0.10

In addition to the values tabulated above, others can be determined, but with less weight, from the two incomplete series of observations; for September 18 we find +0.15, and for March 17, +0.30. The mean for four maxima is +0.24, showing that in the mean and three times out of four the star is bluer at maximum than at minimum, and corroborating from other data the conclusion expressed above that the photographic range exceeds the photovisual. The value of the color-index indicates an average spectrum more nearly of type F than of type A as classified at Harvard.[1]

In connection with the study of XX Cygni it was thought of interest to investigate the color of the surrounding stars. Fig. 3 shows all those within a distance of 11 minutes of arc from the variable, brighter than magnitude 13.7 photo-visually—the limit of visibility on our plates. Table VI contains in the first column the adopted designation of the stars, in the second Kron's designation for such of them as he has named, and in the third Kron's visual magnitude (on an extension of the Potsdam scale). In the last three columns

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[1] *Harvard Annals*, **56**, 194, 1912.

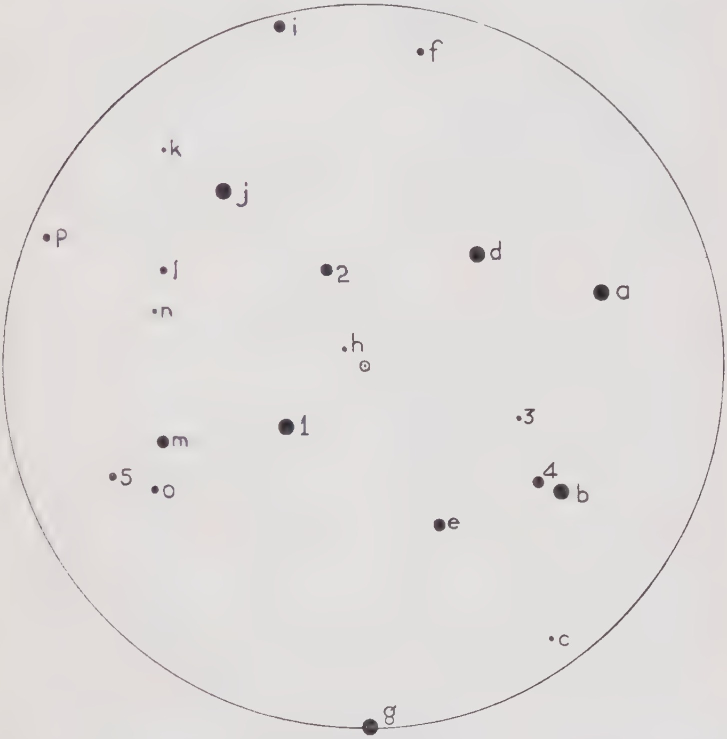

Fig. 3.—Field of XX Cygni, $20^{\mathrm{h}}1^{\mathrm{m}}.3$, +58°40′ (1900), giving all stars brighter than magnitude 13.7 photo-visually, within 11′ of the variable.

are the mean Mount Wilson photographic and photo-visual magnitudes, and the color-index derived from them. These magnitudes were determined from the eight polar comparison plates (Table I), which were taken in four pairs, each consisting of one isochromatic and one Seed 27 plate. Since in each case the plates compared were taken in quick succession, the color-indices are free from errors due to possible light-variations in the stars themselves. On comparing Kron's visual results with the Mount Wilson photo-visual magnitudes, we find that the two scales are parallel throughout this range of 2.3 magnitudes, and, correcting for the systematic difference in zero-point of 0.12 mag., the average difference for the 12 stars is ±0.05 mag.

TABLE VI

COLOR-INDICES OF STARS NEAR XX CYGNI

Star	Kron		Mount Wilson		
	Designation	Visual Mag.	Photog. Mag.	Photo-vis. Mag.	Color-Index
1..........	11	10.56	11.10	10.76	+0.34 mag.
2..........	9	11.21	12.56	11.33	+1.23
3..........	....	....	13.35	13.15	+0.20
4..........	....	....	12.35	11.77	+0.58
5..........	5	11.94	12.38	12.23	+0.15
a..........	8	10.80	11.21	10.83	+0.38
b..........	A	10.15	11.21	10.27	+0.94
c..........	....	....	12.80	12.94	−0.14
d..........	B	10.29	11.52	10.39	+1.13
e..........	7	11.64	12.14	11.80	+0.34
f..........	....	....	12.70	12.77	−0.07
g..........	D	9.72	10.36:	9.83	+0.53:
h..........	....	....	14.22	13.62	+0.60
i..........	....	....	11.76	11.37	+0.39
j..........	E	10.33	11.36	10.40	+0.96
k..........	....	....	13.70	12.99	+0.71
l..........	6	11.84	13.19	11.86	+1.33
m..........	4	11.54	12.31	11.76	+0.55
n..........	....	....	13.76	13.29	+0.47
o..........	3	12.03	12.46	12.19	+0.27
p..........	12	....	13.18	12.58	+0.60

The color-indices of these stars near XX Cygni were compared with those of North Polar stars within the same limits of photovisual magnitude.[1] The results, given in Table VII, show that the average color-index of all the stars considered is less in the field of XX Cygni than near the Pole. Perhaps because the former is in the Milky Way we should expect to find these stars bluer.

It was noted above that the light-curves of XX Cygni appeared to have an entirely different shape on August 21 and 23, 1914. Such a difference from night to night is found in a much more striking degree on investigating the observations published by Kron. The variations from the mean curve are certainly much larger than the errors of observation. Three pairs of curves are reproduced in Figs. 4, 5, and 6 to illustrate the extent of this irregularity of form. The first pair is taken from Guthnick's photometric measures, the second from Schwab's visual estimates, and the third from Kron's observations with the Potsdam photometer. These diagrams give examples of extreme types of maxima; all intermediate stages between them are found among the curves examined. Of the two Mount Wilson curves illustrated, that of August 21 is of the intermediate type, slightly narrower than the mean, while August 23 shows a broad round-topped curve nearing the extreme of that type. In the light of these wide variations in the form of the curves, the observed differences in range and color on different nights are much more plausible.

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[1] *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

TABLE VII

COMPARISON OF THE COLOR OF STARS IN THE FIELD OF XX CYGNI WITH THE COLOR OF STARS AT THE NORTH POLE

LIMITS OF MAG.	NORTH POLE		FIELD OF XX CYGNI	
	No. Stars	Average Color-Index	No. Stars	Average Color-Index
10.0 to 11.0.........	7	+0.57 mag.	6	+0.71 mag.
11.0 " 12.0.........	3	+0.45	6	+0.74
12.0 " 13.0.........	14	+1.01	6	+0.25
13.0 " 14.0.........	29	+0.98	3	+0.42
10.0 " 14.0.........	53	+0.90	21	+0.55

An attempt was made to see if the change from one type of curve to another occurred at regular intervals, but from the observations published by Kron no periodicity could be established. It was noted, however, that there are no instances in which consecutive maxima showed a great difference in type (on account of the three-hour period of the star, it was on some nights observed throughout two or more successive periods). From one night to the next, on the other hand, there is often considerable change, and in the instances where observations are available for many successive nights, there is a rough indication of a periodicity of possibly a week. As all of these curves are reasonably smooth, only a part of the difference can be attributed to errors of observation. All that can be said at present, however, is that the variation in the shape of the curve definitely exists, and that the change from one type to another appears to be gradual. The successive maxima of XX Cygni, then, are obviously not exact repetitions of the same phenomenon. Whether the minima, as well as the maxima, vary in shape, it is impossible definitely to say, for they have not been sufficiently observed; but the runs of residuals during those minima that have been observed indicate the existence of different types.

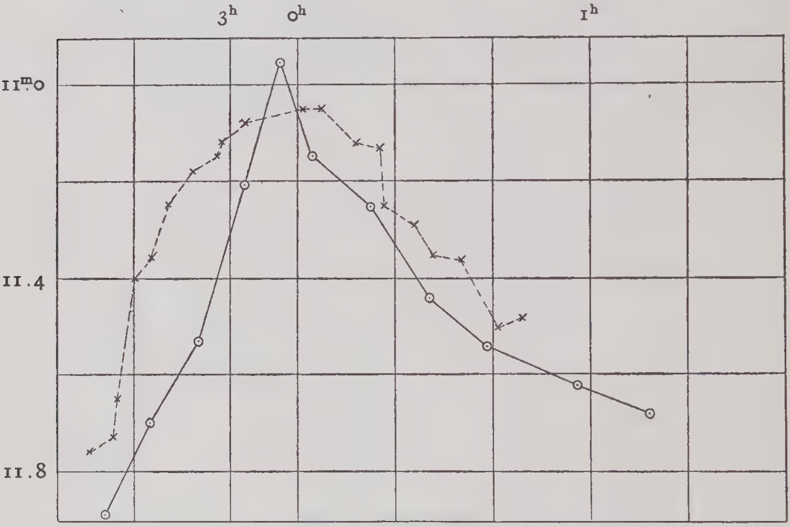

FIG. 4.—Guthnick's visual light-curves of XX Cygni for September 22, and October 5 (broken line), 1908, showing two extreme types of maximum.

To give still more evidence (if that is needed) for the reality of the different types of light-curve, Table VIII has been prepared to show that the many large residuals from the mean curves do not represent accidental errors. The deviations in each case are from the mean curve for that observer. If the variations from a mean are distributed according to the Gaussian law of error, there should be in the long run as many changes as persistences of sign. A glance at the table shows that here this is plainly not the case. These persistences of sign apparently verify from the measures of

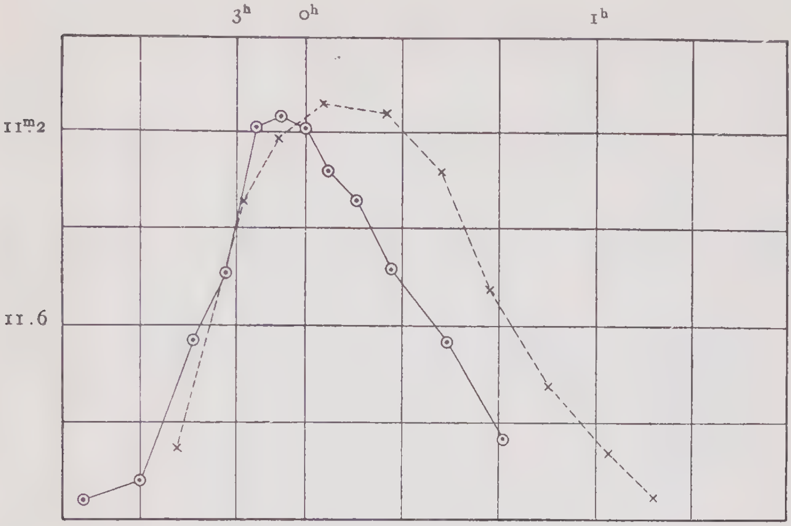

FIG. 5.—Schwab's visual light-curves of XX Cygni for January 9 (broken line) and March 13, 1905.

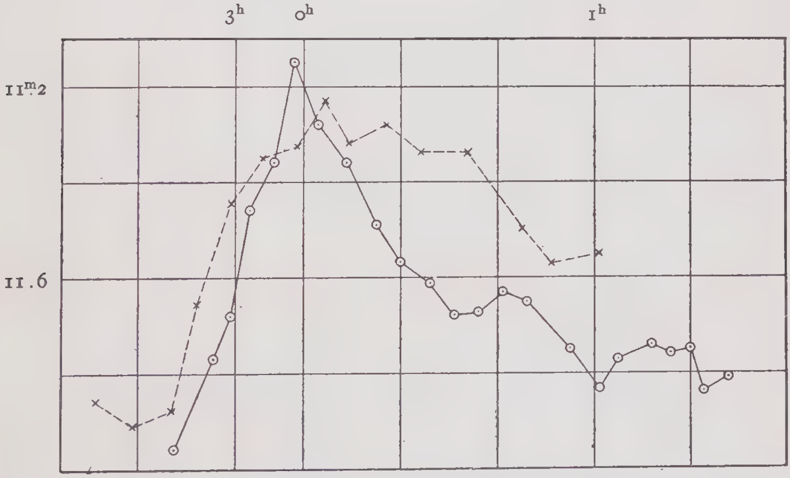

FIG. 6.—Kron's visual light-curves of XX Cygni for September 21 (broken line) and December 3, 1909.

each observer the existence of entirely different types of light-curve. They cannot be due to any appreciable extent to ordinary night error or to inconstancy of the light of comparison stars, as individual examination of each curve shows.

TABLE VIII

Systematic Runs of Residuals from Mean Curves

Observer	No. Persistences of Sign	No. Changes of Sign	Ratio
Blažko	431	142	3.0
Schwab	330	123	2.7
Parkhurst and Jordan	42	16	2.6
Graff	64	37	1.7
Luizet	157	50	3.1
Guthnick	239	84	2.8
Nijland	105	69	1.5
Kron	338	127	2.7
Hartwig	13	9	1.4
Total	1719	657	2.6

The conclusion must be that, whatever the cause, there is a short-period though possibly irregular change in the actual shape of the curve, and also probably a short-period oscillation in the time of maximum, though the mean period remains sensibly constant.

SUMMARY

1. To test the supposition that its visual range exceeds the photographic, and to investigate the general question of regularity in Cepheid variation, a study has been made at Mount Wilson of the light-curve of XX Cygni, the variable with the shortest known period. More than 300 exposures were made for this purpose with the 60-inch reflector.

2. It is found that, in agreement with all results for other Cepheids, the visual range does not exceed the photographic, though from the data now at hand we cannot say definitely that the photographic range is greatly in excess (Table V).

3. The stars in the field of XX Cygni within eleven minutes of arc of the variable yield an average color-index considerably less than that of North Polar stars of the same magnitudes.

4. Though a short-period oscillation in the time of maximum is suspected, the mean period of XX Cygni is sensibly constant. The secular term in Kron's elements is perhaps unnecessary; at least the light-elements which contain no secular term predict times of maximum in better accordance with the Mount Wilson observations in 1914 and 1915.

5. An examination of observations made by many observers and published by Kron shows definitely the existence of several types of maximum (Figs. 4, 5, and 6). The Mount Wilson observations confirm this (Figs. 1 and 2).

6. The maxima of XX Cygni, therefore, are not exact repetitions of the same phenomenon, but rather disturbances in the star's brightness occurring at regular intervals but varying in character.

MOUNT WILSON SOLAR OBSERVATORY
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THE RADIAL VELOCITIES OF FIVE HUNDRED STARS

By WALTER S. ADAMS

The program of radial velocity work for the Cassegrain spectrograph during the past few years has consisted for the most part of observations on the following classes of stars:

1. A- and B-type stars, mainly between magnitudes 5 and 6.5, a knowledge of whose motions is of particular interest as aiding in the determination of the elements of the two principal star-streams.

2. A, F, G, K, and M stars of magnitudes 5.5 to 6.5 which have very small astronomical proper motions. These may in general be considered as very distant stars of high luminosity, and are of interest as regards both their radial velocities and certain characteristics of their spectra.

3. Stars with measured parallaxes, most of which have very large proper motions. The magnitudes of these stars are chiefly between 5.5 and 8.5.[1]

In addition to these lists a number of brighter stars have been observed, for which determinations of radial velocity have been published from other observatories.

It seems desirable to make the results so far obtained available for the use of astronomers who are engaged in the discussion of stellar motions, and accordingly values are given in this communication for five hundred stars for which, with a few exceptions, three or more observations have been secured. Many other stars have been observed once or twice, and results for these will be published as soon as additional material has been obtained.

Several different optical combinations have been employed in the spectrograph during the course of these observations. The principal consideration which governs the dispersion to be used is, of course, the character of the spectrum of the star, but this has been modified in many cases by other factors. For example, the spectra of the small proper-motion stars have in almost all cases been photographed with low dispersion, although most of them are of the solar type, and so are well adapted for the use of high dispersion. It seemed desirable in their case to sacrifice accuracy to some extent in order to secure statistical material more rapidly, and to make it possible to institute direct comparisons between their spectra and those of the fainter stars of large proper motion and measured parallax. The different combinations used in the spectrograph may be summarized as follows. The linear scale denotes the number of Ångström units per millimeter. The 18-cm camera has been used in the case of only three of the published results.

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[1] The radial velocities of 100 of these stars were published in *Mt. Wilson Contr.*, No. 79; *Astrophysical Journal*, **39**, 341, 1914.

TABLE I

No. Prisms	Camera	Linear Scale at Hγ	Stars Observed
2	38 and 46 cm	21 and 18 A	A, B, and brighter parallax stars
1	102	16	
1	46	36	Small p.m. and parallax stars
1	18	92	Parallax stars fainter than 8.5

Table II contains values for the individual stars. In view of the importance of the *Preliminary General Catalogue* of Boss for determinations of proper motion it has seemed preferable to designate the stars which occur in his catalogue by their numbers rather than to give a heterogeneous collection of names and catalogue numbers. The stars with measured parallaxes have the designations given in *Groningen Publication*, No. 24. Additional stars are indicated by the Lalande number so far as possible, the *B.D.* number being used only in a very few cases. The magnitudes are those of Harvard, with the exception of such as are given in parentheses, which are from miscellaneous sources.

The spectral classification has been made from the Mount Wilson negatives, and most of the determinations, particularly for the A and B stars, are due to Mr. Kohlschütter. Especial attention should be called to the M stars which are marked "peculiar." The peculiarity in nearly all cases consists in the combination of hydrogen lines of an intensity corresponding to that in G- and K-type stars with the bands of the M stars. Some of these stars, classified according to the intensity of their hydrogen lines, have been discussed by Adams and Kohlschütter in a previous communication.[1]

The total proper motion μ is in most cases derived from the values given by Boss. For the parallax stars it is taken from *Groningen Publication*, No. 24. The angle λ is the angle between the star and the sun's apex. The co-ordinates used for the apex are those adopted by Kapteyn,

$$\alpha=17^{\mathrm{h}}59^{\mathrm{m}},\quad \delta=+30\overset{\circ}{.}8\,,$$

and the values both for μ and λ are taken from a list calculated under his direction for all of the stars given in Boss's catalogue.

The first of the two columns in Table II denoted by v contains the means of the observed radial velocities; the second the corresponding values published by other observatories. The following abbreviations are used: A, Allegheny Observatory; L, Lick Observatory; Y, Yerkes Observatory.

The final column of the table contains the values of v corrected for the solar motion. The values are given by the equation

$$v'=v+V\cos\lambda$$

in which the value 20 km has been assumed for V, the sun's motion in space.

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[1] *Mt. Wilson Contr.*, No. 89; *Astrophysical Journal*, **40**, 385, 1914.

TABLE II

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 5..	$0^h\ 1^m.2$	+63°38′	5.5	B8	0″.009	63°	− 6.8	....	+ 2.3
18..	0 4.9	+10 35	5.4	B8	0.034	86	+13.5	....	+14.9
41..	0 11.6	+60 59	5.8	G2	0.002	65	− 4.1	....	+ 4.4
43..	0 11.9	+38 8	4.6	A2	0.052	74	+ 0.1	+ 6.2L	+ 5.6
56..	0 15.5	+ 7 38	5.6	K0	0.015	89	+16.4	....	+16.7
81..	0.22.8	+17 20	5.3	Mbp	0.117	86	+ 6.8	....	+ 8.2
90..	0 24.9	− 4 31	6.0	K5	0.011	98	+ 5.2	....	+ 2.4
118..	0 30.6	+53 37	5.2	B7	0.021	70	+ 2.8	....	+ 9.6
124..	0 31.6	+14 41	5.9	B3	0.028	89	−18.3	....	−18.0
125..	0 32.0	+34 51	5.6	G0	0.019	79	− 0.7	....	+ 3.1
Pi. 0^h130..	0 32.2	−25 19	5.7	G5	1.36	109	+15.5	+18:L	+ 9.0
Boss 131..	0 33.6	+48 48	5.7	K1	0.019	73	− 9.3	....	− 3.5
54 Piscium ..	0 34.2	+20 43	6.1	K1	0.59	87	−33.9	....	−32.9
Boss 138..	0 35.7	+38 55	5.4	G6	0.007	78	− 8.6	....	− 4.4
Lal. 1198..	0 39.9	+ 1 15	8.1	K4	0.63	100	+ 6.9	....	+ 3.4
Boss 165..	0 42.2	+ 6 12	6.2	G6	0.016	96	+14.8	....	+12.7
169..	0 43.1	−22 16	5.3	A1	0.036	110	+19.1	....	+12.3
Groom. 145..	0 43.2	+69 54	8.0	K0	0.44	65	−28.0	....	−19.6
Boss 183..	0 45.2	+50 58	6.5	F5	0.128	73	+ 1.8	....	+ 7.6
198..	0 50.6	+26 40	5.9	a1	0.006	86	− 8.8	....	− 7.4
209..	0 52.4	+28 27	5.6	Kop	0.017	86	0.0	....	+ 1.4
210..	0 52.7	+13 9	6.4	G4	0.022	95	+15.7	....	+14.0
217..	0 54.6	+ 5 57	6.3	Map	0.023	99	−14.5	....	−17.6
Groom. 211..	0 55.6	+44 55	7.0	G4	0.105	78	−71.1	....	−66.9
Boss 223..	0 57.3	+40 48	5.8	A6	0.024	80	+ 3.3	....	+ 6.8
224..	0 57.3	+31 16	5.4	B9	0.039	85	+ 9.7	....	+11.4
Lal. 1799..	0 57.2	+ 4 31	8.0	K6	0.48	100	+20.2	....	+16.7
Boss 252..	1 3.2	+ 5 7	5.5	A4	0.326	101	+ 6.6	....	− 3.8
261..	1 4.5	+19 7	5.6	F3	0.004	94	− 8.5	....	− 9.9
263..	1 5.0	+63 40	5.5	B9	0.038	69	− 6.2	....	+ 1.0
267..	1 5.4	+ 1 55	6.2	K1	0.004	103	− 2.1	....	− 6.6
284..	1 8.8	+15 36	5.7	B2p	0.028	96	−16.1	....	−18.2
295..	1 12.6	+ 3 5	5.4	A2	0.056	104	+ 4.6	....	− 0.2
Lal. 2450..	1 16.9	+18 10	8.1	G0	0.57	96	0	....	− 2
Boss 305..	1 17.5	− 0 58	6.5	K0	0.017	107	+14.9	....	+ 9.1
349..	1 30.5	+72 32	5.5	G5	0.008	67	− 6.7	....	+ 1.1
355..	1 31.6	+57 28	5.7	G7	0.004	76	− 7.6	....	+ 2.8
Lal. 3022..	1 33.9	+27 36	7.8	G7	0.50	94	+57.0	....	+55.6
Boss 375..	1 36.0	+29 32	6.0	G4	0.013	93	+ 5.7	....	+ 4.7
379..	1 36.3	+34 44	5.5	B9	0.053	90	+ 0.2	....	+ 0.2
107 Piscium...	1 37.1	+19 47	5.3	G8	0.72	99	−34.2	....	−37.3
Boss 410..	1 44.6	+21 47	5.9	G9	0.018	99	+ 3.3	....	+ 0.2
414..	1 45.7	+10 33	5.8	F2	0.074	106	+10.6	....	+ 5.1
420..	1 47.3	+40 14	5.6	K1	0.009	88	− 6.4	....	− 5.7
430..	1 50.0	+36 47	6.1	K1	0.005	91	+ 7.0	....	+ 6.7
432..	1 50.2	+36 46	5.8	K0	0.180	91	+59.0	....	+58.7
434..	1 50.3	+23 5	6.0	G8	0.011	100	+14.1	....	+10.7
457..	1 55.6	+63 54	5.7	B8	0.011	74	−20.7	....	−15.2
466..	1 57.1	+32 48	5.5	A1	0.023	94	+ 1.5	....	+ 0.1
472..	1 58.2	+17 46	6.4	K2	0.024	104	+10.4	....	+ 5.6
478..	2 1.7	+57 57	5.9	A4	0.009	78	−36.8	....	−32.6
Lal. 3922..	2 2.5	− 1 5	7.5	G0	0.51	117	−40.7	....	−49.8
Boss 488..	2 4.5	+57 10	6.4	B8	0.013	79	−36.8	....	−33.0
493..	2 5.5	+25 28	6.2	K4	0.011	101	−18.3	....	−22.1

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 498..	2^{h} $6^{m}.6$	+66° 3′	6.2	F4	0.″003	73°	− 12.6	....	− 6.8
508..	2 10.0	+32 54	5.3	A0	0.035	96	− 3.2	....	− 5.3
521..	2 12.6	+19 26	5.5	B9	0.015	106	+ 11.6	....	+ 6.1
526..	2 13.2	+28 11	5.3	A2	0.012	100	+ 4.4	....	+ 0.9
529..	2 14.2	+46 51	6.1	B7	0.005	88	− 0.8	....	− 0.1
536..	2 16.6	+40 57	5.8	F0	0.128	92	− 35.0	....	−35.7
539..	2 16.8	− 0 4	5.6	Map	0.006	119	+ 23.3	....	+13.6
551..	2 21.1	−12 44	4.9	B8	0.027	127	+ 0.8	....	−11.2
572..	2 26.3	+ 1 49	5.4	K4	0.013	120	+ 27.0	....	+17.0
581..	2 29.5	+36 52	5.9	K0	0.010	96	− 5.8	....	− 7.9
616..	2 37.1	+10 19	6.2	A2p	0.040	117	+ 4.7	....	− 4.4
619..	2 37.6	+43 52	5.6	F9	0.004	92	− 3.8	....	− 4.5
648..	2 46.0	+14 40	5.4	B5	0.045	115	+ 16.9	....	+ 8.5
654..	2 47.4	+37 56	5.7	F2	0.10	98	+ 0.6	....	− 2.2
660..	2 50.2	+17 56	5.9	Mc	0.018	113	+ 46.5	....	+38.7
666..	2 51.6	− 4 7	5.1	B9	0.054	128	+ 1.7	....	−10.6
674 N	2 53.5	+20 56	4.7	A3	0.017	112	− 7.7	....	−15.2
674 S	2 53.5	+20 56		A3	0.017	112	− 5.3	....	−12.8
677..	2 53.7	− 3 11	5.1	A2	0.070	128	− 4.4	....	−16.7
W.B. 2^{h}927..	2 55.2	+ 5 36	8.2	G8	0.68	123	+ 66.7	....	+55.8
Boss 707..	3 1.6	− 6 29	5.6	Map	0.002	132	+17.2	....	+ 3.8
719..	3 6.3	+26 53	5.6	A0	0.019	109	+ 12.0	....	+ 5.5
724..	3 8.1	+56 46	5.9	A2	0.004	85	− 14.9	....	−13.2
W.B. 3^{h}113..	3 9.4	+ 8 37	7.8	K0	0.62	123	− 21.0	....	−31.9
Boss 742..	3 11.5	+49 51	5.3	B3	0.045	91	− 0.6	....	− 0.9
757..	3 14.7	+42 58	5.1	A1	0.058	97	− 4.6	+ 2.0L	− 7.0
767..	3 16.1	+48 51	5.4	B4	0.035	92	+ 3.4	....	+ 2.7
768..	3 16.2	+27 15	5.6	G8	0.020	110	+ 6.7	....	− 0.1
790..	3 22.2	+49 10	4.5	B5	0.044	93	− 1.6	....	− 2.6
791..	3 22.4	+55 6	5.1	B9p	0.033	88	+ 1.0	+ 6.0L	+ 1.7
800..	3 24.9	−13 1	5.5	A1	0.017	140	+ 14.7	....	− 0.6
801..	3 24.9	+11 0	5.1	B8p	0.021	124	+ 19.8	....	+ 8.6
802..	3 25.1	+47 41	5.5	B9	0.040	94	− 2.9	....	− 4.3
817..	3 29.4	+47 52	4.2	B7p	0.044	94	+ 0.7	....	− 0.7
832..	3 34.5	+59 39	6.0	G8	0.002	84	− 9.9	....	− 7.8
W.B. 3^{h}617..	3 35.3	− 3 32	7.2	F5	0.78	137	+113.7	....	+99.1
Boss 838..	3 35.8	+47 28	3.1	B5	0.046	96	+ 0.7	....	− 1.4
845..	3 38.0	+36 9	5.6	A2	0.055	105	+ 21.8	....	+16.6
849..	3 38.8	−10 48	5.7	A3	0.020	142	+ 16.2	....	+ 0.4
B.D. 23°535..	3 41.4	+23 25	7.9	F3	....	116	− 10.5	....	−19.3
Boss 896..	3 48.6	+62 47	5.0	B9	0.005	83	+ 4	+ 5:L	+ 6
898..	3 48.8	+47 35	5.3	B5	0.039	96	+ 9.9	....	+ 7.8
933..	3 58.4	+23 50	5.6	F8p	0.022	118	+ 18.3	....	+ 8.9
956..	4 4.8	−16 39	5.3	B5	0.013	151	+ 13.3	....	− 4.2
960..	4 5.5	− 7 11	5.6	G6	0.010	144	− 12.1	....	−28.3
977..	4 9.6	+80 35	5.6	G7	0.020	67	− 8.6	....	− 0.8
989..	4 11.4	+20 20	4.9	A3	0.066	123	+ 15.8	+16.8L	+ 4.9
997..	4 13.5	+20 54	5.4	B9	0.063	122	+ 13.8	....	+ 3.2
1014..	4 16.5	+20 35	6.1	G9	0.011	123	− 8.5	....	−19.4
1024..	4 18.7	− 3 59	5.3	A0	0.072	144	− 0.3	....	−16.5
1039..	4 21.3	+22 46	5.4	B5	0.019	121	+ 10.5	....	+ 0.2
1064..	4 27.0	+64 3	5.8	A2	0.029	83	− 15.7	....	−13.3
1069..	4 28.8	+ 5 22	5.7	A2	0.028	138	− 7.2	....	−22.1
1084..	4 32.1	+ 0 48	5.3	B5	0.018	142	+ 22.6	....	+ 6.8

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 1088..	$4^{h}32^{m}.6$	− 2°40′	5.3	A5	$0''.067$	146°	+ 18.2	....	+ 1.6
1089..	4 33.4	+15 36	5.2	A2	0.074	129	+23.0	....	+10.4
1093..	4 33.9	+48 6	5.7	A0	0.052	99	+22.8	....	+19.7
1097..	4 34.5	+12 0	5.3	B9	0.027	133	+ 18.1	....	+ 4.5
1103..	4 35.8	+43 10	5.4	A0	0.067	104	+ 5.6	....	+ 0.8
1128..	4 42.7	+63 20	5.8	Ma	0.111	85	−35.5	....	−33.8
1136..	4 44.0	+15 44	6.3	K0	0.017	130	+13.5	....	+ 0.6
1146..	4 45.7	+42 25	5.6	A1	0.013	105	− 2.4	....	− 7.6
1163..	4 49.4	+10 0	4.7	B9	0.145	136	+17.0	....	+ 2.9
B.D. 35°930..	4 49.7	+36 1	6.2	B3	....	111	−11	....	−18
Boss 1165..	4 50.1	+14 53	5.6	B9	0.025	131	+ 9	....	− 4
1176..	4 51.8	+53 0	6.4	K3	0.012	95	− 1.8	....	− 3.5
1182..	4 53.4	+39 15	6.0	F1	0.013	109	+ 5.7	....	− 0.8
1183..	4 53.5	+39 30	6.9	K5	0.010	108	−23.4	....	−29.6
W.B. $4^{h}1189$..	4 55.9	− 5 52	6.5	K9	1.25	151	+20.1	+31:L	+ 2.6
Boss 1195..	4 57.4	+58 50	5.4	B3p	0.007	89	−13.2	....	−12.9
1221..	5 2.9	+19 44	6.6	G3	0.018	127	+ 6.0	....	− 6.0
1234..	5 5.9	+15 55	5.4	K6	0.004	131	− 7.0	....	−20.1
1247..	5 9.4	+ 5 2	5.8	K1	0.017	142	− 7.0	....	−22.8
1268..	5 13.4	+33 51	5.1	A4	0.016	114	− 6.0	....	−14.1
1281..	5 16.2	−21 20	4.7	A0	0.020	166	+29.2	+31.5L	+ 9.8
1295..	5 18.6	− 0 15	5.5	B3	0.015	148	+22.4	....	+ 5.4
1309..	5 20.7	+62 59	5.8	K1p	0.005	86	−18.0	....	−16.6
1310..	5 20.7	+30 7	5.7	B9	0.019	118	+13.6	....	+ 4.2
1318..	5 22.0	+15 47	5.5	B8	0.031	132	+13.6	....	+ 0.2
1332..	5 26.0	+ 3 13	5.6	B3	0.009	145	+21.6	....	+ 5.2
1334..	5 26.3	+74 59	6.4	K5	0.019	74	− 2.5	....	+ 3.0
1348..	5 28.4	+54 22	6.0	K2	0.007	95	+ 1.4	....	− 0.3
1354..	5 29.3	+23 58	5.1	B3	0.032	125	+23.2	....	+11.7
Groom. 990..	5 30.4	+51 23	8.1	K0	0.56	98	−43.8	....	−46.6
Boss 1380..	5 32.4	+65 39	5.8	K3	0.024	83	−18.6	....	−16.2
Pi. $5^{h}146$..	5 33.2	+53 26	6.4	K0	0.55	96	+ 1.7	....	− 0.4
Boss 1394..	5 34.9	+61 26	6.4	G5	0.002	88	− 3.1	....	− 2.4
Lal. 10797..	5 39.2	+37 15	7.3	K2	0.72	112	−30.4	....	−37.9
Boss 1424..	5 41.6	+17 41	5.3	F0	0.010	131	+ 6.6	....	− 6.5
1441..	5 44.5	+ 9 50	5.9	G4	0.014	139	+43.7	....	+28.6
1444..	5 44.7	+27 56	5.6	G8	0.009	121	+ 8.1	....	− 2.2
1453..	5 46.5	+55 41	4.8	A2	0.014	93	−15.6	−13.6L	−16.6
1479..	5 52.5	+45 56	4.6	Mbp	0.011	103	+ 0.9	+ 1.3L	− 3.6
1513..	5 59.6	+ 5 26	5.8	G4	0.007	144	+20.4	....	+ 4.2
1514..	5 59.7	+ 4 10	5.7	G3	0.012	144	+32.7	....	+16.5
1523..	6 1.7	− 4 11	5.4	B6p	0.007	153	+16.1	....	− 1.7
1560..	6 8.7	+61 33	5.3	Map	0.005	87	+13.9	....	+14.9
1568..	6 9.7	+16 10	5.2	B9	0.021	133	+29.5	....	+15.9
1572..	6 10.1	+12 35	5.4	B9	0.017	136	+12.6	....	− 1.8
1573..	6 10.2	+24 0	6.1	G4	0.029	125	−20.4	....	−31.9
1575..	6 10.8	+59 3	4.5	A1	0.022	90	− 2.2	−3L, −6Y	− 2.2
1578..	6 10.9	+23 46	6.3	B6	0.014	125	+11.6	....	+ 0.1
1599..	6 15.0	− 2 54	5.2	Ma	0.276	152	+48.3	....	+30.6
1608..	6 18.1	+58 28	5.5	K3	0.011	91	− 4.6	....	− 4.9
1627..	6 22.1	+58 14	6.0	G7	0.33	91	+35.7	....	+35.4
1632..	6 22.6	+46 45	6.0	K3	0.007	103	−46.8	....	−51.3
1643..	6 24.9	+78 5	5.9	K6	0.019	71	−13.9	....	− 7.4

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 1672..	6h29m.1	+56° 56′	5.8	A0	0″.013	92°	+ 0.4	....	− 0.3
23 Hev. Cam..	6 29.2	+79 40	5.6	A2	0.64	69	+12.0	....	+19.2
Boss 1704..	6 35.0	+28 17	6.5	G1	0.017	121	− 3.8	....	−14.1
1739..	6 42.3	−14 19	5.3	B8	0.012	161	+17.3	....	− 1.6
1751..	6 44.1	+16 19	5.8	B9p	0.023	132	+12.8	....	− 0.6
1756..	6 44.8	+13 32	5.9	G9	0.013	134	+26.8	....	+12.9
97 Monoc.....	6 45.7	− 0 25	5.8	A5	0.20	148	−17.2	....	−34.2
Boss 1788..	6 50.9	+10 5	6.0	B8	0.027	137	+33.0	....	+18.4
Lal. 13427..	6 54.0	+48 32	8.2	K1	0.71	100	−22.3	....	−25.8
Boss 1846..	7 5.6	+51 36	5.7	Map	0.019	96	−49.3	....	−51.4
1868..	7 9.7	+28 4	5.9	K3	0.020	119	+22.2	....	+12.5
1873..	7 10.2	+ 0 1	6.5	G4	0.019	145	− 9.6	....	−26.0
Lal. 14146..	7 11.3	−12 53	7.3	F9	0.56	156	+57.0	....	+38.7
Boss 1894..	7 13.5	+60 5	6.3	A7	0.011	88	+ 6.2	....	+ 6.9
1897..	7 14.1	+45 25	5.7	A4	0.036	102	+24.6	....	+20.4
1916..	7 16.5	+81 6	6.5	G5	0.004	68	− 1.5	....	+ 6.0
1926..	7 18.3	+27 50	5.7	F0	0.020	118	− 5.0	....	−14.4
1930..	7 19.4	+11 52	5.3	A2	0.028	133	+ 6.3	....	− 7.3
1935..	7 20.1	−16 0	5.1	B5p	0.034	156	− 7.6	....	−25.9
1956..	7 23.1	+28 19	5.0	A2	0.067	118	+42.8	+28:A	+33.4
2020..	7 36.4	+14 27	5.8	Mbp	0.015	130	−15.6	....	−28.5
28 Hev. Cam..	7 39.8	+80 31	6.5	G6	0.49	68	− 7.8	....	− 0.3
Boss 2054..	7 42.6	+23 23	6.2	F1	0.020	121	− 4.9	....	−15.2
2144..	8 0.4	+22 55	6.2	Map	0.024	119	+26.7	....	+17.0
2148..	8 2.5	+42 43	6.4	K2	0.07	101	+38.4	....	+34.6
2150..	8 2.9	+68 46	5.5	G3	0.006	78	− 9.5	....	− 5.3
2159..	8 4.9	−15 57	5.5	B3	0.014	148	+32.7	....	+15.7
2178..	8 7.4	+60 41	6.4	A2	0.017	85	−16.2	....	−14.5
2203..	8 14.3	+60 57	6.5	G8	0.008	84	− 5.4	....	− 3.3
2220..	8 17.9	+42 20	6.2	K1p	0.011	101	+27.1	....	+23.3
2236..	8 20.6	+45 59	6.3	G1	0.021	126	−34.3	....	−36.7
2245..	8 21.2	+12 59	5.8	Map	0.116	124	− 7.0	....	−18.2
2246..	8 21.5	− 3 40	5.5	A6	0.070	137	+25.5	....	+10.9
2293..	8 31.9	+53 4	6.0	K3	0.04	90	+26.6	....	+26.6
Lal. 16904..	8 33.1	+56 2	8.1	G3	0.44	87	+36.8	....	+37.8
Boss 2335..	8 38.8	− 6 52	4.6	G0	0.007	136	+30.3	....	+15.9
2338..	8 39.2	+31 4	6.1	G3	0.020	107	−12.4	....	−18.2
2357..	8 42.2	− 1 32	5.2	B9	0.046	132	+ 1.6	....	−11.8
2378..	8 46.5	+28 38	6.3	Mbp	0.022	108	+11.8	....	+ 5.6
2400..	8 51.7	+15 42	5.1	A3	0.062	118	− 0.8	....	−10.2
2407..	8 53.0	+12 15	4.1	A3	0.054	121	− 8.3	−15L, A	−18.6
2410..	8 53.5	+18 31	6.6	Mbp	0.089	115	+21.7	....	+13.3
2413..	8 54.2	+42 11	4.1	F5	0.504	97	+23.9	+27.3L	+21.5
2449..	9 3.6	+22 27	5.2	G6	0.008	111	− 6.7	....	−13.9
2455..	9 4.6	+22 24	6.1	G5	0.010	111	− 6.6	....	−13.8
2461..	9 5.8	+73 22	6.0	A3	0.100	71	+ 1.9	....	+ 8.4
81 π Cancri...	9 6.9	+15 24	6.4	G5	0.58	116	+45.9	....	+37.1
Boss 2465..	9 7.3	+43 38	5.4	A0p	0.052	94	+26.6	+16A	+25.2
2490..	9 11.7	− 5 56	5.5	K5	0.004	129	− 7.3	....	−19.9
2492..	9 11.8	− 8 20	5.5	B9	0.040	130	+11.0	....	− 1.9
2584..	9 32.1	+40 41	5.2	A1	0.020	93	− 5.8	....	− 6.8
2598..	9 35.5	+79 36	6.2	A5	0.029	65	− 6.5	....	+ 1.9
Lal. 19022..	9 37.1	+43 10	8.2	K5	0.80	91	−12.9	....	−13.2
Boss 2612..	9 38.3	+14 29	5.6	Map	0.014	111	+ 8.2	....	+ 1.0

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 2614..	$9^h39^m.4$	+57° 35′	5.4	Mbp	0″.025	80°	+ 9.4	....	+12.9
Lal. 19229..	9 43.5	+14 14	8.4	A2p	0.83	110	−23.2	....	−30.0
Boss 2647..	9 47.0	+ 2 55	5.9	A2	0.204	116	+95.8	....	+87.0
2650..	9 47.6	− 7 38	5.1	A1	0.076	122	+15.4	....	+ 4.8
2701..	10 5.0	+41 9	6.5	K0	0.017	88	+13.4	....	+14.1
Lal. 19896..	10 8.9	+ 3 39	7.7	G0	0.47	111	−24.7	....	−31.9
Boss 2726..	10 10.8	+65 36	5.8	A3	0.090	72	− 1.8	....	+ 4.4
2737..	10 13.4	+69 15	5.9	A2	0.065	70	+ 3.9	....	+10.7
2745..	10 15.2	+84 46	5.7	A2	0.130	62	+10.0	....	+19.4
2756..	10 17.3	+34 25	5.9	A1	0.027	91	−12.7	....	−13.0
2773..	10 21.5	+42 7	5.9	A1	0.102	86	+ 5.2	....	+ 6.6
2787..	10 24.3	+39 26	5.9	A1	0.011	88	+ 8.2	....	+ 8.9
2795..	10 25.7	+81 1	6.6	G6	0.016	63	−11.5	....	− 2.4
2808..	10 27.8	+35 30	5.6	A1	0.040	88	+ 2.6	....	+ 3.3
2813..	10 28.7	+57 36	5.2	A8	0.074	75	− 9.2	....	− 4.0
2819..	10 30.6	+36 51	6.2	A6	0.050	87	−24.2	....	−23.2
2822..	10 31.6	−11 42	5.7	F1	0.67	115	− 9.0	....	−17.4
2829..	10 33.1	+32 30	4.8	G0	0.008	90	− 7.6	− 6.3L	− 7.6
2887..	10 44.4	+28 30	6.1	A7	0.021	89	+ 2.4	....	+ 2.7
2909..	10 50.2	+25 17	4.3	A3	0.077	91	+ 6.1	....	+ 5.8
2910..	10 50.2	+34 2	5.9	G8	0.15	86	−23.4	....	−22.0
2915..	10 50.8	+ 6 43	6.0	Mcp	0.023	101	−12.3	....	−16.1
2921..	10 54.0	+36 38	6.2	Ma	0.095	83	−22.7	....	−20.3
Lal. 21258..	11 0.5	+44 2	8.9	Ma	4.46	78	+65	....	+69
Boss 2976..	11 9.9	+23 38	4.9	Map	0.018	88	+15.2	+16.0L	+15.9
2983..	11 12.1	+ 2 34	5.4	K8	0.159	99	−57.5	....	−60.6
2987..	11 13.7	+38 44	4.8	A0	0.102	80	+ 5.7	....	+ 9.2
83 Leonis Br..	11 21.7	+ 3 33	6.5	G8	0.76	96	− 2.4	....	− 4.5
83 Leonis Ft..	11 21.7	+ 3 33	7.6	K8	0.76	96	+ 2.2	....	+ 0.1
Boss 3045..	11 29.5	+17 21	5.8	B3	0.011	88	+17.8	....	+18.5
3055..	11 31.6	− 9 15	5.0	B9	0.061	100	− 8.0	....	−11.5
3063..	11 33.0	+44 11	5.6	A3	0.154	74	+ 5.6	....	+11.1
3067..	11 33.3	+ 8 41	5.5	Mb	0.013	91	+ 1.4	....	+ 0.9
Groom. 1822..	11 40.3	+48 14	7.9	G0	0.67	70	+23.9	....	+30.7
Boss 3089..	11 40.7	+ 7 5	4.2	Map	0.188	90	+51.2	+51.2L	+51.2
3125..	11 50.8	+57 9	5.9	G9	0.012	66	+13.1	....	+21.2
3137..	11 55.6	− 9 53	6.4	G6	0.47	96	0.0	....	− 2.1
3143..	11 57.0	+43 36	4.9	A5	0.323	69	+ 8.2	....	+15.4
3150..	11 59.2	+22 1	5.9	A5	0.044	79	+ 4.3	....	+ 8.1
3177..	12 6.5	+82 16	6.3	K4	0.019	59	−26.0	....	−15.7
3178..	12 6.6	+ 4 37	(7.2)	A8	0.006	86	− 6.0	....	− 4.6
W.B. 12^h69..	12 7.4	− 2 32	7.3	G4	0.74	90	+11.3	....	+11.3
Lal. 22908..	12 8.1	+11 24	7.5	G3	0.59	82	−30.0	....	−27.2
Boss 3183..	12 8.3	+10 49	5.8	A8	0.096	82	+ 3.4	....	+ 6.2
3193..	12 11.1	+41 13	5.7	K3	0.051	68	−14.4	....	− 6.9
3206..	12 14.3	+23 35	6.1	A7	0.038	75	+ 1.2	....	+ 6.4
3215..	12 15.4	−21 40	5.4	B8	0.092	98	−20.9	....	−23.7
3217..	12 15.8	−13 1	5.5	K2	0.002	93	+12.5	....	+11.5
3234..	12 20.3	+57 20	6.0	Map	0.028	62	−16.7	....	− 7.3
3248..	12 22.8	+56 16	5.8	Ma	0.031	62	+17.6	....	+27.0
3266..	12 26.0	+25 7	5.4	A2	0.020	71	− 0.5	....	+ 6.0
3290..	12 31.6	− 5 17	5.9	A0	0.040	84	− 3.6	....	− 1.5
3294..	12 33.3	+ 2 24	6.0	Mb	0.090	81	−15.0	....	−11.9
3309..	12 36.8	+10 47	4.8	B8	0.135	76	+ 3.5	....	+ 8.3

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 3310..	$12^h36^m.9$	+7° 21′	5.4	A0	0″.078	78°	+ 2.2	....	+ 6.4
3331..	12 42.8	+ 4 7	6.7	Ma	0.013	78	+ 8.7	....	+ 12.9
3332..	12 43.0	+63 20	5.8	A4	0.022	58	− 18.0	....	− 7.4
3334..	12 43.2	+14 6	6.4	A0	0.068	73	− 0.2	....	+ 5.6
3336..	12 43.5	+67 20	5.6	G5	0.008	58	+ 9.0	....	+ 19.6
3337..	12 43.9	+14 40	5.7	A0	0.051	74	− 7.0	....	− 1.5
3338..	12 44.1	+49 1	6.1	A5	0.055	60	− 2.4	....	+ 7.6
3339..	12 44.4	+28 6	5.7	A0	0.095	68	+ 1.4	....	+ 8.9
3348..	12 47.2	+17 37	6.5	G9	0.023	71	− 0.4	....	+ 6.1
3360..	12 48.8	+12 58	6.2	A3	0.067	73	− 3.8	....	+ 2.0
3367..	12 50.6	+ 3 56	3.7	Mbp	0.479	77	− 17.4	−17.6L	− 12.9
3382..	12 56.4	+56 54	4.9	A2p	0.101	57	− 3.1	....	+ 7.8
3406..	13 4.2	+10 33	6.0	G7	0.020	70	+ 0.1	....	+ 6.9
3408..	13 4.5	− 9 48	6.2	K6	0.023	81	− 6.8	....	− 3.7
3442..	13 11.5	+81 0	6.3	G5	0.008	56	− 10.0	....	+ 1.2
3462..	13 17.1	+ 5 41	5.8	A1	0.080	70	− 11.4	....	− 4.6
3478..	13 20.3	+24 23	5.8	A1	0.014	61	+ 1.6	....	+ 11.3
Lal. 25012..	13 26.6	− 1 49	7.5	G6	0.94	72	− 53.9	....	− 47.7
Boss 3499..	13 26.8	− 5 44	4.9	K5	0.112	75	− 19.1	−19.2L	− 13.9
3506..	13 29.1	+ 4 10	5.0	A2p	0.052	68	− 8.4	− 6:L	− 0.9
3534..	13 36.4	− 8 12	6.2	Ma	0.018	74	− 32.7	....	− 27.2
3542..	13 39.1	−15 41	5.7	F8	0.009	78	+ 0.2	....	+ 4.4
3580..	13 46.7	+35 16	6.6	A2	0.033	53	− 12.3	....	− 0.3
3585..	13 47.4	+12 40	5.9	A1	0.038	60	− 17.1	....	− 7.1
3589..	13 48.5	+65 13	4.8	Map	0.004	51	− 10.4	− 9.9L	+ 2.2
3629..	14 3.7	− 9 52	6.5	G8	0.016	70	− 19.6	....	− 12.8
3653..	14 9.9	−17 44	5.5	B9	0.044	74	− 15.7	....	− 10.2
3654..	14 9.9	+52 15	4.4	A5	0.067	46	− 15.9	−19:L	− 2.0
3663..	14 11.4	+19 23	5.9	A7	0.055	52	+ 6.3	....	+ 18.6
Lal. 26196..	14 14.4	− 4 41	7.6	K1	0.68	64	− 13.0	....	− 4.2
Boss 3684..	14 15.7	+39 15	6.0	A0	0.030	46	− 12.6	....	+ 1.3
3703..	14 21.4	+38 51	6.3	K1	0.019	45	+ 25.8	....	+ 39.9
3706..	14 22.2	− 5 40	6.1	A1	0.098	63	− 15.6	....	− 6.5
3734..	14 31.7	−11 53	6.0	F5	0.97	66	− 70.5	....	− 62.4
3743..	14 35.1	+54 27	5.5	A1	0.027	43	+ 4.2	....	+ 18.8
3756..	14 37.4	−24 34	5.6	B9	0.026	74	− 4.2	....	+ 1.3
Lal. 27298..	14 52.4	+54 4	7.9	K0	1.08	40	− 14.4	....	+ 0.9
A.Oe. 14320..	15 4.7	−15 54	9.2	G0	3.76	63	+290	....	+299
Boss 3867..	15 7.5	+19 21	6.0	Mbp	0.004	40	− 34.2	....	− 18.9
3875..	15 8.9	−17 24	6.3	B8	0.029	63	− 25.9	....	− 16.8
3883..	15 10.3	+29 32	5.2	A2	0.078	36	− 18.8	....	− 2.6
3885..	15 10.7	+ 0 45	5.7	A2	0.109	50	− 5.5	....	+ 7.4
3893..	15 13.5	+67 44	5.2	F3	0.459	44	− 45.4	....	− 31.0
3918..	15 18.6	− 0 40	6.0	A3	0.077	50	− 2.7	....	+ 10.2
3942..	15 25.0	−16 16	5.9	K0	0.019	60	− 1.2	....	+ 8.8
3955..	15 29.0	− 8 51	5.1	B7p	0.029	54	− 4.3	....	+ 7.5
3985..	15 35.1	+47 8	5.8	A8	0.162	32	− 2.3	....	+ 14.7
4007..	15 40.4	+ 5 46	5.5	A2	0.033	41	− 7.3	....	+ 7.8
4022..	15 45.2	+55 41	5.9	F2	0.010	34	− 1.9	....	+ 14.7
4026..	15 45.8	+ 4 47	3.8	A6	0.136	40	− 10.6	−10.0L	+ 4.7
39 Serpentis..	15 48.5	+13 31	6.2	F8	0.56	34	+ 38.7	....	+ 55.3
Boss 4070..	15 55.4	− 8 8	5.4	A0	0.034	49	− 19.4	....	− 6.3
4096..	16 2.0	−26 4	5.6	Map	0.122	63	− 21.2	....	− 12.1
4103..	16 3.6	+ 8 48	5.9	Mbp	0.024	34	− 21.6	....	− 5.0

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 4119..	16^h 6^{m}5	− 9°48′	5.1	A2	0″027	48°	− 1.7	− 8.9L	+ 11.7
4120..	16 6.7	− 8 17	5.5	A2	0.041	47	− 6.4	....	+ 7.2
4122..	16 7.0	+16 55	5.9	A2	0.014	29	− 12.6	....	+ 4.9
4125..	16 7.4	+23 45	6.0	Mbp	0.028	26	− 24.2	....	− 6.2
4131..	16 8.3	−11 35	5.5	K6	0.020	50	− 26.3	....	− 13.4
4134..	16 9.1	− 3 26	3.0	Map	0.161	43	− 17.5	−19.5L	− 2.9
4137..	16 10.2	− 8 6	5.9	G1	0.53	47	+ 11.2	....	+ 24.8
4138 Ft.	16 10.9	+34 7	6.8	F5	0.29	23	− 18.1	....	+ 0.3
4146..	16 12.7	+29 24	5.6	A1	0.028	24	+ 6.4	....	+ 24.7
4159..	16 15.6	+60 0	5.6	Map	0.022	34	− 35.2	....	− 18.6
4182..	16 20.8	+14 16	4.4	A0	0.079	28	− 2.4	− 6.1L	+ 15.3
4188..	16 22.3	− 7 22	5.4	Map	0.176	44	+ 97.1	....	+111.5
B.D. 51°2097..	16 23.0	+51 22	7.5	A1	0.109	27	− 17.6	....	+ 0.2
Boss 4210..	16 27.7	+ 5 44	5.5	A0	0.023	32	− 26.9	....	− 9.9
4221..	16 31.0	+61 2	5.8	A1	0.021	34	− 11.6	....	+ 5.0
Lal. 30271..	16 32.6	+31 9	7.2	F2	0.46	18	− 8.0	....	+ 11.0
Boss 4228..	16 33.3	+46 49	6.0	G9	0.020	23	− 16.8	....	+ 1.6
4229..	16 33.8	+53 6	5.7	A0	0.026	28	− 7.0	....	+ 10.7
4291..	16 47.5	+15 9	6.3	B9	0.011	23	− 22.8	....	− 4.4
4300..	16 49.2	+31 52	5.3	A6	0.098	15	− 21.4	....	− 2.1
4303..	16 50.3	−16 39	6.5	G8	0.089	51	− 2.2	....	+ 10.4
4316..	16 53.4	+25 30	6.7	G5	0.009	20	+ 9.1	....	+ 27.9
4332..	16 57.9	+33 43	5.3	A5	0.012	13	− 13.2	....	+ 6.3
Lal. 31055..	16 59.8	− 4 54	7.9	K5	1.47	38	+ 27.5	....	+ 43.3
Boss 4395..	17 15.2	−12 45	4.3	A1	0.036	45	+ 3.8	+ 8L	+ 17.9
4418..	17 20.0	+16 24	5.7	A8	0.044	17	+ 10.8	....	+ 29.9
W.B. 17^{h}322..	17 20.8	+ 2 14	7.8	K9	1.36	30	− 27.8	....	− 10.5
Boss 4427..	17 22.5	+20 10	5.4	B6	0.016	13	− 28.2	....	− 8.7
4438..	17 26.7	+26 11	4.5	K2	0.021	9	− 25.2	−26.1L	− 5.4
Lal. 31905..	17 26.5	+ 1 45	7.2	K4	....	30	− 15.4	....	+ 1.9
W.B. 17^{h}514..	17 29.9	+ 6 4	8.6	F1	0.58	26	−148	....	−131
Boss 4461..	17 31.7	+21 4	5.8	A4	0.028	11	− 17.5	....	+ 2.1
4494..	17 41.9	+53 51	5.6	A0	0.043	23	+ 1.8	....	+ 20.2
4514..	17 47.4	+48 25	6.4	B2p	0.008	17	− 16.3	....	+ 2.8
4530..	17 51.6	+22 29	5.7	K4	0.002	8	− 43.1	....	− 23.3
4546..	17 55.6	−17 9	6.3	K0	0.012	48	− 21.5	....	− 8.1
4547..	17 55.6	+16 45	4.7	G8	0.012	14	− 24.4	−21.5L	− 5.0
4554..	17 56.9	+72 0	5.5	F2	0.009	41	− 5.9	....	+ 9.2
4592..	18 4.6	+20 2	5.2	A3	0.027	11	− 16.1	....	+ 3.5
Lal. 33439..	18 6.3	+38 27	6.7	K2	0.65	8	− 17.7	....	+ 2.1
Boss 4601..	18 6.5	+36 27	5.9	G7	0.009	6	− 25.9	....	− 6.0
4620..	18 12.5	+42 8	5.2	B8	0.009	12	− 20.4	....	− 0.8
4629..	18 15.1	+24 24	5.5	K0	0.016	8	+ 0.5	....	+ 20.3
4630..	18 15.4	−24 58	6.4	Mbp	0.011	56	+ 3.9	....	+ 15.1
B.D. 8°3689..	18 21.4	+ 8 44	7.7	G1	0.50	23	− 22.5	....	− 4.1
Boss 4668..	18 22.1	−17 52	6.0	B8	0.007	49	− 34.9	....	− 21.8
4685..	18 25.6	−18 28	5.2	B8	0.035	49	− 37.8	....	− 24.7
4686..	18 25.7	+65 30	5.0	G9	0.104	35	+ 30.0	+33.4L	+ 46.4
4702..	18 29.0	+30 29	5.4	B9	0.007	7	− 11.2	....	+ 8.7
4707..	18 30.9	+56 58	5.1	F8	0.011	27	− 9.9	−10.4L	+ 7.9
4719..	18 32.5	− 0 24	5.8	A3	0.026	32	+ 13.3	....	+ 30.3
4724..	18 34.6	+77 28	5.8	K0	0.006	46	+ 1.6	....	+ 15.5
4740..	18 39.8	+ 1 57	4.9	B7	0.022	30	− 2.2	....	+ 15.1
4748..	18 41.0	+39 34	6.7	A0	0.06	12	− 37.1	....	− 17.5

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 4750..	$18^{h}41^{m}2$	−10° 14′	5.8	F5	0″005	42°	+ 9.8	....	+ 24.7
4758..	18 42.0	+26 33	4.9	K0	0.029	10	− 17.2	−16.6L	+ 2.5
4763..	18 43.1	+60 57	6.2	G5	0.010	31	− 24.2	....	− 7.1
Lal. 34986..	18 43.8	+10 39	8.1	K5	0.45	23	− 17.4	....	+ 1.0
Boss 4772..	18 46.0	+32 42	5.8	B3	0.014	10	− 16.1	....	+ 3.6
4780..	18 48.0	−21 29	5.8	G3	0.012	53	− 3.8	....	+ 8.2
4783..	18 49.0	−15 44	5.0	B7	0.023	48	− 6.3	....	+ 7.1
4805..	18 51.7	+41 28	5.6	G5	0.007	15	− 8.2	....	+ 11.1
4816..	18 53.8	−12 59	5.4	B8p	0.025	46	− 12.5	....	+ 1.4
4842..	18 57.7	+50 23	5.1	B3	0.025	22	− 19.3	....	− 0.8
4866..	19 2.5	+24 6	5.6	A4	0.056	15	− 23.0	....	− 3.7
4873..	19 3.7	+35 57	5.2	B6	0.009	14	− 30.5	....	− 11.1
4898..	19 10.8	+14 55	5.6	K1	0.020	23	− 22.4	....	− 4.0
4899..	19 11.0	+21 3	5.5	A2	0.044	19	− 23.9	....	− 5.0
4910..	19 12.7	+49 54	6.3	G6	0.014	24	+ 6.5	....	+ 24.8
4912..	19 12.9	+37 57	4.5	G8	0.010	17	− 30.7	−30.3L	− 11.6
4914..	19 13.1	+11 25	5.1	A3	0.011	26	− 16.2	....	+ 1.8
4917..	19 13.5	+22 51	5.2	B3	0.014	18	0.0	....	+ 19.0
4919..	19 13.7	+ 0 9	6.3	K1	0.009	36	− 28.0	....	− 11.8
4942..	19 18.8	+26 4	4.8	B8p	0.015	18	− 12.2	−12.2L	+ 6.8
4967..	19 22.1	+19 42	6.1	K8	0.057	22	− 35.1	....	− 16.6
4974..	19 24.0	+ 1 45	5.7	B8	0.037	35	+ 16.8	....	+ 33.2
4976..	19 24.5	+24 28	4.6	K5	0.170	20	− 86.6	−85.0L	− 67.8
4978..	19 24.8	+24 34	6.0	G6	0.012	20	− 26.4	....	− 7.6
5024..	19 36.2	+42 35	5.4	A0	0.032	23	− 38.8	....	− 20.4
5044..	19 40.5	−20 0	5.1	G9	0.165	56	+ 16.5	+23L	+ 27.7
5063..	19 45.9	+38 27	6.2	G2	0.016	23	+ 10.6	....	+ 29.0
5073..	19 47.9	+18 25	6.3	B2p	0.007	28	− 10.9	....	+ 6.8
5088..	19 49.2	− 8 30	6.5	B7	0.029	48	− 13.4	....	0.0
5096..	19 51.2	+36 44	5.8	F5	0.008	24	− 24.2	....	− 5.9
5122..	19 54.7	+30 43	5.4	B8	0.031	25	− 0.2	....	+ 17.9
5125..	19 55.5	+17 15	5.6	Map	0.015	30	− 16.9	....	+ 0.4
5134..	19 57.8	+24 39	5.2	A3p	0.099	27	− 33.8	....	− 16.0
5142..	19 59.2	− 0 59	5.8	K2	0.119	43	+ 0.8	....	+ 15.4
B.D. 36°3883..	20 3.5	+36 16	(7.1)	Map	......	26	− 32.0	....	− 14.0
Boss 5177..	20 7.6	+26 31	5.8	G9	0.020	28	− 22.2	....	− 4.5
5188..	20 10.8	+36 30	5.1	B9	0.094	28	− 13.4	−22:L	+ 4.3
5213..	20 14.8	+34 40	5.2	F0	0.017	29	− 4.0	....	+ 13.5
5218..	20 15.9	+55 5	5.8	A2	0.021	34	− 2.8	....	+ 13.8
5220..	20 16.6	+39 5	6.1	A0	0.022	29	0.0	....	+ 17.5
5224..	20 17.8	+24 8	5.4	B8p	0.018	31	− 9.1	....	+ 8.0
A.Oe. 20452..	20 17.7	−21 40	8.1	G0p	1.21	62	−179	....	−170
Boss 5240..	20 21.6	−18 32	5.1	B8p	0.016	61	− 18.4	....	− 8.7
5258..	20 25.5	+36 7	5.9	A1	0.014	31	− 18.0	....	− 0.9
5267..	20 27.2	+36 36	6.3	F7	0.013	31	− 22.7	....	− 5.6
5284..	20 31.5	− 2 54	5.2	K2	0.011	50	− 10.9	....	+ 2.0
5296..	20 33.5	+31 10	6.4	A3	0.069	33	+ 1.8	....	+ 18.6
5301..	20 34.1	+20 51	4.7	A1	0.058	36	− 19.2	−15L	− 3.0
5307..	20 34.4	+15 29	5.9	B3	0.025	39	− 2.0	....	+ 13.5
5316..	20 36.0	+45 19	6.5	B6	0.010	34	− 15.1	....	+ 1.5
5317..	20 36.6	+14 14	6.2	K2	0.012	39	− 31.1	....	− 15.6
5319..	20 37.0	+31 57	5.7	G7	0.018	33	− 28.0	....	− 11.2
5325..	20 39.1	+49 59	5.4	B3	0.006	36	− 2.6	....	+ 13.6
5366..	20 46.9	− 5 53	5.5	A0p	0.011	54	− 4.2	....	+ 7.6

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v′
Boss 5373..	$20^{\mathrm{h}}47^{\mathrm{m}}.8$	$+26^{\circ}43'$	4.8	G8	$0''.103$	37°	+ 3.3	− 0.2L	+19.3
Lal. 29208..	20 50.6	+40 19	6.5	B8p	0.025	36	−21.3	....	− 5.1
Fed. 3638..	20 52.4	+74 23	7.8	G4	0.70	49	−30.4	....	−17.3
Boss 5389..	20 52.5	+47 2	5.7	B8	0.006	37	−15.3	....	+ 0.7
5397..	20 53.8	+21 56	5.6	K5	0.005	40	−27.3	....	−12.0
5417..	20 58.7	−20 15	5.0	A2	0.060	67	+23.7	+26L	+31.5
5420..	20 59.2	+38 16	6.2	G7	0.012	37	− 2.5	....	+13.5
W.B.20^{h}1454..	20 59.1	+ 2 36	8.0	F3	0.56	51	− 9.3	....	+ 3.3
Boss 5422..	20 59.6	+ 5 6	5.9	K6	0.019	50	−15.2	....	− 2.3
5432..	21 2.3	+30 47	5.9	F5	0.010	39	− 6.0	....	+ 9.5
B.D. 38°4362..	21 5.2	+38 19	7.9	K0	....	39	− 8.1	....	+ 7.4
Boss 5456..	21 9.9	−21 4	5.4	G8	0.007	69	− 6.9	....	+ 0.3
5481..	21 16.5	+58 12	5.8	F8	0.014	43	−21.1	....	− 6.5
5486..	21 16.8	+76 35	6.2	K2	0.020	51	+15.5	....	+28.1
Lal. 30218..	21 18.5	+13 36	(6.6)	B6p	....	49	− 9.0	....	+ 4.1
Boss 5498..	21 19.5	+23 51	5.8	A6	0.124	45	−18.9	....	− 4.8
5512..	21 21.7	+36 14	5.9	B3	0.011	42	+ 1.7	....	+16.6
5522..	21 25.4	+23 12	4.8	Ma	0.017	46	−19.4	−17.5L	− 5.5
5542..	21 30.1	− 4 26	5.8	G5	0.021	61	− 0.3	....	+ 9.4
5546..	21 30.7	+38 5	5.0	G8	0.148	44	−63.9	−65.5L	−51.1
5550..	21 32.4	− 0 50	6.2	A0	0.026	60	+16.9	....	+26.9
5555..	21 33.1	+18 52	5.3	A1	0.100	49	−38.6	....	−25.5
5558..	21 34.4	+19 49	5.8	A3	0.117	49	−13.4	....	− 0.3
5583..	21 39.1	+40 42	5.6	K3	0.026	45	−25.9	....	−11.8
5589..	21 39.7	− 9 33	5.3	G4	0.010	67	− 6.5	....	+ 1.3
5590..	21 39.8	+16 53	4.3	G7	0.029	52	−25.5	−22.1L	−13.2
5599..	21 41.5	+22 29	5.4	G0	0.016	50	−10.7	....	+ 2.2
5614..	21 44.5	+60 14	5.6	K4	0.006	47	−19.4	....	− 5.8
5650..	21 53.8	+63 9	5.4	Map	0.012	49	−19	....	− 6
5655..	21 56.0	+ 0 7	5.8	K1	0.006	64	+ 7.2	....	+16.0
5657..	21 56.2	+ 7 47	5.8	K2	0.022	60	−22.8	....	−12.8
B.D. 61°2233..	21 57.4	+61 59	6.5	B6	0.013	49	−21.0	....	− 7.9
Boss 5664..	21 58.2	+52 24	5.7	B9	0.004	48	−22.1	....	− 8.7
5669..	21 58.9	+44 10	5.3	A1	0.039	48	+ 0.9	....	+14.3
Groom. 3689..	22 3.1	+52 39	8.1	G8	0.61	49	−35.6	....	−22.5
Boss 5749..	22 11.9	− 5 53	5.8	G5	0.020	71	+ 6.9	....	+13.4
5757..	22 15.4	+ 5 17	5.2	B7	0.019	65	−15.5	....	− 7.1
5769..	22 18.3	− 7 42	6.1	G4	0.006	73	−13.7	....	− 7.9
5805..	22 25.4	+57 54	(7.5)	B8	0.028	52	−25.7	....	−13.4
5858..	22 37.1	+28 47	4.8	A3	0.039	59	+ 7.1	+ 8.7L	+17.4
5868..	22 39.6	+38 56	6.1	K4	0.016	57	−26.6	....	−15.7
5904..	22 49.3	−16 21	3.5	A2	0.052	84	+13.4	+22L	+15.5
5920..	22 53.5	+ 8 50	6.5	F5	0.420	71	−26.9	....	−20.4
5923..	22 54.3	−13 36	6.3	K2	0.014	84	+13.2	....	+15.3
Lal. 45028..	22 56.6	− 4 23	7.8	K2	0.50	79	−50.7	....	−46.9
Boss 5940..	22 58.9	+27 32	2.6	Map	0.234	64	+10.6	+ 8.4L	+19.4
5962..	23 4.5	+ 8 8	5.4	Mbp	0.004	74	+13.4	....	+18.9
5967..	23 5.0	+ 9 17	5.4	B9	0.024	74	+11.8	....	+17.3
5969..	23 5.5	+58 47	5.6	A3	0.015	57	−11.8	....	− 0.9
5972..	23 5.8	+43 0	5.8	F2	0.275	60	−43.9	....	−33.9
5973..	23 6.7	+ 8 11	5.1	B9p	0.017	75	+ 5.6	....	+10.8
5981..	23 10.7	− 9 38	4.5	G8	0.367	85	−28.4	−26.9L	−26.7
5982..	23 10.9	+27 42	6.5	K0	0.015	67	+ 3.5	....	+11.3
6016..	23 18.1	+59 35	5.9	G9	0.003	59	−11.2	....	− 0.9

TABLE II—*Continued*

Star	α 1900	δ 1900	Mag.	Spec.	μ	λ	v	v	v'
A.Oe. 25685..	$23^h26^m.5$	+58°37′	7.5	K1	1″.08	60°	−24.7	....	−14.7
B.D. 62°2244..	23 28.2	+62 36	7.3	G4	0.44	60	+ 8.0	....	+18.0
Boss 6063..	23 29.7	+39 41	5.4	A0p	0.042	66	+13.2	....	+21.3
6089..	23 38.3	+ 9 47	5.4	Map	0.007	81	−35.2	....	−32.1
6105..	23 42.1	+56 54	5.8	K0	0.021	62	− 5.4	....	+ 4.0
6106..	23 42.2	+58 6	5.1	G9	0.087	62	−22.9	−20.3L	−13.4
6111..	23 44.0	+61 40	5.6	A3p	0.010	61	−55.7	....	−46.0
6113..	23 44.3	+ 0 31	5.8	A3	0.027	86	+ 9.4	....	+10.7
6123..	23 46.8	+ 2 22	5.8	K1	0.020	86	+ 0.9	....	+ 2.3
6133..	23 48.0	+ 1 32	6.2	A1	0.013	87	+ 9.7	....	+10.7
6135..	23 49.4	+56 57	5.0	F9p	0.007	63	−42.6	−42.1L	−33.5
6145..	23 52.0	+42 6	6.0	F2p	0.012	69	− 7.7	....	− 0.5
6166..	23 56.5	+60 40	5.7	A6	0.007	63	−22.4	....	−13.3
6176..	23 57.5	+65 33	5.8	K0	0.018	62	−16.9	....	− 7.5
6180..	23 59.1	+61 44	6.0	A0	0.008	63	−18.9	....	− 9.8

ACCURACY OF THE OBSERVATIONS

The great variety of spectral types among the stars of Table II involves a wide range in the accuracy of the determinations of radial velocity. Many of the A- and B-type stars have vague and very ill-defined lines, and for such stars the accuracy necessarily is low. In some cases as many as seven or eight determinations have been made to guard against the inclusion of possible spectroscopic binaries, and the range among the individual plates occasionally amounts to more than 10 km. On the other hand, the results for spectra having well-defined lines are usually in excellent agreement. The accompanying short table (III) shows the average of the probable errors of v for ten stars of each type selected at random from Table II.

TABLE III

Type	Quality for Measurement	Average No. Plates	Linear Scale of Plates	Probable Error
			per mm	
A and B.........	Poor	5	16 A	±1.16 km
A and B.........	Good	3	16	0.73
F...............	Good	3	36	0.98
G and K.........	Good	3	36	0.97
M...............	Good	3	36	±1.09

For the sake of uniformity it has seemed preferable to retain the fractional part of the kilometer for v wherever three observations are available, although it can have little significance in the case of individual stars photographed with such relatively low dispersion. The fact that the linear scale of the spectra of the A and B stars is over twice that of the F, G, K, and M stars aids in counteracting the effect of the poorer quality of their lines, and so tends to make the accuracy of the determinations for all of the stars in Table II more nearly the same.

COMPARISON WITH RESULTS OF OTHER OBSERVERS

There are fifty stars in the list for which determinations of radial velocity have been published by other observers, a very large proportion being from the Lick Observatory photographs. The Lick spectrograms were in most cases taken with a dispersion of three prisms, and have a linear scale about three times that employed for most of the F, G, K, and M stars of Table II. A comparison by spectral types with the Lick Observatory results gives the values shown in Table IV.

TABLE IV

Type	No. Stars	Lick−Mount Wilson
B and A.........	21	+0.9 km
F and G.........	14	+1.6
K and M........	12	+0.4

The star W.B. 4^h1189 has been omitted from this comparison, as it seems probable that the large difference between the two results may be due to the fact that the star has a variable velocity. The same remark may apply to one or two other stars in the list, particularly Boss 5904 and 5044. The exclusion of these stars would reduce the difference for the B and A stars from +0.9 to +0.4 km, and for the F and G stars from +1.6 to +1.1 km.

A large number of observations on the two stars *α* Boötis and *α* Tauri have been made during the period covered by the results shown in Table II. The values for these stars are given in Table V.

The evidence seems to indicate a small systematic difference in the direction of larger negative or smaller positive values for the Mount Wilson results, but it is probably no larger than may be accounted for by the wave-lengths of the lines employed. A slight difference might arise from the fact that the iron arc has been used for comparison purposes at Mount Wilson, and that Rowland's wave-lengths have been utilized both for comparison lines and for such stellar lines as appear in the sun. The adopted values of the laboratory wave-lengths used for the helium lines of type B and the magnesium line λ 4481 of type A may also differ to some extent. In view of the fact that the Mount Wilson results are based mainly on comparatively low-dispersion photographs, the agreement with the Lick Observatory values must be considered as quite satisfactory.

TABLE V

Star	No. Plates	Mount Wilson	Lick	Yerkes
α Boötis.........	31	− 4.3 km	− 3.9 km	−4.5 km
α Tauri..........	16	+54.0	+55.1	....

SOME INDIVIDUAL STARS

Among the stars with exceptionally high velocities the following are of especial interest:

	v'		v'
A.Oe. 14320........	+299 km	Lal. 21258........	+69 km
A.Oe. 20452........	−170	Boss 2647........	+87.0
W.B. 17^h514.......	−131		

The first of these stars has a proper motion of 3″76 and a parallax, as determined by Russell, of +0″035. Its motion in space as based on these values and its radial velocity would amount to 577 km, directed toward the vertex $\alpha = 189^\circ$, $\delta = -70^\circ$. At a distance of 5′ there is a second star which shares in the proper motion. The spectrum of this star is G0.

The star Lalande 21258 has a proper motion of 4″46 and a parallax of 0″20. Its absolute brightness is extremely small, its magnitude being 10.4 (sun=5.5). In proper motion, absolute magnitude, and spectrum it resembles very strongly Lalande 21185, but the radial velocities of the two stars, though both large, are of opposite sign.

Boss 2647 is one of the very few stars of type A with a high radial velocity.

A star of exceptional interest because of the character of its spectrum is Lalande 19229. The spectral type is A2, but the line λ 4481, usually so prominent in stars of this type, is either absent or very faint. Two stars with a very similar spectrum had been found previously in the list of those having large proper motions. The data for the three stars are given in Table VI.

TABLE VI

Star	Mag.	μ	π	Spectrum
Lal. 5761.......	8.0	0″90	+0″039	A3p
Lal. 19229.......	8.4	0.83	−0.046	A2p
Lal. 28607.......	7.3	1.17	+0.029	A2p

The hydrogen lines in these stars are exceptionally narrow and well defined. Although the measured parallaxes are small, it seems probable that these stars are of comparatively low luminosity, and the suggestion may be made that the normal A-type spectrum is modified in this way in the case of stars of small absolute brightness. If such is the case, these spectral peculiarities should serve as a valuable criterion for the discovery of stars of this character. On physical grounds the absence of the spark line of magnesium at λ 4481, which is associated in the laboratory with high vapor-density and probably high temperature, and the narrowness and sharpness of the hydrogen lines, which would indicate a hydrogen atmosphere of low density, would be in harmony with this hypothesis.

Attention was called in the publication already referred to on the radial velocities of 100 stars with measured parallaxes[1] to the marked preponderance of the negative sign among the highest velocities. There seems to be no such noticeable effect in the case of the velocities given in Table II. The number of positive and negative velocities is essentially equal if $v' = 50$ km is set as a limit. Between 45 and 50 km, however, there are six negative velocities and only one with the positive sign.

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[1] *Mt. Wilson Contr.*, No. 79; *Astrophysical Journal*, **39**, 341, 1914.

RADIAL VELOCITY AND PROPER MOTION

It is well known that, in general, the proper motions of the stars of type B are extremely small, those of type A considerably larger, and those of types F, G, and K larger still. The M-type stars have proper motions averaging about the same as the A stars. An observing list of stars of different types selected on the basis of apparent magnitude alone would, therefore, contain material which would not be homogeneous as regards the distances of the stars. Since large proper motions when treated statistically indicate not only small distance, but also high velocity, as is shown clearly by the values for stars of large proper motion,[1] the tendency would be in such an observing list to compare rapidly moving stars of one type with slowly moving stars of another type.

Most of the F, G, K, and M stars and some of the A stars which appear in Table II have been selected for observation because of their small proper motions. A knowledge of their radial velocities enables us to institute a comparison between the average velocities of groups of stars having these spectra with those of types B and A of the same average proper motion. In Table IV are collected the radial velocities of all of the stars in Table II, for which the proper motion is less than 0".030 annually. One K-type star and one M star with velocities exceeding 50 km have been omitted. This makes it possible to compare directly with a similar table published by Professor Campbell based on his velocities of stars of all types.[2] For the present purpose Campbell's first table based on 1034 stars is used, no constant correction *K* having been applied to these results. The proper motions for Campbell's stars have been taken from Boss's catalogue for the individual stars published in *Lick Observatory Bulletins*, Nos. 195, 211, and 229. Not all of these stars are used in Campbell's table, and, accordingly, the average proper motions derived are not strictly correct. In view of the large number of stars used, however, it does not seem probable that the values can be materially in error.

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[1] The average value of the radial velocity (corrected for the sun's motion) of 135 stars of large proper motion, $\mu = 0''.82$, as determined at Mount Wilson is 24.3 km. Stars with velocities exceeding 100 km are omitted.

[2] *Lick Observatory Bulletin*, No. 196.

The peculiar feature of this comparison is the relatively close agreement of the A and B stars and the large difference for the other stars. The question at once arises whether this may not be associated with the great increase in proper motion for Campbell's stars between type A and type F. In a recent publication by Kapteyn and Adams,[1] Professor Kapteyn has made a computation of the relationship between radial velocity and proper motion for the K stars, using as a basis Campbell's published values of radial velocity and some of the Mount Wilson observations. The stars were selected in such a way as to eliminate so far as possible the effect of stream motion, and the components of the linear velocities were computed by aid of the mean parallaxes for stars of known proper motion and magnitude given in *Groningen Publication*, No. 8. If we assume that the results of this computation for the K stars may be applied to stars of other types, we have Table VIII connecting proper motion and radial velocity.

TABLE VII

SPECTRAL TYPES	CAMPBELL				MOUNT WILSON		
	No. Stars for v'	No. Stars for P.M.	Proper Motion	v' km	No. Stars	Proper Motion	v' km
O and B......	141	224	0″.031	8.99	61	0″.016	8.23
A............	133	206	0.094*	9.94	55	0.019	10.04
F............	159	192	0.234	13.90	20	0.011	10.14
G and K.....	529	549	0.202	15.15	119	0.014	11.03†
M...........	72	78	0.074	16.55	27	0.015	12.56

* The omission of 5 stars would reduce this value to 0″.079.

† The separate values of the G and K stars are G: 63, 0″.013, 10.60; K: 56, 0″.014, 11.53.

TABLE VIII

	v'	μ	v'
0″.000 to 0″.025......	12.1 km	0″.100 to 0″.119......	14.3 km
0.026 “ 0.039......	12.5	0.120 “ 0.149......	14.8
0.040 “ 0.059......	12.9	0.150 “ 0.199......	15.9
0.060 “ 0.079......	13.3	0.200 “ 0.299......	17.7
0.080 “ 0.099......	13.7	≧ 0.300	24.5

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[1] *Communications to the National Academy of Sciences*, No. 1; *Proceedings of the National Academy of Sciences*, 1, 14, 1915.

The use of these values gives the following corrections to the radial velocities for the proper motions of the Campbell stars in Table VII in order to reduce to the average proper motion 0″.031 of the O and B stars:

A, −1.3; *F*, −4.9; *G* and *K*, −4.1; *M*, −1.0 km.

Table IX shows the values with these corrections applied, and also with the reductions applied to correct for stream motion which have been calculated by Eddington.[1]

TABLE IX

Type	v' km	Campbell v' Corrected for Stream Motion	Proper Motion	v' km	Mount Wilson v' Corrected for Stream Motion	Proper Motion
O and B...	9.0	9.0 km	0″.031	8.2	8.2 km	0″.016
A........	8.6	6.8	"	10.0	7.7	0.019
F........	9.0	7.8	"	10.1	8.8	0.011
G........	11.0	9.6	"	10.6	9.2	0.013
K........				11.5	10.0	0.014
M........	15.6	13.6	"	12.6	10.9	0.015

In his definitive solutions of the solar motion for the several spectral types[2] Campbell has given the average radial velocity for each type with a constant correction K applied to the velocity of each star. This constant has a value ranging from about zero for the F and G stars to over 4 km for the B stars. If we treat these values in the same way as those of Table VII we obtain Table X.

TABLE X

Type	v'	v' Corrected for Stream Motion	Proper Motion
O and B.....	6.5 km	6.5 km	0″.031
A...........	9.6	7.4	"
F...........	9.5	8.3	"
G..........	9.1	7.9	"
K..........	13.2	11.5	"
M.........	16.1	14.0	"

The value of the constant *K* as used by Campbell is the average velocity v' taken according to sign for the stars of the several spectral types, and is, of course, dependent upon the value of the solar motion V as derived for each type. Since the same value of V has been used for all of the Mount Wilson stars, no direct comparison is possible. It is, however, of interest to note how the value $V=$ 20 km satisfies the stars of the several types. The average velocity v' taken according to sign for the stars of Table VII is as follows:

---

[1] *Stellar Movements*, p. 157.

[2] *Lick Observatory Bulletin*, No. 196.

B, +1.26; *A*, −0.24; *F*, −0.86; *G*, +0.05; *K*, −1.18; *M*, +0.31 km.

A change in the value of V from 20 to 19 km would reduce the residual for the B stars from +1.26 to +1.06 km. These quantities must be regarded as very moderate in size. The number of stars used is not very large, however, and hence the values might be changed materially by the inclusion of additional velocities. Thus if all of the M stars both of large and of small proper motion in Table II are included, together with one or two stars for which only a single observation is available, we obtain the following result:

No. Stars	μ	v'	v' According to Sign
43	0.″058	14.54 km	−0.97 km

A similar computation for the B stars gives:

No. Stars	μ	v'	v' According to Sign
113	0.″028	8.89 km	+1.62 km

The value +1.62 km would be reduced about 10 per cent by employing a value of the solar motion $V=19$ km.

The Mount Wilson results of Table IX seem to indicate, if interpreted directly, that among the very distant stars the change of velocity with spectral type is slight, and Campbell's results, except perhaps in the case of the M stars, point to the same conclusion when allowance has been made for the effect of the large number of relatively near stars included among his F- to M-type spectra. This would be in agreement with the hypothesis put forward by Eddington in 1911,[1] but later entirely disproved, as he considered, by the evidence of the A stars,[1] that the relation between velocity and spectral type might be a relation between velocity and distance, the stars nearest the sun, which are mainly of types F to K, moving more rapidly than the distant stars. The evidence which Eddington regarded as conclusive in disproving this hypothesis was provided by an analysis according to proper motion of the A-type stars for which velocities had been published. No increase of radial velocity with proper motion was indicated by the results. It has already been stated in this communication that such a conclusion is by no means tenable in the case of the K stars, for which Kapteyn has found from the Lick and Mount Wilson values an increase of velocity of from 10.9 km for stars having an average proper motion of about 0″.020 to 26.7 km for stars with a proper motion exceeding 0″.30. The following evidence derived entirely from the Mount Wilson observations for the other types of spectra will be of interest in this connection. The effect of stream motion has not been eliminated.[2]

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[1] *British Association Report*, 1911.

TABLE XI

	No. Stars	μ	v'	No. Stars	μ	v'
B..........	61	0″.016	8.2 km	52	0″.041	9.6 km
A..........	55	0.019	10.0	104	0.067	10.7
F..........	20	0.011	10.1	45	0.53	24.6
G..........	63	0.013	10.6	69	0.67	24.9
M..........	27	0.015	12.6	12	0.17	17.6

The agreement of these results with those obtained from the K-type stars is surprisingly close, and suggests that the empirical law connecting proper motion and radial velocity derived by Kapteyn may be applied to the other types of spectra quite as well. Only a few A-type stars of very large proper motion have been observed at Mount Wilson. Of those for which μ exceeds 0″.20, two have velocities exceeding 150 km; one has a velocity of 87 km; and the average for the other six is 20 km.

The main feature of interest resulting from this comparison of proper motion and radial velocity is the low average velocity found for the very distant stars of types F to M. The selection of stars on the basis of small proper motion means, of course, the selection not alone of distant stars but also of those which have small intrinsic velocities as well as those whose motion is mainly in the line of sight. These factors will affect the results to some extent, especially when comparatively small numbers of stars are used. On the other hand, the direct comparison of the average velocities of groups of stars of greatly different average proper motions means a comparison in part between stars of widely different distance, and in part between slowly moving stars of one type and rapidly moving stars of another. If the rate of change of velocity with spectral type is as gradual as seems probable from these results, a very accurate knowledge of the stream motions for the different types of stars will be essential for a determination of its true value.

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[1] *Stellar Movements*, p. 161.

[2] Velocities exceeding 100 km have been omitted.

No attempt is made here to discuss the well-known investigation by Kapteyn,[1] in the course of which he first analyzed the relationship of radial velocities and proper motions to spectral types; nor the work of Boss,[2] in which he deduced the linear cross-motions of the stars of his catalogue according to spectral type. There can, of course, be no doubt that among the stars selected on the basis of apparent brightness those of the solar type are moving more rapidly than those of types A and B. The question which is raised is whether there exists any such marked difference for the stars of the solar type with distances comparable to those of types A and B.

The small proper-motion stars of types F to M whose motions are considered here are on the average stars of very high absolute luminosity. The possible existence of a relationship between absolute brightness and velocity has been discussed in the communication by Kapteyn and Adams, to which reference has already been made. The observational material essential to an investigation of this question would necessarily be much more extensive than that given here, and should be selected with this purpose in view. It may, however, be noted in passing that the average radial velocity of the stars of very low absolute luminosity is extraordinarily great.

[1] *Mt. Wilson Contr.*, No. 45; *Astrophysical Journal*, **31**, 258, 1910.

[2] *Astronomical Journal*, **26**, 187, Nos. 623–624, 1911.

Of the stars in the Groningen list of parallaxes with absolute magnitudes of 8 or fainter (sun = 5.5) sixteen have been observed to some extent at Mount Wilson. The average velocity of these stars (corrected for the sun's motion) is 36 km; eight have velocities exceeding 40 km, although none have been included with values higher than 100 km. It is difficult to think of these stars as other than stars of small mass, and the results for their velocities would be in agreement with the hypothesis suggested by Halm[1] that the motions of stars are a function of their masses.

I am greatly indebted to several of my colleagues at the Observatory, and particularly to Dr. Kohlschütter, for much of the observational material upon which these results are based. Several of the members of the Computing Division have assisted in measuring and reducing the photographs.

MOUNT WILSON SOLAR OBSERVATORY
June 1915

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[1] *Monthly Notices*, **71**, 634, 1911.

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